International Journal of Natural and Engineering Sciences (6)1: 41-44, 2012
ISSN: 1307-1149, E-ISSN: 2146-0086, www.nobel.gen.tr

How to Manage Risk in Life Insurance

Ali MOHSENIYAN SISAKHT Mohamad REZA ABBASI
Payame Noor University, Social Science and Management Department, Tehran, IRAN

***Corresponding Author**
e-mail: mohsenabbasi260@yahoo.com

Received : October 11, 2011
Accepted : November 18, 2011

Abstract

In life insurance, insurer promises to pay a designated beneficiary a sum of money (the "benefits") upon the death of the insured person. Naturally, managing risk especially valuation is one of the most important tasks for insurer. In this research, we try to response the most important of questions of insurers. How a contract must at least be priced such that the insurance company is treated fairly? In this research we use expectation principle but the expectation is taken due to an equivalent martingale measure. Based on this assumption we proposed a model for valuation and implemented the model for an example.

Keywords: insurance management, expectation principle, fairly price, risk

INTRODUCTION

In life insurance, insurer promises to pay a designated beneficiary a sum of money (the "benefits") upon the death of the insured person. Naturally, managing risk especially valuation is one of the most important tasks for insurer. We can manage risk by determining the fair value of contract. One common way to price insurance contracts at fair value is to use contingent claims pricing theory, which is based on the works of Black and Scholes [1] and Merton [2]. Babbel and Merrill [3], used particular adequacy of arbitrage models to provide a discussion of economic valuation models for insurers.

Many researches were done about this area for example: [4-11]. The life insurance contract studied by Grosen and Jørgensen [4] features cliquet-style annual surplus participation. In this type of contract, the greater of the guaranteed interest rate or a fraction of the asset return is annually credited to the policy and in turn becomes part of the guarantee. A bonus account is introduced that serves for a smoothing mechanism for the participation in asset returns. The authors decompose the contract into a risk-free bond, a bonus part, and a surrender option.

Gatzert and Kling [8] propose a method that considers both pricing and risk measurement and thus increases information on insurance liabilities. They examine the effect of fair valuation on the insurer's risk situation, i.e., the actual likelihood and extent of a shortfall for fair contracts with the same market value. Key risk drivers are identified by comparing the results for different types of contracts, including cliquet-style and point-to-point guarantees. Gatzert and Schmeiser [10] develop a model framework for a contract that includes an interest-rate guarantee, cliquet-style annual surplus participation, and offers paid-up and resumption options. The valuation is not based on assumptions about particular exercise strategies, but an upper bound to the option price is provided that is independent of the policyholder's exercise behavior. Using this approach, the impact of guaranteed interest rate, annual surplus participation, and investment volatility on the values of the premium payment options is analyzed.

Kling, Richter, and Ruß [11] present a general framework for contracts containing cliquet-style guarantees, which are common in Germany, and evaluate them by taking into consideration the German regulatory framework. They analyze the interaction of various contract parameters such as management decisions concerning surplus participation rates and guaranteed interest rates.

METERIAL AND METHODS

In this we first discuss assumptions made in the valuation of the policy holder's option to exercise the life insurance contract before its maturity. Second, a model set up for a sample contract is presented and, finally, a numerical example is presented to illustrate efficiency of algorithm.

Principles Of Life Insurance Mathematics

In this section we provide a more detailed description of the life insurance contracts and pension plan products which we will analyze. Furthermore, we introduce the basic model to be used in the analysis and valuation of these contracts, especially the valuation of various embedded option elements.

The evaluation of insurance contracts including rights to early exercise such as premium payment or surrender options is complex, as it strongly depends on assumptions about policyholder exercise behavior and the underlying asset model. We consider the following eight assumptions as crucial assumptions for a modern theory of life insurance mathematics. At first the principles are given in an informal manner. Then the mathematically precise formulation follows later.

- *Independence of biometric and financial events*
- *Large classes of similar individuals*
- *Similar individuals can not be distinguished*
- *No-arbitrage pricing*
- *Principle of Equivalence*
- *Complete, arbitrage-free financial markets*
- *Minimum fair prices allow hedging such that mean balances converge to zero almost surely*
- *Biometric states of individuals are independent*

Model

Let $(F, \mathcal{F}_T, \text{d})$be a probability space equipped with the purification$(\mathcal{F}_t)_{t\in\mathbb{T}}$, where $\mathbb{T} = \{0,1,2,\ldots,T\}$ denotes the discrete finite time axis. Assume that $\mathcal{F}_0$ is trivial, i.e. $\mathcal{F}_0 = \{\emptyset, F\}$. Let the price dynamics of d securities of a frictionless financial market be given by an adapted $\mathbb{R}^d$ -valued process $S = (S_t)_{t\in\mathbb{T}}$. The d assets with price processes $(S_t^0)_{t\in\mathbb{T}}, \ldots, (S_t^{d-1})_{t\in\mathbb{T}}$ are traded at times $t \in \mathbb{T}\{0\}$. The first asset with price process $(S_t^0)_{t\in\mathbb{T}}$ is called the *money account* and has the properties $S_0^0 = 10 = 1$ and $S_t^0 > 0$ for $t \in \mathbb{T}$. The tuple $M^F = (F, (\mathcal{F}_t)_{t\in\mathbb{T}}, (\mathcal{F}_t)_{t\in\mathbb{T}}, \mathbb{T}, S)$ is called a *securities market model*. A portfolio due to M^F is given by a d-dimensional vector $\theta = (\theta^0, \ldots, \theta^{d-1})$ of real-valued random variables θ^i $(i = 0, \ldots, d-1)$ on $(F, \mathcal{F}_T, \text{d})$. A *t-portfolio* is a portfolio θ_t which is $\mathcal{F}_t$-measurable. As usual, $\mathcal{F}_t$ is interpreted as the information available at time t. By considering the available information, a *trading strategy* is a vector $\theta_{\mathbb{T}} = (\theta_t)_{t\in\mathbb{T}}$ of t-portfoliosθ_t. The discounted total gain (or loss) of such a strategy is given by $\sum_{t=0}^{T-1} < \theta_t, \bar{S}_{t+1} - \bar{S}_t >$ where $\bar{S} := (\frac{S_t}{S_t^0})_{t\in\mathbb{T}}$ denotes the price process discounted by the money account and $<.,.>$ denotes the inner product on $\mathbb{R}^d$. One can now define

$$G = \{\sum_{t=0}^{T-1} < \theta_t, \bar{S}_{t+1} - \bar{S}_t >: each\ \theta_t\ is\ a\ t-portfotil\}$$

G is a subspace of the space of all real-valued random variables $L^0(F, \mathcal{F}_T, \text{d})$ where two elements are identified if they are equal dd-a.s. The process S satisfies the so-called *no-arbitrage condition* (NA) if$G \cap L_+^0 = \{0\}$, where L_+^0 are the non-negative elements of $L^0(F, \mathcal{F}_T, \text{d})$ (Delbaen, 1999). The Fundamental Theorem of Asset Pricing states that the price process S satisfies (NA) if and only if there is a probability measure Q equivalent to F such that under Q the process S is a martingale. Q is called equivalent martingale measure (EMM), then. Moreover, Q can be found with bounded Radon-Nikodym derivative$dQ/d\text{d}$.

A valuation principle π^F on a set Θ of portfolios due to M^F is a linear mapping which maps each $\theta \in \Theta$ to an adapted $\mathbb{R}$ -valued stochastic process (price process) $\pi^F(\theta) = (\pi_t^F(\theta))_{t\in\mathbb{T}}$ such that

$$\pi^F(\theta) =< \theta, S_t >= \sum_{i=0}^{d-1} \theta^i S_t^i$$

for any $t \in \mathbb{T}$ for which θ is F_t-measurable.

A t-claim with payoff C_t at time t is a t-portfolio of the form $\frac{C_t}{C_t^0} e_0$ where C_t is a F_t-measurable random variable and e_0 denotes the first canonical base vector in $\mathbb{R}^d$. A cash flow over the time period $\mathbb{TT}$ is a vector $(\frac{C_t}{C_t^0})_{t\in\mathbb{T}}$ of t-claims. In classical life insurance mathematics, the financial market is deterministic. We realize the assumption by $|\mathcal{F}_T| = 2|$, i.e. $\mathcal{F}_T = \{\emptyset, F\}$, and identify $(M, (\mathcal{M}_t)_{t\in\mathbb{T}}, \mathbb{P})$, with $(B, (\mathcal{B}_t)_{t\in\mathbb{T}}, \mathbb{B})$,. As the market is assumed to be free of arbitrage, all assets must show the same dynamics. Hence, we can assume$S = (S_t^0)_{t\in\mathbb{T}}$, i.e. d = 1 and the only asset is the money account as a deterministic function of time. In the classical framework, it is common sense that the fair present value at times of a $\mathbb{B}$ -integrable payoff C_t at t is the conditional expectation of the discounted payoff due to $\mathcal{B}_s$, i.e. for a t-claim ${C_t}/{S_t^0}$.

$$\pi_s\left({C_t}/{S_t^0}\right) := S_s^0 . E_{\mathbb{B}}\left[{C_t}/{S_t^0} | \mathcal{B}_s\right], \quad s \in \mathbb{T}$$

As the discounted price processes are $\mathbb{B}$=martingales, the classical financial market together with a finite number of classical price processes of life insurance policies is free of arbitrage opportunities.

Now, consider the set of life insurance contracts $\{({}^i\gamma_t, {}^i\delta_t)_{t\in\mathbb{T}} : i \in \mathbb{N}^+\}$with the deltas being defined in analogy to the gammas above. Since for the company a contract can be considered as a vector $({}^i\gamma_t - {}^i\delta_t)_{t\in\mathbb{T}}$ of portfolios, the analogous hedge is given by $(E_{\mathbb{B}}[{}^i\gamma_t], E_{\mathbb{B}}[{}^i\delta_t])_{t\in\mathbb{T}}$. Therefore the contract has value zero. From the Expectation Principle we therefore obtain for all

$$\sum_{t=0}^{T} \pi_0 (E_{\mathbb{B}}[{}^i\delta_t], E_{\mathbb{B}}[{}^i\gamma_t]) = \sum_{t=0}^{T} \pi_0({}^i\gamma_t, {}^i\delta_t) = 0$$

Hence, a life insurance company can (without any costs at time 0) pursue a hedge such that the mean balance per contract at any time t converges to zero almost surely for an increasing number of individual contracts:

$$\frac{1}{m}\sum_{i=1}^{m}({}^i\delta_t - {}^i\gamma_t - E_{\mathbb{B}}[{}^i\delta_t] + E_{\mathbb{B}}[{}^i\gamma_t]).S_t^0 \xrightarrow{m\to\infty} 0 \quad \mathbb{B}-a.s.$$

As a direct consequence, the mean of the final balance converges, too:

$$\frac{1}{m}\sum_{i=1}^{m}\sum_{t=1}^{T}({}^i\delta_t - {}^i\gamma_t - E_{\mathbb{B}}[{}^i\delta_t] + E_{\mathbb{B}}[{}^i\gamma_t]).S_t^0 \xrightarrow{m\to\infty} 0 \quad \mathbb{B}-a.s.$$

RESULT AND DISCUSSION

In this section we consider traditional contracts with stochastic interest rates to evaluate the proposed algorithm. Now, consider a man of age x = 30 years and the time axis $\mathbb{T}$ ={0, 1, . . . , 10} (in years). The absolute values at the starting point (September 1972) are ${}^{i}d/{}^{i}c$=0.0607 for the endowment, respectively ${}^{i}d/{}^{i}c$ =0.0023 for the life insurance. The premiums of the endowment seem to be much more subject to the fluctuations of the interest rates than the premiums of the traditional life insurance. For instance, the minimum fair annual premium ${}^{i}d$ for the 10-years endowment with a benefit of ${}^{i}c$ = 100, 000 Euros was 6,296.58 Euros at the 31st July 1984 and 7,065.26 at the 31st January 2003. For the traditional life insurance (with the same benefit), one obtains ${}^{i}d$= 232.46 Euros at the 31st July 1984 and 168.11 at the 31st January 2003.

If one assumes a discrete technical rate of interest R'_{tech}, e.g. 0.045, which is the mean of the interest rates legally guaranteed by German life insurers, one can compute technical quotients ${}^{i}d_{tech}/{}^{i}c$ by computing the technical values of zero-coupon bonds, i.e. $p_{tech}(t,\zeta) = (1+R'_{tech})^{-\zeta}$,. If a life insurance company charges the technical premiums ${}^{i}d_{tech}$ instead of the minimum fair premiums ${}^{i}d$ and ${}^{i}f$ one considers the valuation principle to be a reasonable choice, the present value of the considered insurance contract at time t is

$$ {}^{i}PV = \left({}^{i}d_{tech} - {}^{i}d \right) . \sum_{\zeta=1}^{T_1-1} p(t,\zeta)_{\zeta}\, p_x(t) $$

due to the Principle of Equivalence. In particular, this means that the insurance company can book the gain or loss in the mean at time 0 as long as proper risk management takes place afterwards. Thus, the present value is a measure for the profit, or simply the expected discounted profit of the considered contract if one neglects all additional costs and the fact that first order mortality tables are used.

All computations from above have also been carried out for a 25-years endowment, respectively life insurance. The corresponding figures are 2.3 and 2.4., the absolute values at the starting point are ${}^{i}d/{}^{i}c$ = 0.012993 for the endowment, respectively ${}^{i}d/{}^{i}c$= 0.002553 for the life insurance. The minimum fair premium id for the 25-years endowment with benefit ${}^{i}c$ = 100, 000 Euros was 708.49 Euros at the 31st July 1984 and 2,176.32 Euros at the 31st January 2003. For the traditional life insurance (with the same benefit), one obtains ${}^{i}d$ = 298.37 Euros at the 31st July 1984 and 303.90 at the 31st January 2003.

Hence, the premium-to-benefit ratio for both types of contracts seems to be more dependent on the yield structure than in the 10-years case. However, compared to the 10-years contracts, the longer running time seems to stabilize the present values of the contracts. Nonetheless, they are still strongly depending on the yield structure.

The examples have shown the importance of realistic valuation principles in life insurance. Any premium calculation method and all related parameters (like e.g. technical rates of interest, which have to be determined in some way) should be carefully examined in order to be properly prepared for the fluctuations of financial markets. There is no doubt that many of the financial problems of life insurance companies that have arisen in the past few years could have been avoided by a proper use of modern valuation principles and -perhaps even more important - modern financial hedging strategies.

CONCLUSION

Managing risk especially valuation is one of the most important tasks for insurer. In this research, we tried to response the most important of questions of insurers. The classical Principle of Equivalence ensures that a life insurance company can accomplish that the mean balance per contract converges to zero almost surely for an increasing number of independent clients. In this research, this idea is adapted to the general case of stochastic financial markets. The implied minimum fair price of general life insurance products is then uniquely determined by the product of the given equivalent martingale measure of the financial market with the probability measure of the biometric state space. This minimum fair price (valuation principle) is in accordance with existing results.

REFERENCE

[1] Black, F., Scholes, M. (1973): The Pricing of Options and Corporate Liabilities, in: Journal of Political Economy, 81(3), 637–654.
[2] Merton, R. C. (1973): Theory of Rational Option Pricing, in: Bell Journal of Economics and Management Science, 4(1), 41–183.
[3] Babbel, D. F. and Merrill, C., 1998: Economic Valuation Models for Insurers, in: North American Actuarial Journal, 2(3), 1–17.
[4] Grosen, A., Jørgensen, P. L. (2000): Fair Valuation of Life Insurance Liabilities: The Impact of Interest Rate Guarantees, Surrender Options, and Bonus Policies, in: Insurance: Mathematics and Economics, 26(1), 37–57.
[5] Hansen, M., Miltersen, K. R. (2002): Minimum Rate of Return Guarantees: The Danish Case, in: Scandinavian Actuarial Journal, 2002(4), 280–318.
[6] Steffensen, M. (2002): Intervention options in life insurance, in: Insurance: Mathematics and Economics, 31 (1), 71–85.
[7] Haberman, S., Balotta, L., Wang, N. (2003): Modeling and Valuation of Guarantees in With-profit and Unitised With-Profit Life Insurance Contracts, in: Actuarial Research Paper No. 146, Cass Business School, City of London.
[8] Gatzert, N., Kling, A. (2005): Analysis of Participating Life Insurance Contracts: A Unification Approach, in: Working Papers on Risk Management and Insurance No. 18, University of St. Gallen.
[9] Ballotta, L., Haberman, S. and Wang, N., 2006: Guarantees in With-Profit and Unitized With-Profit Life Insur-

ance Contracts: Fair Valuation Problem in Presence of the Default Option, in: Journal of Risk and Insurance, 73(1), 97–121.

[10] Gatzert, N., Schmeiser, H. (2006): Assessing the Risk Potential of Premium Payment Options in Participating Life Insurance Contracts, in: Working Papers on Risk Management and Insurance No. 22, University of St. Gallen.

[11] Kling, A., Richter, A., Ruß, J. (2006): The Interaction of Guarantees, Surplus Distribution, and Asset Allocation in With Profit Life Insurance Policies, to appear in: Insurance: Mathematics and Economics.