

## Supplier and Distributor Selection in Supply Chain and Optimizing The Selective Criterias Which Affect Production-Distribution Planning Due to This Selection by Using Fuzzy Optimization

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### Abstract

This paper addresses the problem of selecting appropriate suppliers for raw material procurement and suitable distributors for delivering final products to different market regions by considering both qualitative and quantitative criteria. To achieve this, the Analytic Hierarchy Process (AHP), a well-established multi-criteria decision-making (MCDM) technique, is employed. The selection process is conducted in two stages using AHP to rank suppliers and distributors. Subsequently, a fuzzy multi-objective linear programming model is developed to optimize the selected criteria, which directly influence production and distribution planning within the supply chain. The novelty of this study lies in the integrated application of AHP and fuzzy optimization to support strategic decision-making under uncertainty. The applicability and effectiveness of the proposed methodology are demonstrated through a numerical example. Finally, conclusions are drawn, and potential directions for future research are discussed.

**Keywords:** Analytic hierarchy process (A.H.P), multi-criteria decision making (M.C.D.M) approach, fuzzy multi-objective linear programming model, supply chain management

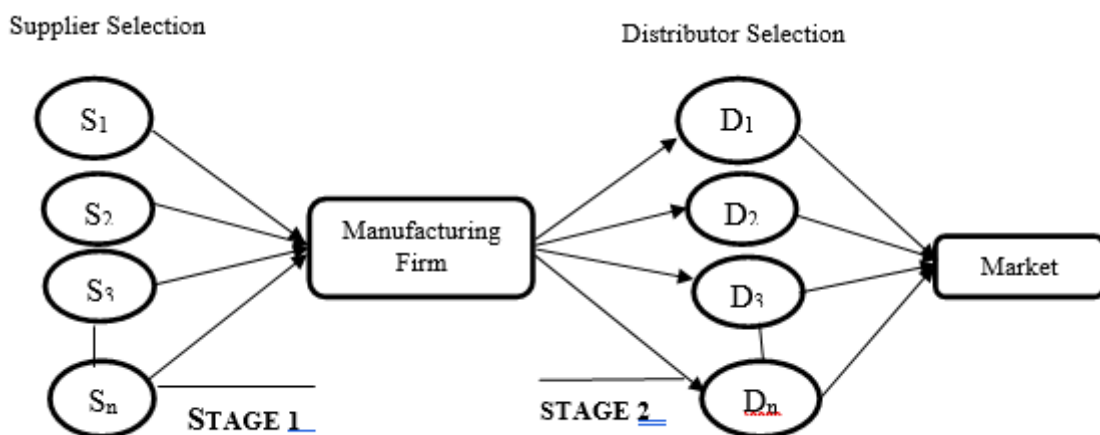
### Introduction

The proper selection of suppliers and distributors by the manufacturing firm always plays a most significant role in reducing the obstacles arising during the process of manufacturing and successively carried out the task of fulfilling the requirement of market over a certain time period which is very necessary for establishing the trustable environment in market, therefore we have to carry out the effective selection of appropriate suppliers and distributors by utilizing the most famous multi-criteria decision-making (M.C.D.M) approach known as Analytic Hierarchy Process (A.H.P). On the other hand, there are some specific criterias through which the selection of suppliers and distributors is carried out and which directly influence the planning of production-distribution in supply chain management. For example, the certain criterias which are considered during selection of appropriate suppliers for acquiring the raw material like per unit price of raw material offered by a supplier, quality of raw material, how much raw material quantity is delivered on time and so on directly influence the production cost, production quality and required time taken for production. Similarly, the certain criterias arising during the appropriate selection of distributors who purchase the bulk amount of product from manufacturer and successively deliver to different areas of a market are required time taken for transportation, amount of product damaged during transportation, how much delivery is carried out on time and so on will always affect the planning of distribution. The proposed hybrid AHP-Fuzzy model is particularly well suited for complex multi-criteria decision-making (MCDM) problems characterized by uncertainty, subjectivity, and qualitative judgments. Compared with conventional methods such as TOPSIS, VIKOR, and Genetic Algorithm (GA)-based models, the hybrid approach offers several conceptual and practical advantages.

In a similar direction we have also highlight some of the existing literature related to supplier and distributor selection in supply chain management, used a fuzzy analytic hierarchy process (FAHP) to identify the most suitable yarn supplier for a Bangladeshi apparel retail manufacturer [1]. Proposed a fuzzy MCDM framework integrating Fuzzy AHP with WASPAS and TOPSIS to evaluate green suppliers based on

environmental and conventional criteria [2]. Addressed green supplier selection for a steel door manufacturing company by identifying relevant environmental and operational criteria and evaluating alternative steel sheet suppliers through managerial assessment [3]. Proposes an integrated fuzzy multi-criteria decision-making framework combining Fuzzy Delphi, Fuzzy AHP, Fuzzy TOPSIS, and DEA to effectively handle uncertainty and select optimal M-Sand suppliers in the construction industry [5]. Compared between crisp and fuzzy ELECTRE techniques for evaluation of suppliers [6]. Proposed an interval type-2 fuzzy AHP (IT2FAHP) model to address ambiguity in green supplier selection and demonstrated its effectiveness through a real case in the home appliance industry. [4]. Utilized a fuzzy TOPSIS approach for evaluated the supplier's environmental performance [7]. Utilized an approach of rough set theory for evaluation of appropriate distributor [8]. Developed the criteria for evaluation of suppliers by utilizing an approach of fuzzy DEMATEL [9]. Addressed an issue of carbon emission during supplier evaluation by using integrated approach of fuzzy A.H.P and a fuzzy multi-objective linear programming [10]. Applied a fuzzy Adaptive Resonance Theory (ART) through which distributors are grouped according their similarity [11]. Evaluated sustainable suppliers by formulate a fuzzy inference system based ranking model [12]. Presented an approach of fuzzy inference system for evaluation of suppliers [13]. Utilized a fuzzy TOPSIS technique for evaluation of distributor in a Chinese agricultural enterprise [14]. Utilized a technique of fuzzy adaptive resonance theory (ART) and AHP-F TOPSIS for evaluation of distributor based on service quality [15]. Handled the issue of evaluation and selection of appropriate distributor by utilizing an intuitionistic fuzzy -TODIM approach [16]. Evaluated suppliers for textile company in Turkish by utilized a TOPSIS approach and intuitionistic fuzzy decision making [17]. Evaluated a suitable distributor by utilized a Grey-based decision-making approach [18].

#### Methodology for effective supplier and distributor selection



**Figure 1.** A Two Stage Supplier and Distributor Selection Model

In fig1. We have presented the methodology which is categorized into two stages. In first stage manufacturer wants to choose the appropriate suppliers from a given list of “n” suppliers for purchasing the required raw materials and in second stage after transforming the raw materials to final product, the manufacturer needs to select the appropriate distributors from given list of “n” distributors for distributing their final manufactured product to different areas of market effectively and within a certain time period.

**Table 1.** Qualitative and quantitative criterias and their sub-criterias for selecting the appropriate suppliers along with their explanations

Criteria	Sub- Criteria	Explanation
Product Reliability ( $s_1$ )	(a) Product quality (b) Fault and scrap % (c) Rejection % (d) Acceptance %	Quality assurance is specified by a certificate. Defective items percentage is expressed by a report. Report explains the percentage of rejected items. Raw material's percentage which is accepted by a manufacturer
Raw Material Supply ( $s_2$ )	(a) On time delivery % (b) Late delivery % (c) Delivery time period (d) Supplier capacity	Raw material percentage delivered by supplier on-time. Raw material's percentage which is late delivered. Required time for raw material delivery. Maximum Supplier's capacity for provided the raw goods.
Price ( $s_3$ )	(a) price of purchasing (b) price of the item	Calculated by purchasing department. Latest price provided by suppliers.
Trade Relationship ( $s_4$ )	(a) Accountability (b) Trust worth (c) Reaction on enquiry (d) Reaction on change	Supplier who accepts full responsibility for his error. A trustworthy supplier is one that can promptly fulfill your requirements on time. Behavior adopted by supplier during enquiry by a manufacturer. Supplier's response time on change of defected item.

**Table 2.** Qualitative and quantitative criterias and their sub-criterias for selecting the appropriate distributors along with their explanation

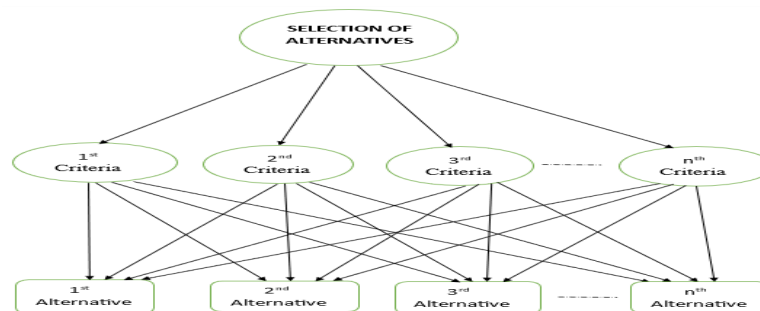
Criteria	Sub- Criteria	Explanation
Marketing Skills ( $d_1$ )	(a) Relationship maintenance (b) Create trust (c) Good representator	Always maintain good relationship. Establishes a trusting environment in the market. Quality to represent any product in a very effective way.
Economic Capability ( $d_2$ )	(a) Delay in payment (b) Price control (c) Financial capability	Time taken for paying a bill to the manufacturer. Criteria which reduce business expenses. Capability to generate profit from market.
Distribution Ability ( $d_3$ )	(a) Transport infrastructure (b) Coverage zone	Transportation facilities for effective distribution. Region covered by distributors for distribution.

Effective Transportation ( $d_4$ )	(a) On-Time Delivery	On-time delivery fulfils the urgent requirement.
	(b) Minimum Late Delivery %	Minimizes the late delivery of the product will satisfy the customers.
	(c) Minimum Transportation Time	Always take minimum time for transporting the product.
	(d) Maximum Transportation Capacity	Maximum amount of product transported in one shift.
	(e) Minimum Damaged Quantity%	Minimum product's quantity damaged during transportation.

Now on the basis of following criterias highlighted in 1<sup>st</sup> and 2<sup>nd</sup> table we have to carry out the appropriate selection of suppliers and distributors by utilizing a most famous and well-known multi-criteria decision-making technique known as Analytic Hierarchy Process (A.H.P) which is described as follows along with their hierarchy structure.

### Materials and Methods

**Analytic hierarchy process (a.h.p):** Thomas L. Saaty [20] was the first person who had designed and created the well-known multi-criteria decision making technique which is known as Analytic Hierarchy Process (AHP) and makes the individuals or team of different decision makers who belong to different areas such as engineering, factories, business, industries etc are capable for evaluating the best alternatives from a given set of alternatives and this evaluation is carried out on the basis of some selective qualitative and quantitative criterias which are initially decided by the individuals or team of decision makers of all such fields. Now the given figure 2. Is a hierarchy structure which demonstrates the general mechanism involved in this decision-making process.



**Figure 2.** Hierarchy Structure of Analytic Hierarchy Process

Now, selection of appropriate suppliers and distributors on the basis of following criterias described in 1<sup>st</sup> and 2<sup>nd</sup> table by utilizing following steps of Analytic hierarchy process (A.H.P) proposed by Thomas L. Saaty [20]:

#### Constructed a pair-wise comparison matrix

A square pair wise comparison matrix " $A=[a_{ij}]_{n \times n}$ " of order " $n \times n$ " is constructed where " $a_{ij}$ " is a comparative weight where " $a_{ij}=\frac{1}{a_{ji}}$ " and " $a_{ii}=1$ " for every  $i$ . and in a given pair-wise comparison matrix  $A$  the weights are originally obtained by utilizing the Saaty Scale proposed by Thomas L. Saaty [20] in 1980 which is given below in table (3).

**Table 3.** Saaty Scale proposed by Thomas L. Saaty

Linguistic Terms	Equally Desirable	Slightly Desirable	Desirable	Most Desirable	Extremely Desirable
Saaty's Scale	1	3	5	7	9

The required intermediate values between any two desirable linguistic terms are 2, 4, 6, 8.

### **Evaluating the geometric mean (R)**

Now obtain the weight of criterias and alternatives with the help of geometric mean (R) which is expressed as " $R = (\prod_{j=1}^n a_{ij})^{1/n}$ ", where " $a_{ij}$ " is a comparative value obtained from pair wise comparison matrix discussed in step (a) and 'n' is the dimension of the comparison matrix. Now by utilizing geometric mean (R), we have calculated the required weights " $w_i$ " by using the following expression " $w_i = \frac{R_i}{\sum_{i=1}^n R_i}$ ".

Remark: If the generated crisp weight satisfied the condition " $\sum w_i = 1$ ", then the weight " $w_i$ " is treated as a local weight.

### **Checking the consistency**

Now evaluated the consistency Index " $C.I = \frac{\lambda_{max} - n}{n-1}$ ", proposed by Thomas L Saaty (1980) and by utilizing the consistency index we have obtained the consistency ratio " $C.R = \frac{C.I}{R.I}$ ", of a pair-wise comparison matrix A discussed in step (a) to analysis the judgment of decision makers where " $\lambda_{max}$ " denoted the largest eigen value of a pair-wise comparison matrix and 'n' is the dimension of required matrix. If  $C.R \leq 0.1$ , it means the comparison done in a pair-wise comparison matrix is acceptable and if  $C.R > 0.1$  then the comparison is not acceptable.

Now the global weight is obtained

Finally, we have obtained the global weights by utilizing the local weights and selecting the appropriate alternatives by using these global weights.

Note: M.S EXCEL software is utilized for performing all these steps and obtaining the weight of the alternatives for taking the right decision.

### **Influence of effective supplier and distributor selection on production-distribution planning in supply chain management**

An appropriate selection of suppliers and distributors plays a vital role in production-distribution planning in supply chain management for fulfilling the requirement of customer in fair and reasonable price and an appropriate time. In real manufacturing environment the valuable supplier selection for purchasing the raw material will directly influence some factors of production planning such as production cost, production quality, production time etc. Similarly, on the other hand, a capable distributor selection is responsible for effective distribution planning and fulfilment of the requirement of customers within a certain time period.

### **Construction of a linear mathematical model**

In this segment, we have constructed a linear mathematical model for simultaneously optimizing all the criterias interpreted in 1st and 2nd category which arise when selection of supplier and distributor is carried out and influences the planning of production-distribution. Now for improving the efficiency and smoothness of planning of production-distribution we have optimized these specific criterias.

Now construct a linear mathematical model by using the following notations for optimizing the criterias which influence the planning of production-distribution:

### ***Index Sets***

$i$ : index for supplier who delivers the raw material, for all  $i=1, 2, \dots, N$

$j$ : index for distributor for distributing the product, for all  $j=1, 2, \dots, J$

### ***Decision Variables***

$q_i$ : amount of raw material delivered by supplier  $i$  for production of any product.

$b_j$ : quantity of an item delivered by a distributor  $j$  to a market place.

### ***Parameters***

$C_i$ : per unit cost of raw material provided by supplier  $i$ .

$L_i$ : raw material's percentage which is late delivered by supplier  $i$ .

$O_i$ : percentage of raw material which is on-time delivered by supplier  $i$ .

$A_i$ : percentage of raw material delivered by supplier  $i$  accepted by a manufacturer for production.

$R_i$ : approximate no. of days taken for delivering the raw material by supplier  $i$  after placing order by the manufacturer.

$D_i$ : defective raw material's percentage delivered by supplier  $i$ .

$F_i$ : the percentage of raw material delivered by supplier  $i$  is of good quality.

$G_j$ : percentage of product which is on-time delivered by a distributor  $j$ .

$E_j$ : percentage of product which is late delivered by a distributor  $j$ .

$T_j$ : approximate time required by a distributor  $j$  for transporting product in one shift.

$S_j$ : percentage of amount of product which is transported by distributor  $j$  in one shift.

$Q_j$ : percentage of quantity which is damaged during transportation by a distributor  $j$ .

$M_i$ : maximum requirement of raw material from supplier  $i$ .

$P$ : total amount of requirement of raw material for production is fulfilled by all the suppliers together.

$B$ : total financial budget of manufacturer for purchasing the raw material.

$Y_j$ : demand's percentage of any product in a specific area of market which is covered by distributor  $j$ .

$K$ : total demand of market for a product which is fulfilled by all the distributors together.

$N$ : No. of suppliers.

$J$ : No. of distributors.

The following objective functions and constraints are constructed for the formulation of the proposed linear mathematical model which is given below from equations (1)-(19):

$$\text{Minimise } Z_1 = \sum_{i=1}^N C_i q_i$$

**Equation 1**

$$\text{Minimise } Z_2 = \sum_{i=1}^N L_i q_i$$

**Equation 2**

$$\text{Maximise } Z_3 = \sum_{i=1}^N O_i q_i$$

**Equation 3**

$$\text{Maximise } Z_4 = \sum_{i=1}^N A_i q_i$$

**Equation 4**

$$\text{Minimise } Z_5 = \sum_{i=1}^N R_i q_i$$

**Equation 5**

$$\text{Minimise } Z_6 = \sum_{i=1}^N D_i q_i$$

**Equation 6**

$$\text{Maximise } Z_7 = \sum_{i=1}^N F_i q_i$$

**Equation 7**

$$\text{Maximise } Z_8 = \sum_{j=1}^J G_j b_j$$

**Equation 8**

$$\text{Minimise } Z_9 = \sum_{j=1}^J E_j b_j$$

**Equation 9**

$$\text{Minimise } Z_{10} = \sum_{j=1}^J T_j b_j$$

**Equation 10**

$$\text{Maximise } Z_{11} = \sum_{j=1}^J S_j b_j$$

**Equation 11**

$$\text{Minimise } Z_{12} = \sum_{j=1}^J Q_j b_j$$

**Equation 12**

Subject to,

$$q_i \leq M_i$$

**Equation 13**

$$\sum_{i=1}^N q_i = P$$

**Equation 14**

$$\sum_{i=1}^N C_i q_i \leq B$$

**Equation 15**

$$\sum_{j=1}^J b_j = K$$

**Equation 16**

$$b_j \leq Y_j$$

**Equation 17**

$$b_j \geq 0 \text{ for all } j=1, 2, \dots, J$$

**Equation 18**

and are integers

$$q_i \geq 0 \text{ for all } i=1, 2, \dots, N$$

**Equation 19**

and are integers

Objective function (1) minimizes total cost of raw material which directly influence the cost of production.

Objective function (2) minimizes the late delivery of raw material quantity which effect the planning of production.

Objective function (3) maximizes the amount of raw material which is on-time delivered.

Objective function (4) maximizes the raw material which is accepted by the manufacturer for production.

Objective percentage (5) minimizes the total time period taken by suppliers for raw material delivery after the order has been placed.

Objective function (6) minimizes total raw material which is going to be defective.

Objective function (7) maximizes the good quality raw material delivered by the suppliers.

Objective function (8) maximizes the on-time delivery of the product for fulfilling the urgent requirement of market.

Objective function (9) minimizes the late delivery of the product will always satisfy the customers.



Objective function (10) minimizes the transporting time period of the product in one shift which is important for timely fulfilment of the requirement of market.

Objective function (11) maximizes the amount of product transported in one shift will help in maintaining the expense of fuel consumption.

Objective function (12) minimizes the damaged quantity of the product during transportation.

Constraint (13) restricted the maximum requirement of raw material from a given supplier.

Constraint (14) presents the fulfilment of total requirement of raw material required for production by all the suppliers together.

Constraint (15) limits the total financial budget of the manufacturer for purchasing the raw material.

Constraint (16) shows that total demand of market is fulfilled by all the distributors together.

Constraint (17) ensures maximum demand percentage of a particular area of market which is covered by a distributor.

Constraint (18) ensures that all variables are integers and are greater than or equal to zero.

Constraint (19) ensures that all variables are integers and are greater than or equal to zero.

### ***Fuzzy linear programming model***

Uncertainties, different types of variations and impreciseness are the most critical issues when we carry out the optimization of some realistic problems because all the parameters and variables involved in any conventional model of linear programming are crisp and precise, so it is not easy to handle all such types of variations and impreciseness which naturally arise in this realistic world and which revolve around different types of uncertainties. So due to all these reasons the fuzzy theory which is proposed by Zadeh [21] is involved in conventional linear programming model and transformed it into fuzzy linear programming model which has the capability to cope with all the fluctuations and impreciseness.

General linear programming model in conventional form which is expressed from equation (20)-(22)

$$\text{Maximize/Minimize } Z=Cy$$

$$\text{Equation 20}$$

Subject to,

$$Ay \text{ “}\leq \text{ or } = \text{ or } \geq\text{” } b$$

$$\text{Equation 21}$$

$$y \geq 0$$

$$\text{Equation 22}$$

Now we have fuzzified the above traditional general linear programming model and obtained the fuzzified version which is expressed by equation (23)-(25).

$$\text{Maximize/Minimize } \tilde{Z}=Cy$$

$$\text{Equation 23}$$

Subject to,

$$Ay \text{ “}\lesseqgtr \text{ or } \cong \text{ or } \gtrsim\text{” } b$$

$$\text{Equation 24}$$

$$y \geq 0$$

$$\text{Equation 25}$$

Now after fuzzing the crisp inequalities “ $\leq$  and  $\geq$ ” which is involved in traditional model of optimization from equations (20)-(22), we have obtained the fuzzy inequalities “ $\lesssim$  and  $\gtrsim$ ” in fuzzified model of optimization from equations (23)-(25) and it is officially known as “fuzzy less than or equal to” and “fuzzy greater than or equal to”. Similarly on the other hand the fuzzy equality “ $\cong$ ” is also obtained after fuzzifying the crisp equality “ $=$ ” of traditional model of optimization from equations (20)-(22).

Now on the basis of the above information the fuzzified version of the proposed linear mathematical model from equation (1)-(19) is represented by the following equation (26)-(44).

$$\widetilde{Z}_1 = \sum_{i=1}^N C_i q_i$$

**Equation 26**

$$\widetilde{Z}_2 = \sum_{i=1}^N L_i q_i$$

**Equation 27**

$$\widetilde{Z}_3 = \sum_{i=1}^N O_i q_i$$

**Equation 28**

$$\widetilde{Z}_4 = \sum_{i=1}^N A_i q_i$$

**Equation 29**

$$\widetilde{Z}_5 = \sum_{i=1}^N R_i q_i$$

**Equation 30**

$$\widetilde{Z}_6 = \sum_{i=1}^N D_i q_i$$

**Equation 31**

$$\widetilde{Z}_7 = \sum_{i=1}^N F_i q_i$$

**Equation 32**

$$\widetilde{Z}_8 = \sum_{j=1}^J B_j b_j$$

**Equation 33**

$$\widetilde{Z}_9 = \sum_{j=1}^J E_j b_j$$

**Equation 34**

$$\widetilde{Z}_{10} = \sum_{j=1}^J T_j b_j$$

**Equation 35**

$$\widetilde{Z}_{11} = \sum_{j=1}^J S_j b_j$$

**Equation 36**

$$\widetilde{Z}_{12} = \sum_{j=1}^J Q_j b_j$$

**Equation 37**

System of constraints

$$q_i \leq M_i$$

**Equation 38**

$$\sum_{i=1}^N q_i = P$$

**Equation 39**

$$\sum_{i=1}^N C_i q_i \leq B$$

**Equation 40**

$$\sum_{j=1}^J b_j = K$$

**Equation 41**

$$b_j \leq A_j$$

**Equation 42**

$$b_j \geq 0 \text{ for all } j=1, 2, \dots, J \text{ and are integers}$$

**Equation 43**

$$q_i \geq 0 \text{ for all } i=1, 2, \dots, N \text{ and are integers}$$

**Equation 44**

### Membership functions

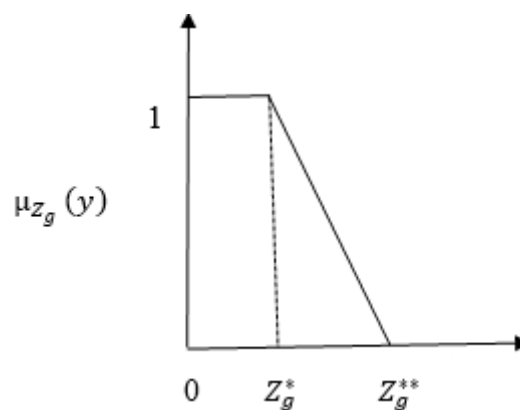
A linear membership function is utilized to demonstrate the fuzzy objective functions  $\widetilde{Z}$  suggested by Zimmermann in 1978 [14] in both minimization and maximization form which is expressed by equation (45) and (46)

$$\mu_{Z_g}(y) = \begin{cases} \frac{Z_g^{**} - Z_g(y)}{D_g(Z)} & \text{if } Z_g^* \leq Z_g(y) \leq Z_g^{**} \\ 1 & \text{if } Z_g(y) \leq Z_g^* \\ 0 & \text{if } Z_g(y) \geq Z_g^{**} \end{cases}$$

**Equation 45**

In above linear membership function (45) of minimum objective function, the difference between the upper bound ( $Z_g^{**}$ ) and lower bound ( $Z_g^*$ ) of all the objective functions  $Z_g(y) = \sum_{g=1}^G a_g y_g$  for all  $g=1, 2, \dots, G$  is expressed as  $D_g(Z)$ .

Now the **Fig. 3** will help to demonstrate the structure of above linear membership function (45) for minimum objective function  $Z_g(y) = \sum_{g=1}^G a_g y_g$  graphically.

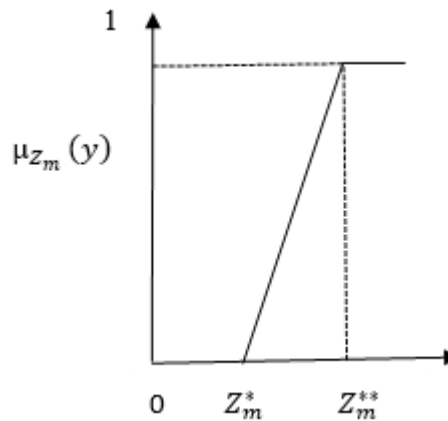
**Figure 3.** Linear membership function for minimum objective function

$$\mu_{Z_m}(y) = \begin{cases} \frac{Z_m(y) - Z_m^*}{D_m(Z)} & \text{if } Z_m^* \leq Z_m(y) \leq Z_m^{**} \\ 1 & \text{if } Z_m(y) \geq Z_m^{**} \\ 0 & \text{if } Z_m(y) \leq Z_m^* \end{cases}$$

**Equation 46**

In above linear membership function (46) of maximum objective function, the difference between the upper bound ( $Z_m^{**}$ ) and lower bound ( $Z_m^*$ ) of all the objective functions  $Z_m(y) = \sum_{m=1}^M b_m y_m$  for all  $m=1, 2, \dots, M$  is expressed by  $D_m(z)$ .

Now the Fig. 4 will help to demonstrate the structure of above linear membership function (46) for maximum objective function  $Z_m(y) = \sum_{m=1}^M b_m y_m$  graphically.



**Figure 4.** Linear membership function for maximum objective function

Now we have transformed the proposed fuzzy optimization model to crisp optimization model for obtaining the required optimal solutions:

Now in this segment for obtaining the required optimal solutions of proposed linear mathematical model, firstly we have obtained the fuzzified form of conventional minimum and maximum objective functions which is involved in proposed linear mathematical model from (1)-(19) by utilizing the linear membership functions provided by equation (45) and (46) and then after we have substituted all the linear membership functions of both fuzzy minimum and maximum objectives and their system of constraints which is deterministic in nature in Zimmermann approach [19] which is presented from equation (47),(48) and then converted it into a crisp linear mathematical model.

Maximize  $\beta$

Subject to,

$$\beta \leq \frac{Z_g^{**} - Z_g(y)}{D_g(Z)} \quad \text{for all } g=1, 2, \dots, G$$

**Equation 47**

$$\beta \leq \frac{Z_m(y) - Z_m^*}{D_m(Z)} \quad \text{for all } m=1, 2, \dots, M$$

**Equation 48**

$$Ay \text{ "}\leq \text{ or } = \text{ or } \geq \text{" } b$$

for all deterministic nature

$$y \geq 0 \quad \text{and are integers}$$

$$\beta \in [0,1]$$

Now according to

Zimmermann [14], we have calculated the maximization and minimization of same objective function "Z" as a linear programming problem under the same set of constraints. for obtaining the optimum upper and lower bound which is expressed by  $Z^{**}$  and  $Z^*$ .

## Results and Discussion

### Numerical example

Now firstly we have to consider a set of four suppliers ( $S_1, S_2, S_3, S_4$ ) from which we have to select the appropriate suppliers for purchasing the raw material by constructing the pair-wise comparison matrices

$(A_1, A_2, A_3, A_4, A_5)$  and establishing the comparison between the qualitative and quantitative criterias which is provided by table (1):

**Matrix ( $A_1$ )**

SUPPLIER SELECTION	Product Reliability ( $s_1$ )	Raw Material Supply ( $s_2$ )	Price ( $s_3$ )	Trade Relationship ( $s_4$ )
Product Reliability ( $s_1$ )	1	1/2	6	4
Raw Material Supply ( $s_2$ )	2	1	5	2
Price ( $s_3$ )	1/6	1/5	1	1/3
Trade Relationship ( $s_4$ )	1/4	1/2	3	1

Now firstly we have to find the geometric mean (R) of a given pair-wise comparison matrix

$$R_1 = (a_{11} \times a_{12} \times a_{13} \times a_{14})^{\frac{1}{4}} = (1 \times 1/2 \times 6 \times 4)^{\frac{1}{4}} = 1.86121$$

In a similar way we have obtained the remaining geometric mean (R) which is defined as follows:

$$R_2 = 2.114743, R_3 = 0.324668, R_4 = 0.782542$$

Now by utilizing the formula  $w_i = \frac{R_i}{\sum_{i=1}^n R_i}$  we have obtained the criteria's weight which is defined as follows:

$$w_1 = 0.366152, w_2 = 0.416029, w_3 = 0.063871, w_4 = 0.153948$$

Here  $\sum_{i=1}^4 w_i = 1$ , then the following weight is considered it as a local weight.

Now finally we have checked the consistency ratio (C.R) of a given pair-wise comparison matrix for checking whether the judgement is correct or not.

Here (C.R) = 0.076454 < 0.1, it means the comparison done in a pair-wise comparison matrix is acceptable.

Now the same procedure is repeated for pair-wise comparison matrix of the suppliers ( $S_i$ ), for all  $i=1,2,3,4$  with respect to first criteria ( $s_1$ ) which is "Product Reliability"

**Matrix ( $A_2$ )**

Product Reliability ( $s_1$ )	Supplier ( $S_1$ )	Supplier ( $S_2$ )	Supplier ( $S_3$ )	Supplier ( $S_4$ )
Supplier ( $S_1$ )	1	4	7	6
Supplier ( $S_2$ )	1/4	1	5	2
Supplier ( $S_3$ )	1/7	1/5	1	1/3
Supplier ( $S_4$ )	1/6	1/2	3	1

Now the required weight of suppliers with respect to criteria ( $s_1$ ) is defined as follows:

$$w_1 = 0.612578, w_2 = 0.213953, w_3 = 0.053154, w_4 = 0.120315$$

Here (C.R) = 0.050052 < 0.1, it means the comparison done in a pair-wise comparison matrix is acceptable.

In a similar way we have constructed the remaining pair-wise comparison matrix with respect to second, third and fourth criterias which is ( $s_2, s_3, s_4$ )

**Matrix ( $A_3$ )**

Raw Material Supply ( $s_2$ )	Supplier	Supplier	Supplier	Supplier
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	( $S_1$ )	( $S_2$ )	( $S_3$ )	( $S_4$ )
Supplier ( $S_1$ )	1	3	6	4
Supplier ( $S_2$ )	1/3	1	7	3
Supplier ( $S_3$ )	1/6	1/7	1	1/2
Supplier ( $S_4$ )	1/4	1/3	2	1

Now the required weight of suppliers with respect to criteria ( $s_2$ ) is defined as follows:

$$w_1 = 0.528783, w_2 = 0.295269, w_3 = 0.059962, w_4 = 0.115986$$

Here (C.R) = 0.055482 < 0.1, it means the comparison done in a pair-wise comparison matrix is acceptable.

**Matrix ( $A_4$ )**

Price ( $s_3$ )	Supplier ( $S_1$ )	Supplier ( $S_2$ )	Supplier ( $S_3$ )	Supplier ( $S_4$ )
Supplier ( $S_1$ )	1	2	3	4
Supplier ( $S_2$ )	1/2	1	3	2
Supplier ( $S_3$ )	1/3	1/3	1	1/2
Supplier ( $S_4$ )	1/4	1/2	2	1

Now the required weight of suppliers with respect to criteria ( $s_3$ ) is defined as follows:

$$w_1 = 0.468731, w_2 = 0.278709, w_3 = 0.102814, w_4 = 0.149746$$

Here (C.R) = 0.036059 < 0.1, it means the comparison done in a pair-wise comparison matrix is acceptable

**Matrix ( $A_5$ )**

Trade Relationship ( $s_4$ )	Supplier ( $S_1$ )	Supplier ( $S_2$ )	Supplier ( $S_3$ )	Supplier ( $S_4$ )
Supplier ( $S_1$ )	1	1/2	6	4
Supplier ( $S_2$ )	2	1	5	2
Supplier ( $S_3$ )	1/6	1/5	1	1/4
Supplier ( $S_4$ )	1/4	1/2	4	1

Now the required weight of suppliers with respect to criteria ( $s_4$ ) is defined as follows:

$$w_1 = 0.36359, w_2 = 0.413117, w_3 = 0.059023, w_4 = 0.16427$$

Here (C.R) = 0.092649 < 0.1, it means the comparison done in a pair-wise comparison matrix is acceptable.

Now we have constructed the table of the criteria's weights and the weights of suppliers with respect to all the given criterias obtained in a pair-wise comparison matrices ( $A_1, A_2, A_3, A_4, A_5$ ):

criteria's weights ( $s_i$ )	supplier's weights with respect to criteria ( $s_1$ )	supplier's weights with respect to criteria ( $s_2$ )	supplier's weights with respect to criteria ( $s_3$ )	supplier's weights with respect to criteria ( $s_4$ )
0.366152	0.612578	0.528783	0.468731	0.36359
0.416029	0.213953	0.295269	0.278709	0.413117

0.063871	0.053154	0.059962	0.102814	0.059023
0.153948	0.120315	0.115986	0.149746	0.16427

Now we have evaluated the global weights of the suppliers through which we will be able to judge which supplier is more appropriate as compared to other suppliers by utilizing the tables which are defined above

Global weights of suppliers ( $S_i$ )

$$S_1 = (0.366152 \times 0.612578) + (0.416029 \times 0.528783) + (0.063871 \times 0.468731) + (0.153948 \times 0.36359)$$

$$S_2 = (0.366152 \times 0.213953) + (0.416029 \times 0.295269) + (0.063871 \times 0.278709) + (0.153948 \times 0.413117)$$

$$S_3 = (0.366152 \times 0.053154) + (0.416029 \times 0.059962) + (0.063871 \times 0.102814) + (0.153948 \times 0.059023)$$

$$S_4 = (0.366152 \times 0.120315) + (0.416029 \times 0.115986) + (0.063871 \times 0.149746) + (0.153948 \times 0.16427)$$

Now after calculation the following global weights of suppliers are defined as follows:

$$S_1 = 0.5301, S_2 = 0.2825, S_3 = 0.06006, S_4 = 0.1271$$

$$S_1 > S_2 > S_4 > S_3$$

Now according to this sequence, we have concluded that  $S_1$  is the most appropriate supplier as compared to  $S_2, S_4$  and  $S_3$ . On the other hand,  $S_2$  is only appropriate supplier as compared to  $S_4$  and  $S_3$ . Similarly  $S_4$  is slightly appropriate supplier as compared to  $S_3$  and  $S_3$  is only a satisfactory supplier. After selection of appropriate suppliers for purchasing the raw materials, we have to consider a set of four distributors ( $D_1, D_2, D_3, D_4$ ) from which selection of appropriate distributors is carried out by constructing the pair-wise comparison matrices ( $M_1, M_2, M_3, M_4, M_5$ ) and establishing the comparison between the qualitative and quantitative criteria which is provided by table (2):

**Matrix ( $M_1$ )**

DISTRIBUTOR SELECTION	Marketing Skills ( $d_1$ )	Economic Capability ( $d_2$ )	Distribution Ability ( $d_3$ )	Effective Transportation ( $d_4$ )
Marketing Skills ( $d_1$ )	1	1/5	2	1/5
Economic Capability ( $d_2$ )	5	1	3	2
Distribution Ability ( $d_3$ )	1/2	1/3	1	1/3
Effective Transportation	5	1/2	3	1

Now firstly we have to find the geometric mean (R) of a given pair-wise comparison matrix

$$R_1 = (a_{11} \times a_{12} \times a_{13} \times a_{14})^{\frac{1}{4}} = (1 \times 1/5 \times 2 \times 1/5)^{\frac{1}{4}} = 0.53183$$

In a similar way we have obtained the remaining geometric mean (R) which is defined as follows:



$$R_2 = 2.340347, R_3 = 0.485492, R_4 = 1.654875$$

Now by utilizing the formula  $w_i = \frac{R_i}{\sum_{i=1}^n R_i}$  we have obtained the criteria's weight which is defined as follows:

$$w_1 = 0.1061, w_2 = 0.466898, w_3 = 0.096855, w_4 = 0.330147$$

Here  $\sum_{i=1}^4 w_i = 1$ , so the following weights is considered as a local weight

Now finally we have checked the consistency ratio (C.R) of a given pair-wise comparison matrix for checking whether the judgement is correct or not.

Here (C.R) = 0.092306 < 0.1, it means the comparison done in a pair-wise comparison matrix is acceptable.

Now the same procedure is repeated for pair-wise comparison matrix of the distributors ( $D_i$ ), for all  $i=1,2,3,4$  with respect to first criteria ( $d_1$ ) which is "marketing skills."

**Matrix ( $M_2$ )**

Marketing Skills( $d_1$ )	Distributor ( $D_1$ )	Distributor ( $D_2$ )	Distributor ( $D_3$ )	Distributor ( $D_4$ )
Distributor ( $D_1$ )	1	1/2	2	1/4
Distributor ( $D_2$ )	2	1	3	2
Distributor ( $D_3$ )	1/2	1/3	1	1/2
Distributor ( $D_4$ )	4	1/2	2	1

Now the required weight of distributors with respect to criteria ( $d_1$ ) is defined as follows:

$$w_1 = 0.156446, w_2 = 0.411789, w_3 = 0.118873, w_4 = 0.312892$$

Here (C.R) = 0.096251 < 0.1, it means the comparison done in a pair-wise comparison matrix is acceptable.

In a similar way we have constructed the remaining pair-wise comparison matrix with respect to second, third and fourth criterias

**Matrix ( $M_3$ )**

Economic Capability ( $d_2$ )	Distributor ( $D_1$ )	Distributor ( $D_2$ )	Distributor ( $D_3$ )	Distributor ( $D_4$ )
Distributor( $D_1$ )	1	1/4	3	1/3
Distributor ( $D_2$ )	4	1	4	2
Distributor ( $D_3$ )	1/3	1/4	1	1/3
Distributor ( $D_4$ )	3	1/2	3	1

Now the required weight of distributors with respect to criteria ( $d_2$ ) is defined as follows:

$$w_1 = 0.142843, w_2 = 0.480464, w_3 = 0.08247, w_4 = 0.294223$$

Here (C.R) = 0.06526 < 0.1, it means the comparison done in a pair-wise comparison matrix is acceptable.

**Matrix ( $M_4$ )**

Distribution Ability ( $d_3$ )	Distributor ( $D_1$ )	Distributor ( $D_2$ )	Distributor ( $D_3$ )	Distributor ( $D_4$ )
Distributor ( $D_1$ )	1	1/3	2	1/4
Distributor ( $D_2$ )	3	1	4	2
Distributor ( $D_3$ )	1/2	1/4	1	1/5

Distributor ( $D_4$ )	4	1/2	5	1
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Now the required weight of distributors with respect to criteria ( $d_3$ ) is defined as follows:

$$w_1=0.127071, w_2=0.440188, w_3 = 0.079081, w_4 = 0.35366$$

Here (C.R) = 0.051841 < 0.1, it means the comparison done in a pair-wise comparison matrix is acceptable.

#### Matrix ( $M_5$ )

Effective Transportation ( $d_4$ )	Distributor ( $D_1$ )	Distributor ( $D_2$ )	Distributor ( $D_3$ )	Distributor ( $D_4$ )
Distributor ( $D_1$ )	1	1/5	4	1/3
Distributor ( $D_2$ )	5	1	5	1
Distributor ( $D_3$ )	1/4	1/5	1	1/4
Distributor ( $D_4$ )	3	1	4	1

Now the required weight of distributors with respect to criteria ( $d_4$ ) is defined as follows:

$$w_1=0.139529, w_2=0.434166, w_3 = 0.064923, w_4 = 0.361382$$

Here (C.R) = 0.082249 < 0.1, it means the comparison done in a pair-wise comparison matrix is acceptable.

Now we have constructed the table of the criteria's weights and the weights of distributors with respect to all the given criteria obtained in a pair-wise comparison matrices ( $M_1, M_2, M_3, M_4, M_5$ ).

criteria's weights ( $d_i$ )	Distributor's weights with respect to criteria ( $d_1$ )	Distributor's weights with respect to criteria ( $d_2$ )	Distributor's weights with respect to criteria ( $d_3$ )	Distributor's weights with respect to criteria ( $d_4$ )
0.1061	0.156446	0.142843	0.127071	0.139529
0.466898	0.411789	0.480464	0.440188	0.434166
0.096855	0.118873	0.08247	0.079081	0.064923
0.330147	0.312892	0.294223	0.35366	0.361382

Now we have evaluated the global weights of the distributors through which we will be able to judge which distributor is more appropriate as compared to other distributors by utilizing the tables which are defined above

#### Global weights of distributors ( $D_i$ )

$$D_1=(0.1061 \times 0.156446) + (0.466898 \times 0.142843) + (0.096855 \times 0.127071) + (0.330147 \times 0.139529)$$

$$D_2=(0.1061 \times 0.411789) + (0.466898 \times 0.480464) + (0.096855 \times 0.440188) + (0.330147 \times 0.434166)$$

$$D_3=(0.1061 \times 0.118873) + (0.466898 \times 0.08247) + (0.096855 \times 0.079081) + (0.330147 \times 0.064923)$$

$$D_4 = (0.1061 \times 0.312892) + (0.466898 \times 0.294223) + (0.096855 \times 0.35366) + (0.330147 \times 0.361382)$$

Now after calculation the following global weights of distributors is defined as follows:

$$D_1 = 0.14166, D_2 = 0.45399, D_3 = 0.08021, D_4 = 0.32413$$

$$D_2 > D_4 > D_1 > D_3$$

Now according to this sequence, we have concluded that  $D_2$  is most appropriate distributor as compared to  $D_4, D_1$  and  $D_3$ . On the other hand,  $D_4$  is only appropriate distributor as compared to  $D_1$  and  $D_3$ . Similarly  $D_1$  is slightly appropriate distributor as compared to  $D_3$  and  $D_3$  is only satisfactory distributor.

Finally, at last, we have simultaneously optimized some selective criterias of this selection which directly influence the planning of production-distribution.

Let us assume that the demand of raw material for production carried out by a manufacturer is approximately about 20,000 units which is fulfilled by all the suppliers together and let us assume that the financial purchasing budget of raw material for production is 30,000 \$ and after finishing the process of manufacturing all the distributors will require to fulfil the market demand of a particular product which is assumed to be 70%. Now we have optimized the criterias of supplier and distributor selection by which planning of production-distribution is influenced

**Table 4.** Consider the following data of supplier and distributor selection

Required Supplier $i$	$C_i$ (\$)	$L_i$ (%)	$O_i$ (%)	$A_i$ (%)	$R_i$ (no. of days)	$D_i$ (%)	$F_i$ (%)	$M_i$
1	0.04	0.2	60	85	5	0.1	90	6000
2	0.02	0.3	70	90	12	0.4	85	7000
3	0.03	0.5	50	80	10	0.3	95	5000
4	0.05	0.4	40	75	15	0.2	80	4000
Required Distributors $j$	$O_j$ (%)	$E_j$ (%)	$T_j$ (Hours)	$S_j$ (%)	$Q_j$ (%)	$A_j$ (%)		
1	85	15	4	60	0.02	60		
2	90	10	6	50	0.01	70		
3	80	20	3	70	0.04	50		
4	95	5	5	80	0.03	40		

Now we have constructed the numerical model with the help of the quantitative data which is provided by Table 4.

$$\begin{aligned}
 Z_1 &= 0.04q_1 + 0.02q_2 + 0.03q_3 + 0.05q_4 \\
 Z_2 &= 0.2q_1 + 0.3q_2 + 0.5q_3 + 0.4q_4 \\
 Z_3 &= 60q_1 + 70q_2 + 50q_3 + 40q_4 \\
 Z_4 &= 85q_1 + 90q_2 + 80q_3 + 75q_4 \\
 Z_5 &= 5q_1 + 12q_2 + 10q_3 + 15q_4 \\
 Z_6 &= 0.1q_1 + 0.4q_2 + 0.3q_3 + 0.2q_4 \\
 Z_7 &= 90q_1 + 85q_2 + 95q_3 + 80q_4 \\
 Z_8 &= 85b_1 + 90b_2 + 80b_3 + 95b_4 \\
 Z_9 &= 15b_1 + 10b_2 + 20b_3 + 5b_4 \\
 Z_{10} &= 4b_1 + 6b_2 + 3b_3 + 5b_4 \\
 Z_{11} &= 60b_1 + 50b_2 + 70b_3 + 80b_4 \\
 Z_{12} &= 0.02b_1 + 0.01b_2 + 0.04b_3 + 0.03b_4 \\
 \text{Subject to}
 \end{aligned}$$

$$\begin{aligned}
q_1 + q_2 + q_3 + q_4 &= 20,000 \\
0.04q_1 + 0.02q_2 + 0.03q_3 + 0.05q_4 &\leq 30,000 \\
q_1 &\leq 6000 \\
q_2 &\leq 7000 \\
q_3 &\leq 5000 \\
q_4 &\leq 4000 \\
b_1 + b_2 + b_3 + b_4 &= 70 \\
b_1 &\leq 60 \\
b_2 &\leq 70 \\
b_3 &\leq 50 \\
b_4 &\leq 40 \\
q_i &\geq 0, \text{ for all } i=1,2,3,4 \text{ and are integers} \\
b_j &\geq 0, \text{ for all } j=1,2,3,4 \text{ and are integers}
\end{aligned}$$

Now according to Zimmermann [14], by minimizing the objective  $Z_1$  under given set of constraints through which the lower bound ( $Z^*$ ) of objective  $Z_1$  is obtained. Similarly, by maximizing the objective  $Z_1$  under the same set of constraints we are able to achieve the upper bound ( $Z^{**}$ ) of objective  $Z_1$ . Now for obtaining the lower and upper bound of the required objectives ( $Z_2, Z_3, Z_4, Z_5, Z_6$ ) by utilizing the same set constraints by repeating the same procedure.

**Table 5.** Lower and upper bounds of required objective functions is described as follows

Serial No.	Objective Function	Upper Bound ( $Z^{**}$ )	Lower Bound ( $Z^*$ )
1	$Z_1$	690	630
2	$Z_2$	7000	6400
3	$Z_3$	1180000	1120000
4	$Z_4$	1690000	1660000
5	$Z_5$	214000	194000
6	$Z_6$	5500	4900
7	$Z_7$	1770000	1740000
8	$Z_8$	6500	5700
9	$Z_9$	1300	500
10	$Z_{10}$	420	230
11	$Z_{11}$	5300	3500
12	$Z_{12}$	2.600	0.700

Now with the help of linear membership function which is expressed by equations (45) and (46), we have obtained minimum and maximum fuzzy objective functions by transforming all the traditional objective function which is involved in proposed linear mathematical model from (1)-(19):

$$\begin{aligned}
\mu_{Z_1} &= \begin{cases} \frac{690 - (0.04q_1 + 0.02q_2 + 0.03q_3 + 0.05q_4)}{60} & \text{if } 630 \leq Z_1 \leq 690 \\ 1 & \text{if } Z_1 \leq 630 \\ 0 & \text{if } Z_1 \geq 690 \end{cases} \\
\mu_{Z_2} &= \begin{cases} \frac{7000 - (0.2q_1 + 0.3q_2 + 0.5q_3 + 0.4q_4)}{600} & \text{if } 6400 \leq Z_2 \leq 7000 \\ 1 & \text{if } Z_2 \leq 6400 \\ 0 & \text{if } Z_2 \geq 7000 \end{cases} \\
\mu_{Z_3} &= \begin{cases} \frac{(60q_1 + 70q_2 + 50q_3 + 40q_4) - 1120000}{60000} & \text{if } 1120000 \leq Z_3 \leq 1180000 \\ 1 & \text{if } Z_3 \geq 1180000 \\ 0 & \text{if } Z_3 \leq 1120000 \end{cases}
\end{aligned}$$

$$\mu_{Z_4} = \begin{cases} \frac{(85q_1+90q_2+80q_3+75q_4)-1660000}{30000} & \text{if } 1660000 \leq Z_4 \leq 1690000 \\ 1 & \text{if } Z_4 \geq 1690000 \\ 0 & \text{if } Z_4 \leq 1660000 \end{cases}$$

$$\mu_{Z_5} = \begin{cases} \frac{214000-(5q_1+12q_2+10q_3+15q_4)}{20000} & \text{if } 194000 \leq Z_5 \leq 214000 \\ 1 & \text{if } Z_5 \leq 194000 \\ 0 & \text{if } Z_5 \geq 214000 \end{cases}$$

$$\mu_{Z_6} = \begin{cases} \frac{5500-(0.1q_1+0.4q_2+0.3q_3+0.2q_4)}{600} & \text{if } 4900 \leq Z_6 \leq 5500 \\ 1 & \text{if } Z_6 \leq 4900 \\ 0 & \text{if } Z_6 \geq 5500 \end{cases}$$

$$\mu_{Z_7} = \begin{cases} \frac{(90q_1+85q_2+95q_3+80q_4)-1740000}{30000} & \text{if } 1740000 \leq Z_7 \leq 1770000 \\ 1 & \text{if } Z_7 \geq 1770000 \\ 0 & \text{if } Z_7 \leq 1740000 \end{cases}$$

$$\mu_{Z_8} = \begin{cases} \frac{(85b_1+90b_2+80b_3+95b_4)-5700}{800} & \text{if } 5700 \leq Z_8 \leq 6500 \\ 1 & \text{if } Z_8 \geq 6500 \\ 0 & \text{if } Z_8 \leq 5700 \end{cases}$$

$$\mu_{Z_9} = \begin{cases} \frac{1300-(15b_1+10b_2+20b_3+5b_4)}{800} & \text{if } 500 \leq Z_9 \leq 1300 \\ 1 & \text{if } Z_9 \leq 500 \\ 0 & \text{if } Z_9 \geq 1300 \end{cases}$$

$$\mu_{Z_{10}} = \begin{cases} \frac{420-(4b_1+6b_2+3b_3+5b_4)}{190} & \text{if } 230 \leq Z_{10} \leq 420 \\ 1 & \text{if } Z_{10} \leq 230 \\ 0 & \text{if } Z_{10} \geq 420 \end{cases}$$

$$\mu_{Z_{11}} = \begin{cases} \frac{(60b_1+50b_2+70b_3+80b_4)-3500}{1800} & \text{if } 3500 \leq Z_{11} \leq 5300 \\ 1 & \text{if } Z_{11} \geq 5300 \\ 0 & \text{if } Z_{11} \leq 3500 \end{cases}$$

$$\mu_{Z_{12}} = \begin{cases} \frac{2.600-(0.02b_1+0.01b_2+0.04b_3+0.03b_4)}{1.9} & \text{if } 0.700 \leq Z_{12} \leq 2.600 \\ 1 & \text{if } Z_{12} \leq 0.700 \\ 0 & \text{if } Z_{12} \geq 2.600 \end{cases}$$

Now with the help of Zimmermann approach which is expressed from equation (47),(48), we have transformed the proposed fuzzy optimization model into its crisp form and the required optimal solutions are obtained.

$$\begin{aligned} & \text{Maximize } \alpha \\ & \text{Subject to,} \\ & \alpha \leq \frac{690-(0.04q_1+0.02q_2+0.03q_3+0.05q_4)}{60} \\ & \alpha \leq \frac{7000-(0.2q_1+0.3q_2+0.5q_3+0.4q_4)}{600} \\ & \alpha \leq \frac{(60q_1+70q_2+50q_3+40q_4)-1120000}{60000} \\ & \alpha \leq \frac{(85q_1+90q_2+80q_3+75q_4)-1660000}{30000} \\ & \alpha \leq \frac{214000-(5q_1+12q_2+10q_3+15q_4)}{20000} \\ & \alpha \leq \frac{5500-(0.1q_1+0.4q_2+0.3q_3+0.2q_4)}{600} \\ & \alpha \leq \frac{(90q_1+85q_2+95q_3+80q_4)-1740000}{30000} \end{aligned}$$

$$\begin{aligned}
\alpha &\leq \frac{(85b_1+90b_2+80b_3+95b_4)-5700}{800} \\
\alpha &\leq \frac{1300-(15b_1+10b_2+20b_3+5b_4)}{800} \\
\alpha &\leq \frac{420-(4b_1+6b_2+3b_3+5b_4)}{190} \\
\alpha &\leq \frac{(60b_1+50b_2+70b_3+80b_4)-3500}{1800} \\
\alpha &\leq \frac{2.600-(0.02b_1+0.01b_2+0.04b_3+0.03b_4)}{1.9} \\
b_1+b_2+b_3+b_4 &= 70 \\
q_1+q_2+q_3+q_4 &= 20000 \\
0.04q_1+0.02q_2+0.03q_3+0.05q_4 &\leq 30000 \\
q_1 &\leq 6000 \\
q_2 &\leq 7000 \\
q_3 &\leq 5000 \\
q_4 &\leq 4000 \\
b_1 &\leq 60 \\
b_2 &\leq 70 \\
b_3 &\leq 50 \\
b_4 &\leq 40 \\
q_i &\geq 0, \text{ for all } i=1, 2, 3, 4 \text{ and are integers} \\
b_j &\geq 0, \text{ for all } j=1, 2, 3, 4 \text{ and are integers}
\end{aligned}$$

Now with the help of linear programming-based software LINGO (Ver 20.0), we have solved the above numerical model for obtaining the following solutions:

$$q_1 = 6000, q_2 = 6334, q_3 = 4949, q_4 = 2717, b_1 = 45, b_2 = 6, b_3 = 0, b_4 = 19$$

The required optimal solution is described as follows:

$$Z_1=651, Z_2=6,661.5, Z_3=1,159,510, Z_4=1,679,755, Z_5=196,253, Z_6=5,161.7, Z_7=1,765,905, Z_8=6,170, Z_9=830, Z_{10}=311, Z_{11}=4, 520, Z_{12}=1.53.$$

## Conclusion

This study makes an original contribution by proposing an integrated two-stage decision-making framework for supplier and distributor selection in a global manufacturing environment. The framework combines the Analytic Hierarchy Process (AHP) with a fuzzy multi-objective linear programming (FMOLP) model to simultaneously address qualitative and quantitative criteria in both procurement and distribution stages. By using a single weighting mechanism (AHP) for evaluating criteria across both stages and embedding these weights into a fuzzy optimization model, the proposed approach directly links supplier–distributor selection with production–distribution planning. The numerical example presented in this paper demonstrates the practicality, applicability, and effectiveness of the proposed methodology in supporting managerial decision-making. Future research can extend this work in several directions. The proposed model may be enhanced by incorporating dynamic or stochastic parameters to better represent real-world uncertainties in demand, lead time, and costs. Alternative multi-criteria decision-making techniques or hybrid approaches could be explored and compared with AHP to improve robustness. Furthermore, large-scale real-world case studies and sensitivity analyses could be conducted to validate the model's performance under different industrial settings. Integrating sustainability, risk management, and digital supply chain considerations also presents promising avenues for future investigation.

## References

- [1] Sheikh M. H, Iqbal M. A, Urmi S. S. (2025). Selecting the Best Yarn Supplier for a Bangladeshi Retail Apparel Industry Using Fuzzy Analytic Hierarchy Process: A Case Study. In International Textile and Apparel Association Annual Conference Proceedings 81(1). Iowa State University Digital Press.
- [2] Tronnebati I, Jawab F, Frichi Y, Arif J. (2024). Green supplier selection using fuzzy ahp, fuzzy tosis, and fuzzy waspas: A case study of the moroccan automotive industry. Sustainability. 16(11): 4580.
- [3] Arslankaya S, Çelik M. T. (2021). Green supplier selection in steel door industry using fuzzy AHP and fuzzy Moora methods. Emerging Materials Research. 10(4): 357-369.
- [4] Ecer F. (2022). Multi-criteria decision making for green supplier selection using interval type-2 fuzzy AHP: a case study of a home appliance manufacturer. Operational Research. 22(1): 199-233.
- [5] Deepika S, Anandakumar S, Bhuvanesh Kumar M, Baskar C. (2023). Performance appraisal of supplier selection in construction company with Fuzzy AHP, Fuzzy TOPSIS, and DEA: A case study based approach. Journal of Intelligent & Fuzzy Systems. 45(6): 10515-10528.
- [6] Sevkli M. (2009). An application of the fuzzy ELECTRE method for supplier selection. International Journal of Production Research. 48(12): 3393-3405.
- [7] Awasthi A, Chauhan S. S, Goyal S. K. (2010). A fuzzy multicriteria approach for evaluating environmental performance of suppliers. International Journal of Production Economics. 126 (2): 370–378.
- [8] Zou Z, SengT. T, Sohn H, Song G, Gutierrez R. (2011). A rough set-based approach to distributor selection in supply chain management. Expert Systems with Applications. 38(1):106-115.
- [9] Chang B, Chang C, Wu C. (2011). Fuzzy DEMATEL method for developing supplier selection criteria. Expert Systems with Applications. 38(3):1850-1858.
- [10] Shaw K, Shankar R, Yadav S. S, Thakur L. S. (2012). Supplier selection using fuzzy AHP and fuzzy multi-objective linear programming for developing low Carbon Supply Chain. Expert Systems with Applications. 39(9): 8182–8192.
- [11] Ghorbani M, Arabzad S. M, Bahrami M. (2012). Applying a neural network algorithm to distributor selection problem. Procedia - Social and Behavioral Sciences. 41:498–505.
- [12] Amindoust A, Ahmed S, Saghafeinia A, Bahreininejad A. (2012). Sustainable supplier selection: A ranking model based on Fuzzy Inference System. Applied Soft Computing. 12(6): 1668–1677.
- [13] Amindoust A, Saghafeinia A. (2014). Supplier evaluation using fuzzy inference systems. Supply Chain Management Under Fuzziness: Recent Developments and Techniques.3-19.
- [14] Haoran S, Wen D, Ling Z. (2014). A fuzzy TOPSIS based approach for distributor selection in supply chain management: An empirical study of an agricultural enterprise in China. Advance Journal of Food Science and Technology. 6(1):112–118
- [15] Ghorbani M, Arabzad S. M, Moghaddam R. T. (2014). Service quality-based distributor selection problem: A hybrid approach using fuzzy ART and AHP-F TOPSIS. International Journal of Productivity and Quality Management. 13(2): 157.
- [16] M. Li, C. Wu, and L. Zhang (2015). An intuitionistic fuzzy-TODIM method to solve distributor evaluation and selection problem. International Journal of Simulation Modelling . 14(3):511–524.
- [17] Nakiboglu G, Bulgurcu B. (2020). Supplier selection in a Turkish textile company by using intuitionistic fuzzy decision-making. The Journal of The Textile Institute. 112(2): 322-332.

- [18] Vats P, Soni G, Rathore A. P, Shukla O. J. (2022). Grey-based decision-making approach for the selection of distributor in a supply chain. *International Journal of Intelligent Enterprise*. 9 (2): 207.
- [19] Zimmermann H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*. 1 (1): 45–55.
- [20] Saaty T.L. (1980). *The Analytic Hierarchy Process*, Mc Graw-Hill, New York, NY.
- [21] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, 8(3), 338-353.