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# Designing Robust Power System Stabilizer Using Pole Placement Technique with the Aid of LMI

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**Abstract**

This paper discusses designing a robust power system stabilizer(PSS) for single machine infinite bus (SMIB) power systems using pole placement technique with the aid of linear matrix inequalities (LMI). The mentioned stabilizer is designed in such a way that in addition to excitation system it send signal to governor too. During the design process all four state variables used in SMIB modeling are considered control inputs. At the end of the process the results are applied to a completed model of SMIB. The simulation results prove the accuracy and capabilities of desiging stabilizer.

**Keywords:** Linear Matrix Inequalities (LMI),robust control ,Power System Stabilizer(PSS),governor ,$\mathbf{H}_{\infty}$

## INTRODUCTION

Nowadays, power system stabilizers are widely used for damping low frequency oscillations in power systems purposes. Conventional power system stabilizers (CPSS) damp the oscillations of the rotor of synchronous generators via generating an additional signal creating some torque elements in phase with the deviation of rotor speed. Indeed, the main object of CPSS is only to enhance the damping of mechanical mode of system [1].

Development of power systems along with various and unpredictable states which may occur in a power system has restricted the efficiency of these stabilizers. In this paper we design a robust stabilizer using pole placement technique with the aid of LMI theory [2]. Beside mechanical mode, this stabilizer minimizes the variations of state variables interacting with controlling output.

Minimizing the variations of state variables requires enhancing both system damping and synchronizer torque. Regarding the rapid development of technology in the field of power systems and also designing and manufacturing rapid governor systems, we design our stabilizer in such a way that in addition to exciter system it broadcasts signal to governor too.

In this plan we used 3rd order of synchronous generator called Heffron-Phillips model[3]. In order to apply LMI control method in our design, in the first stage the poles of closed- loop are defined via assignment procedure in such a way that they are located in a given region of LMI. Then, through minimizing the infinite norm of function and also using LMI theory, robust control in the presence of indefinite is defined in such a way that the system remains stable even in the presence of indefinites. At the next stage, we combine the advantages of the two previous approaches and define control low by introducing a multi-purpose problem in such a way that both poles of closed-loop system are placed in a predefined region and robust of system in the presence of indefinites is enhanced. In order to assess the efficiency of the obtained control low, the results of this method are compared with the results of CPSS. Section. In continuous describe different strategies for designing the stabilizer. Eventually we present simulation results and conclusions.

### Power system modeling

In order to consider a synchronous generator in SMIB modeling process, we use 3rd order synchronous generator model called Heffron-Philips model [3]. This model contains 3 state variables: $\Delta\omega_r, \Delta\delta, \Delta E'_q$ . Considering the exciter model will lead to the introduction of the fourth state variable i.e. $\Delta E_{fd}$ to equations. In this model, governing differential equations are linear around operating point.

Fig. 1 shows block-diagram of linear mode of Heffron-Phillips model along with exciter and AVR [4].

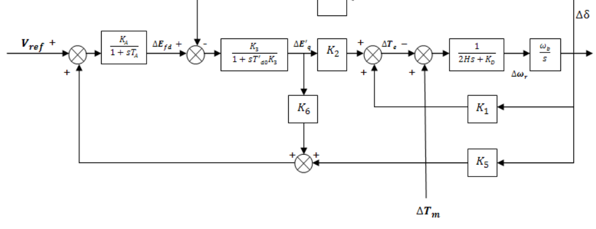



**Fig.1.** Block-diagram of the Heffron-Phillips model of SMIB

Based on this block-diagram we obtain the following equations [4]:

$$\begin{bmatrix}\Delta\dot{\omega}_r\\ \Delta\dot{\delta}\\ \Delta\dot{E}'_q\\ \Delta\dot{E}_{fd}\end{bmatrix} = \begin{bmatrix} -\frac{K_D}{2H} & -\frac{K_1}{2H} & -\frac{K_2}{2H} & 0\\ \omega_b & 0 & 0 & 0\\ 0 & -\frac{K_4}{T'_{d0}} & -\frac{1}{T'_{d0}K_3} & \frac{1}{T'_{d0}} \\ 0 & -\frac{K_AK_5}{T_A} & -\frac{K_AK_6}{T_A} & -\frac{1}{T_A}\end{bmatrix}\begin{bmatrix}\Delta\omega_r\\ \Delta\delta\\ \Delta E'_q\\ \Delta E_{fd}\end{bmatrix} + \begin{bmatrix}0 & \frac{1}{2H}\\ 0&0\\0&0\\ \frac{K_A}{T_A} & 0\end{bmatrix}\begin{bmatrix}V_{ref}\\ \Delta T_m\end{bmatrix} \quad (1)$$

**Conventional Power System Stabilizer (CPSS)**

The architecture of the conventional power system stabilizer (CPSS) is shown in the fig. 2 .

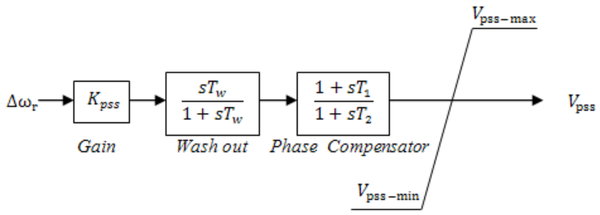



**Fig.2.** Block diagram of the conventional power system stabilizer (CPSS)

Block diagram of the CPSS has 3 blocks:

a) Phase compensation block which provides proper phase lead property in order to compensate the phase lag between the exciter input and electric torque of the generator.

b) The washout block which acts as a high pass filter with time constant, $T_w$, which is enough to perform the task.

c) Gain block which ascertains the generated damping induced by the PSS.

CPSS has the following conversion function:

$$\frac{V_{PSS}(s)}{\Delta\omega_r(s)} = K_{PSS}\frac{sT_w}{1+sT_w}\frac{1+sT_1}{1+sT_2} \quad (2)$$

Fig.3 shows the structure of the single machine infinite bus power system including PSS.

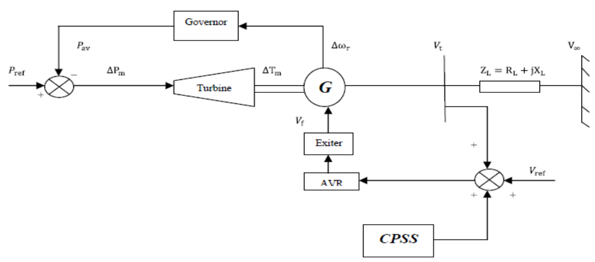



**Fig.3.** Schematic structure of SMIB by CPSS

The mentioned power system contains synchronous generator, exciter and transmission line connected to the infinite bus. PSS control signal is applied to AVR (Automatic Voltage Regulator) inputs.

**Desiginng stabilizer via assigning poles inside LMI region**

In addition to stabilizing closed-loop systems, we expect a stabilizer to have a rapid time response and appropriate damping rate. A typical method for meeting these criteria is to place the poles of closed-loop system in a proper region on the left half plane of image axis. This region is called LMI region. Later on we describe this region and its characteristics and we will show how the LMI could serve as a useful tool for optimal assignment purposes.

***LMI region; definition and introduction***

Every D subset of a complex plan which is defined as follows:

$$\mathcal{D} = \{z \in \mathbb{C} : L + zS + z^*S^T < 0\} \quad (3)$$

In which $S, L=L^T$ are real matrices is called LMI region. Also function $D$

$$f_{\mathcal{D}}(z) = L + zS + z^*S^T \quad (4)$$

is called the characteristic function. We can introduce some LMI regions as follows:

I. Half plan: $\mathrm{Re}(z) < -\alpha$ :

$$f_{\mathcal{D}}(z) = z + z^* + 2\alpha < 0 \quad (5)$$

II.A disk with the center of $(-\gamma, 0)$ and radius of β:

$$f_{\mathcal{D}}(z) = \begin{bmatrix} -\beta & \gamma + z\\ \gamma + z^* & -\beta\end{bmatrix} \quad (6)$$

III. A conic sector whose apex lies in center and its internal radius is $2\theta$ :

$$f_{\mathcal{D}}(z) = \begin{bmatrix} \sin\theta(z+z^*) & \cos\theta(z-z^*)\\ \cos\theta(z^*-z) & \sin\theta(z+z^*)\end{bmatrix} \quad (7)$$

Here are some reasons why LMI regions are very important:

1.The intersection of LMI regions is a LMI region itself

2.Each convex and symmetric area to real axis could be assessed with acceptable accuracy through a LMI region.

Matrix $F$ is D-stable ، In the other words, the whole eigenvalues of this matrix lie inside $D$ region, which is a LMI region, if and only if there is a positive and symmetric matrix like $X$ :

$$L \otimes X + S \otimes (XA) + S^T \otimes (A^TX) < 0 \quad (8)$$

In which ⊗ refers to Kronecker product [5] of the two matrices. This factor has been described in detail in Appendix A. We can consider this problem a generalization of Lyapunov stability theory in which stable region is defined as

$$f_{\mathcal{D}}(z) = z + z^* < 0 \quad (9)$$

and (8), is expressed as follows:

$$1 \otimes (XA) + 1 \otimes (A^TX) = A^TX + XA < 0 \quad (10)$$

***Desigining stabilizer for power systems through pole placement with the aid of LMI***

Fig. 4 shows the schematic diagram of the stabilizer designed for SMIB systems through pole assignment and using governor.

As we can see in Fig. 4 this stabilizer has been designed in

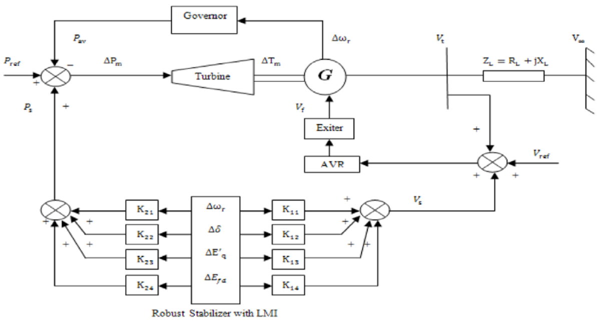



**Fig.4.**Schematic structure of SMIB of proposed stabilizer with the sent signal to Exiter & governer

such a way that in addition to exciter system it broadcasts signal to governor too.

According to block-diagram shown in Fig. 1 and equation (1), the SMIB can be expressed through the following state equations:

$$\begin{cases} \dot{\mathbf{X}} = \mathbf{AX} + \mathbf{Bu} \\ \mathbf{y} = \mathbf{CX} \end{cases} \tag{11}$$

In which X, U, A, B, C are respectively:

$$\mathbf{A} = \begin{bmatrix} 0 & -\frac{K_1}{7} & -\frac{K_2}{7} & 0 \\ 100\pi & 0 & 0 & 0 \\ 0 & -\frac{K_4}{T'_{d0}} & -\frac{1}{T'_{d0}K_3} & \frac{1}{T'_{d0}} \\ 0 & -800K_5 & -800K_6 & -20 \end{bmatrix} \tag{12}$$

$$\mathbf{B} = \begin{bmatrix} 0 & \frac{1}{7} \\ 0 & 0 \\ 0 & 0 \\ 800 & 0 \end{bmatrix} \tag{13}$$

$$\mathbf{C} = [1 \quad 0 \quad 0 \quad 0] \tag{14}$$

$$\mathbf{X} = [\Delta\omega_r \quad \Delta\delta \quad \Delta E'_q \quad \Delta E_{fd}]^{T} \tag{15}$$

$$\mathbf{u} = [V_{ref} \quad \Delta T_m]^{T} \tag{16}$$

The values of $\mathbf{K_1}$ to $\mathbf{K_6}$ and also $\mathbf{T'_{d0}}$ have been expressed in parametrical form in order to estimate the resistance of the designed stabilizer. Nominal values are presented in Appendix B.

At the start of the designing process, we select a combination of two LMI regions inside which we wish to place the poles of closed-loop system. Fig. 5 shows this region.

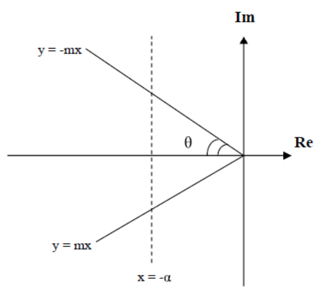



**Fig.5.** Proposed regoin for the closed loop poles

The vertical line $\mathbf{x = -\alpha}$ causes the poles of closed-loop system to be placed in an appropriate distance from image axis. This enhances settling time. Existence of two lines $\mathbf{y = \pm mx}$ ensures that overshoot has a proper value provided that we select appropriate damping ratio.

The poles of the closed-loop lie inside the area shown in Fig. 5 if and only if there is a positive and symmetric matrix like $\mathbf{Q}$ in such a way that it materializes the following inequalities [6]:

$$(\mathbf{A} + \mathbf{BK_{pol}})\mathbf{Q} + \mathbf{Q}(\mathbf{A} + \mathbf{BK_{pol}})^{T} + 2\,\alpha\,\mathbf{Q} < 0 \tag{17}$$

$$\begin{bmatrix} \sin(\theta)\left[(A + BK_{pol})Q + Q(A + BK_{pol})^{T}\right] & \cos(\theta)\left[(A + BK_{pol})Q - Q(A + BK_{pol})^{T}\right] \\ \cos(\theta)\left[(A + BK_{pol})Q - Q(A + BK_{pol})^{T}\right] & \sin(\theta)\left[(A + BK_{pol})Q + Q(A + BK_{pol})^{T}\right] \end{bmatrix} < 0 \tag{18}$$

$$\mathbf{Q} > 0 \tag{19}$$

The above relation could not be considered LMI due to the existence of $\mathbf{K_{pol}Q}$ multiplication term and its transposed matrix. In order to convert this relation to an LMI, we define the variable $\mathbf{Y}$as follows:

$$\mathbf{Y} = \mathbf{K_{pol}Q} \tag{20}$$

By varying this new variable, the nonlinear equation of (17-19) is converted to the following LMI problem:

$$\mathbf{AQ} + \mathbf{QA^{T}} + \mathbf{BY} + \mathbf{Y^{T}B^{T}} + 2\,\alpha\,\mathbf{Q} < 0 \tag{21}$$

$$\begin{bmatrix} \sin(\theta)\,[AQ + QA^{T} + BY + Y^{T}B^{T}] & \cos(\theta)\,[AQ - QA^{T} + BY - Y^{T}B^{T}] \\ \cos(\theta)\,[AQ - QA^{T} + BY - Y^{T}B^{T}] & \sin(\theta)\,[AQ + QA^{T} + BY + Y^{T}B^{T}] \end{bmatrix} < 0 \tag{22}$$

$$\mathbf{Q} > 0 \tag{23}$$

Unknowns $\mathbf{Y}$and $\mathbf{Q}$ could be obtained through solving possibility problem. Using values obtained from solving the LMI problem, the considered stabilizer is obtained through the following relation:

$$\mathbf{K_{pol}} = \mathbf{YQ^{-1}} \tag{24}$$

**Desiging robust stabilizer**

Consider a stable and controllable system with $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$ state space pattern. Then :

$$\|\mathbf{T}\|_{\infty} = \left\|\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}\right\|_{\infty} < \gamma \tag{25}$$

If and only if the system meets one of the following conditions [7]:

$$\begin{bmatrix} \mathbf{AY} + \mathbf{YA^{T}} & \mathbf{B} & \mathbf{YC^{T}} \\ \mathbf{B^{T}} & -\gamma\mathbf{I} & \mathbf{D^{T}} \\ \mathbf{CY} & \mathbf{D} & -\gamma\mathbf{I} \end{bmatrix} < 0, Y = \mathbf{Y^{T}} > 0 \tag{26}$$

$$\begin{bmatrix} \mathbf{YA} + \mathbf{A^{T}Y} & \mathbf{YB} & \mathbf{C^{T}} \\ \mathbf{B^{T}Y} & -\gamma\mathbf{I} & \mathbf{D^{T}} \\ \mathbf{C} & \mathbf{D} & -\gamma\mathbf{I} \end{bmatrix} < 0, Y = \mathbf{Y^{T}} > 0 \tag{27}$$

In order to use this theory, the system should be controllable. In other words the controllability matrix $[\mathbf{B}, \mathbf{AB}, \dots, \mathbf{A^{n-1}B}]$ should be full rank.

The main object of controller designer $\mathbf{u} = \mathbf{K_{\infty}X}$ is to stabilize the closed-loop system even in the presence of indefinites. Assuming that the system is realizable, we would have[8]:

$$\begin{cases}\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \\ y(t) = Cx(t)\end{cases} \tag{28}$$

In the above equation $A, B, C$ are system nominal realization matrices. We introduced them in before section and $B = B_w$. The object of control low is to minimize the infinite norm of function from input $w$ to output. By replacing this $u = K_\infty x(t)$ in equation V and using different variation $Y = K_\infty Q$ system LMIs are described as follows:

$$\begin{bmatrix} AQ + QA' + BY + Y'B & B & QC' \\ B' & -I & 0 \\ CQ & 0 & -\beta I \end{bmatrix} < 0 \tag{29}$$

$$Q > 0 \tag{30}$$

In contrast with the previous problem this problem has been converted to an optimization problem in which we attempt to minimize $\beta$. Whenever $\beta$ meets the following inequality:

$$\|T_{yx}\|_\infty < \sqrt{\beta} \tag{31}$$

The more minimizing of $\beta$ will lead to improving system robust.

Matrices $Q$and $Y$can be computed through solving this minimizing problem. So, the optimal control low would be:

$$K_\infty = YQ^{-1} \tag{32}$$

**Designing robust stabilizer along with pole placement with the aid of LMI**

As saw in two recent sections, in the first method we designed the stabilizer through assigning the poles of closed-loop in LMI region. In the second method, we empower the stabilizer against indefinites. In other words, we designed a robust stabilizer. Now, we combine the methods of the two previous sections and design a robust stabilizer using pole assignment technique with the aid of LMI [8]:

1.System can withstand indefinites

2. The damping rate of system is an appropriate value. For this, it is necessary for the poles of closed-loop system to lie inside a region shown in Fig. 5.

Therefore, this problem is a multipurpose problem and could be formulated as follows [9]:

$$\begin{bmatrix} \sin(\theta)\,[AQ + QA^T + BY + Y^TB^T] & \cos(\theta)\,[AQ - QA^T + BY - Y^TB^T] \\ \cos(\theta)\,[AQ - QA^T + BY - Y^TB^T] & \sin(\theta)\,[AQ + QA^T + BY + Y^TB^T] \end{bmatrix} < 0 \tag{33}$$

$$\begin{bmatrix} AQ + QA' + BY + Y'B & B & QC' \\ B' & -I & 0 \\ CQ & 0 & -\beta I \end{bmatrix} < 0 \tag{34}$$

$$Q > 0 \tag{35}$$

$$K_{Robust+LMI} = YQ^{-1} \tag{36}$$

## SIMULATION AND RESULTS

In the previous sections we designed a stabilizer through three control methods i.e. pole placement technique using LMI and robust control method in before stage. In the 6th stage, which is the main object of this paper, we combined these methods and designed a robust stabilizer using pole assignment technique with the aid of LMI. In the designing process, we used Heffron-Phillips model which is a reduced order model. In order to estimate our designs through simulations, we use complete model of SMIB containing synchronous generator, exciter system, governor, turbine, 3-phase transformer, transmission line, load and infinite bus. For comparison purposes, we compare the variations of $\Delta\omega_r$ before and after 3-phase fault occurring in the middle of transimission line. Three-phase fault occurs at 1 sec. and is gone within 1.1 sec.

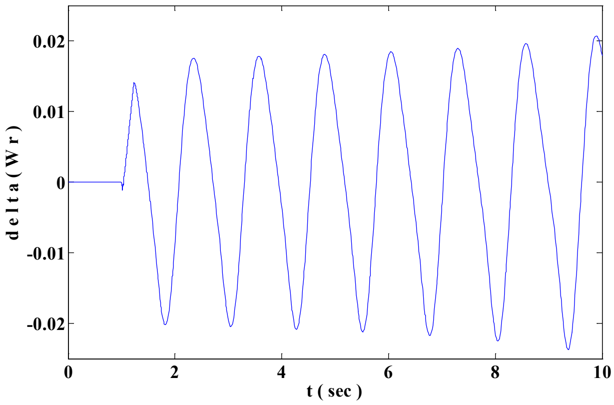



**Fig.6.** a) variations of $\Delta\omega_r$ before and after 3-phase fault without PSS

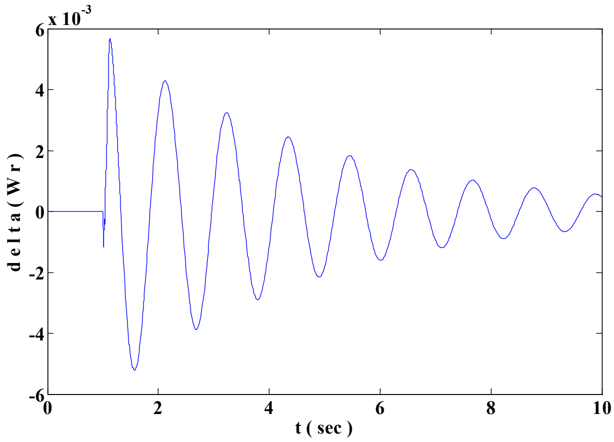
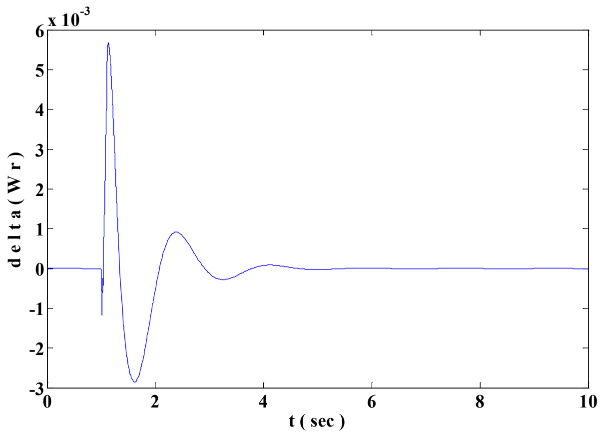
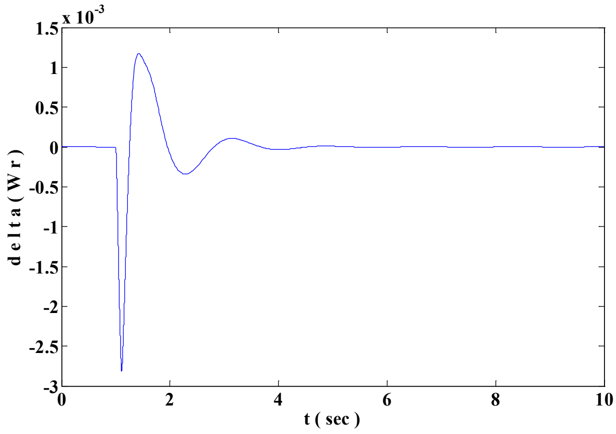


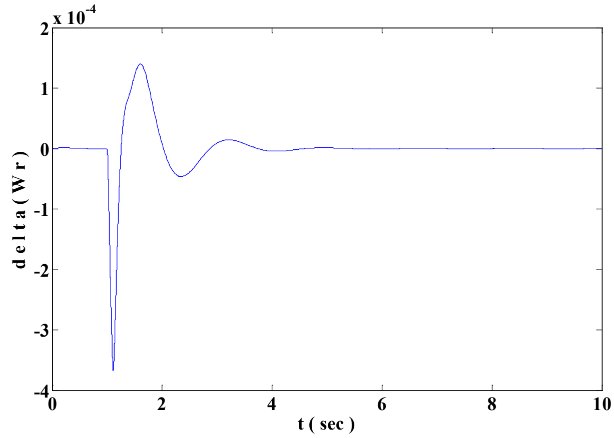

**Fig.7.** Variations of $\Delta\omega_r$ before and after 3-phase fault




(a)




(b)




(c)

**Fig 8.** Variations of $\Delta\omega_r$ before and after 3-phase fault with designed stabilizer by: a) pole placement with LMI, b) robust control, c) robust +LMI

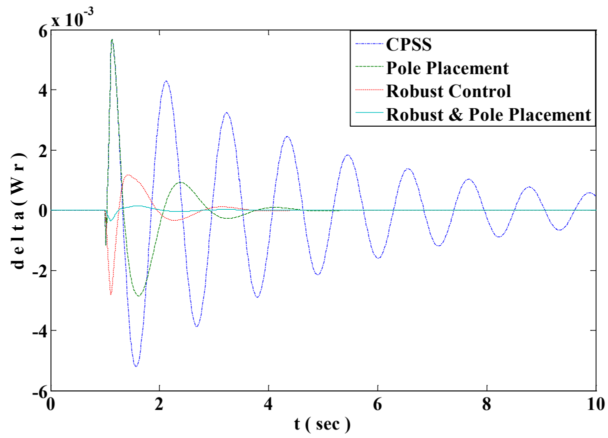



**Fig.9.** Comparison the results

## CONCLUSION

As we can see the simulation results of the stabilizer, designed based on pole placement technique using LMI , robust stabilizer method and combined method, in which in addition to the exciter, signal is sent to governor too, show a meaningful difference with the results of conventional power system stabilizer (CPSS).

In order to have a better comparison between the three methods, regardless of the simulation results of CPSS, Fig. 9 shows the results of the stabilizers designed based on the three methods. Also, in order to have an accurate comparison between these three design methods, the values of settling time, overshoot and infinity norm of wave shape, $\Delta\omega_r$, resulted from the simulations of the designed stabilizers, have been included in Table 1.

According to Fig 9 and Table 1, we can see that the value of overshoot in the combined method is lower than other two methods. In order to analyze the permissible indefinite rate correctly, we use small gain hypothesis. According to this hypothesis the lower value of infinite norm will lead to higher robust capability of a system. Based on this hypothesis and according to the obtained results it is clearly obvious that the robust capability of pole placement technique using LMI is lower than the other two methods. In addition to this parameter, the stabilizer designed based on combination method, which combines two other methods i.e. pole placement using LMI method and robust control method, has been enhanced compared with mere robust control method. This proves the capabilities of the design based on combined method compared with the other two methods.

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**Table1.** Comparison the results

| Design Method | Setteling Time $(t_s)$ | Overshoot | Infinity Norm |
|---|---|---|---|
| Pole placement with LMI | $3.12$ sec | $5.8 \times 10^{-3}$ | $2.4721 \times 10^{-3}$ |
| $H_\infty$ (Robust Control) | $2.45$ sec | $-2.8 \times 10^{-3}$ | $1.5243 \times 10^{-3}$ |
| $H_\infty$ + LMI | $1.09$ sec | $-3.8 \times 10^{-4}$ | $3.3596 \times 10^{-4}$ |

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**Appendix**

***Kronecker Product***

Given two matricies $A \in \mathbb{C}^{m\times n}, B \in \mathbb{C}^{k\times l}$,the Kronecker product of $A$ and $B$ is the $mk \times nl$ matrix :

$$A \otimes B = [A_{ij}B]_{ij} = \begin{bmatrix} A_{11}B & \cdots & A_{1n}B \\ \vdots & & \vdots \\ A_{m1}B & \cdots & A_{mn}B \end{bmatrix}$$

***Coefficients $\mathbf{k_1}$ to $\mathbf{k_6}$ and $\mathbf{T'_{d0}}$ :***

$K_1$=0.7636,$K_2$=0.8644,$K_3$=0.3231,$K_4$=1.4189,$K_5$=-0.1463,$K_6$=0.4167,$T'_{d0}$=8 sec

10.3.Synchronous Generator Parameters

$S_B$= 500 MVA , $V_B$= 22 Kv , $f_s$= 50 Hz

H= 3.5 $\frac{MWsec}{MVA}$ , $K_D$= 0

Stator Resistance (pu) :

$R_s$= 0.003

Reactances (pu) :

$X_d$= 1.81, $X'_d$= 0.3, $X''_d$= 0.23, $X_q$= 1.76, $X'_q$=0.65, $X''_q$= 0.25

Time Constants (sec) :

$T'_{d0}$= 8, $T''_{d0}$= 0.03 , $T'_{q0}$= 1 , $T''_{q0}$= 0.07