

A Novel Approach for Defuzzification Based on Ambiguity- Preserving

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Abstract

The problem of defuzzification is examined in this paper from a broader perspective as a special way of dealing with the general problem of retranslation. The paper includes an overview of different formulations of the problem of defuzzification, as well as an overview of methods that have been suggested in the literature for dealing with the problem. Our own approach to defuzzification, which is described in the paper in more details, is based on relevant measures of uncertainty-based information. This paper is a companion to our recent paper that addresses the general problem of retranslation in computing with perception (Saneifard 2011). Another application of this defuzzification noticed in end of article.

Key Words: Fuzzy sets; Fuzzy intervals; Defuzzification; Correlation.

INTRODUCTION

The emergence of computer technology in the second half of the 20th century opened many new possibilities for machines. These possibilities have been discussed in various contexts quite extensively in the literature. One aspect of considerable interest has been a comparison of existing and prospective capabilities of machines with those of human beings. An overall observation at this time is that the range of machine capabilities has visibly expanded over the years, from numerical computation to symbol manipulation, processing of visual data, learning from experience, etc. Moreover, machines have become superior to humans in some specific capabilities, such as large-scale processing of numerical data, massive combinatorial searches of various kinds, complex symbol manipulation, or sophisticated graphics. Some important new areas have emerged due to these machine capabilities, such as fractal geometry, cellular automata or evolutionary computing. In spite of the impressive advances of machines, they are still not able to match some capabilities of human beings.

Perhaps the most exemplary of them are the remarkable and very complex perceptual abilities of the human mind, which allow humans to use perceptions in purposeful ways to perform complex tasks. Although current machines are not capable of reasoning and acting on the basis of perceptions, a feasible research program for developing this capability was recently proposed by Zadeh [1]. The crux of this program is to approximate perceptions by statements in natural language and, then, to use fuzzy logic to represent these statements and deal with them as needed. This approach to developing perception-based machines is referred to in the literature as computing with words, which is a name suggested also by Zadeh [2]

Approximating statements in natural language by propositions in fuzzy logic may be viewed as a translation from natural language to a formalized language. Alternatively, it may be viewed as a linguistic approximation of the first kind. As is well known, this translation (or approximation) is strongly context dependent. Once it is accomplished in the context of a given application, all available resources of fuzzy logic in the broad sense can be utilized to emulate the ordinary (commonsense) human reasoning that pertains to the application (Yager et al. [3], Bezdek et al. [4]). It is quite obvious that any relevant background knowledge should also be utilized in the reasoning. Regardless of the nature of the reasoning process, its consequents are fuzzy propositions, each of which involves one or more fuzzy sets. In order to connect these fuzzy propositions to perceptions (i.e. to convey appropriate perceptions), we need to approximate them by statements in natural language. This means, in turn, that we need to express each of the fuzzy sets involved by a linguistic expression in natural language that has an understandable meaning in the given context. These issues pertain to the second kind of linguistic approximation, which may conveniently be called a retranslation. While the problem of translation has been extensively studied and discussed in the literature, the problem of retranslation is far less developed.

Prior to the late 1990s, this problem had been recognized only by a few authors, among them Eshragh and Mamdani [5] and Novak [6]. More recently, the problem has been addressed more substantially by Dvorak [7], Yager [8], Delgado et al. [9], Saneifard [10]. In our previous paper [15], we discuss the retranslation problem and explore some approaches to dealing with it. In this paper, our primary objective is to look at the well-known problem of defuzzification as one way, perhaps the simplest one, of dealing with retranslation. On this occasion,

we present a fairly comprehensive overview of the literature dealing with the defuzzification problem, and we consider this overview our secondary objective of this paper. We deal only with linguistic variables whose base variables are numerical.

Basic Definitions And Notations

In this paper, we assume that the reader is familiar with basics of fuzzy set theory and fuzzy logic in the broad sense. For the sake of completeness, we introduce in this section only those concepts that are relevant to our discussion of the retranslation problem. We denote all fuzzy sets in this paper by capital letters. Classical sets are viewed as special fuzzy sets, called crisp sets, and are thus denoted by capital letters as well. For the sake of simplicity, we consider only numerical linguistic variables whose states are expressed by normal and convex fuzzy sets that are defined on some given closed interval, $X = [x_1, x_2]$ of real numbers. These fuzzy sets, usually referred to as fuzzy intervals, are viewed as concave functions from X to $[0,1]$ whose maxima are 1.

Definition 2.1.[11]. For any fuzzy interval $A: X \rightarrow [0,1]$, its α -cut, A^α , is for each $\alpha \in [0,1]$ the closed interval as follows:

$$A^\alpha = \{x \in X \mid A(x) \geq \alpha\}.$$

Definition 2.2.[11]. For each given fuzzy interval, A , the canonical form is as follows,

$$A(x) = \begin{cases} a_L(x) & \text{when } x \in [a, b], \\ 1 & \text{when } x \in [b, c], \\ a_R(x) & \text{when } x \in (c, d], \\ 0 & \text{otherwise.} \end{cases} \quad (2.1)$$

Where $x \in X = [x_1, x_2]$ and a, b, c, d are real numbers in X such that $a \leq b \leq c \leq d$, a_L is a continuous and increasing function from $a_L(a) = 0$ to $a_L(b) = 1$, and a_R is a continuous decreasing function from $a_R(c) = 1$ to $a_R(d) = 0$.

For each value $\alpha \in [0,1]$, the α -cut of A , A^α , is a closed interval of real numbers defined by the formula

$$A^\alpha = [a_L^\alpha, a_R^\alpha]. \quad (2.2)$$

where a_L^α and a_R^α are the inverse function of a_L and a_R , respectively. The crisp sets

$$Supp(A) = \{x \in X \mid A(x) > 0\},$$

$$Core(A) = \{x \in X \mid A(x) = 1\}$$

are called, respectively, a Support of A and a Core of A . Clearly, $supp(A) = (a, d)$ and $re(A) = [b, c]$.

Definition 2.3.[14]. A function symmetric around

$$f: [0,1] \rightarrow [0,1]$$

$$f\left(\frac{1}{2} - \alpha\right) = f\left(\frac{1}{2} + \alpha\right)$$

for all

$$\alpha \in \left[0, \frac{1}{2}\right]$$

which reaches its minimum in $\frac{1}{2}$, is called the bi-symmetrical weighted function. Moreover, the bi-symmetrical weighted function is called regular if

$$f\left(\frac{1}{2}\right) = 0,$$

$$f(0) = f(1) = 1,$$

$$\int_0^1 f(\alpha) d\alpha = \frac{1}{2}$$

In most examples in this paper, we use trapezoidal fuzzy intervals, T , in which a_L and a_R are linear functions. That is

$$a_L(x) = \frac{(x-a)}{(b-a)}$$

and

$$a_R(x) = \frac{(d-x)}{(d-c)}$$

Then, for each

$$\alpha \in (0, 1)$$

$$T^\alpha = [a + (b - a)\alpha, d - (d - c)\alpha], \quad (2.3)$$

Every trapezoidal fuzzy interval T is thus uniquely characterized via the quadruple $T = \langle a, b, c, d \rangle$. A special case in which $b = c$, which is called a triangular fuzzy interval, is also employed in this paper. An important concept for dealing with the problem of retranslation is the degree of subsection, $s(A \subseteq B)$ of fuzzy set A in fuzzy set B (both defined on the same interval X), which is expressed by the

$$s(A \subseteq B) = \frac{\int_X \min\{A(x), B(x)\} dx}{\int_X A(x) dx}. \quad (2.4)$$

The minimum operator in this formula represents the standard intersection of fuzzy sets. As is well known, this is the only intersection of fuzzy sets that is cutworthy in the sense that $(A \cap B)^\alpha = A^\alpha \cap B^\alpha$, hold for all $\alpha \in [0, 1]$, [15]. To deal with the retranslation problem, we also need to measure the non-specificity and fuzziness of the fuzzy sets involved.

Definition 2.4. For any given normal and convex fuzzy set A , we define a well-justified measure of non-specificity, NS , as follows:

$$NS(A) = \int_0^1 f(\alpha) \text{Log}_2[1 + L_m(A^\alpha)]^2 d\alpha \quad (2.5)$$

Where $f: [0, 1] \rightarrow [0, 1]$ is a bi-symmetrical (regular) weighted function [13]. One can, of course, propose many regular bi-symmetrical weighted functions and hence obtain different bi-symmetrical weighted distances. Further on we will consider mainly a following function

$$f(\alpha) = \begin{cases} 1 - 2\alpha & \text{when } \alpha \in \left[0, \frac{1}{2}\right], \\ 2\alpha - 1 & \text{when } \alpha \in \left[\frac{1}{2}, 1\right]. \end{cases} \quad (2.6)$$

In Eq. (2.5), $L_m(A^\alpha)$ denotes for each $\alpha \in [0, 1]$ the Lebesgue measure of A^α . In this case, $L_m(A^\alpha)$ is the length of the interval A^α for each $\alpha \in [0, 1]$. The reason for choosing logarithm base 2 in this formula is to measure ambiguity in a convenient measurement unit: $NS(A) = 1$ when $L_m(A^\alpha) = 1$. Calculating ambiguity is more complicated for fuzzy sets defined on R^n when $n > 1$, but this is beyond the scope of this paper. Function NS is a special case of a more general measure of non-specificity (applicable to convex subsets of the n-dimensional Euclidean space), which is called a Hartley-like measure [8].

Definition 2.5. Given a convex fuzzy set A , its fuzziness, $f(A)$, can be measured by the overlap of A and its complement. Using standard operations of complementation and intersection of fuzzy sets, we have

$$f(A) = \int_X \min\{A(x), 1 - A(x)\} dx \tag{2.7}$$

Clearly, $f(A) = 0$ if and only if A is a crisp (classical) set, and the maximum degree of fuzziness is obtained for the unique fuzzy set in which $A(x) = 1 - A(x) = 0.5$ for all $x \in X$.

Two criteria that are considered in this paper as essential are validity and informativeness.

Definition 2.6. The degree of validity of choosing a standard fuzzy interval F that has a linguistic interpretation to represent a given convex fuzzy set G , $v(F|G)$, as the degree to which G is contained in F define as follows,

$$v(F|G) = \frac{\int_X \min\{G(x), F(x)\} dx}{\int_X G(x) dx} \tag{2.8}$$

For any pair of standard fuzzy intervals, F_1 and F_2 , that compete for representing a given fuzzy set G , clearly, if $v(F_1|G) \geq v(F_2|G)$ then F_1 is preferable to F_2 according to validity.

Definition 2.7. The degree of informativeness of F , $i(F)$, is concerned, it is define it as the normalized reduction of non-specificity with respect to the non-specificity of X as follows:

$$i(F) = 1 - \frac{NS(F)}{\text{Log}_2[1 + L_m(X)]} \tag{2.9}$$

Clearly, if $i(F_1) \geq i(F_2)$ then F_1 is preferable to F_2 according to informativeness.

Example 2.1. Let G is a fuzzy number with membership function as follows that $X = [-10, 10]$ (2.10)

$$G(x) = \begin{cases} \frac{x}{2} & \text{when } x \in [0, 2), \\ 2.4 - 0.7x & \text{when } x \in [2, 3), \\ 0.3 & \text{when } x \in [3, 4), \\ 1.5 - 0.3x & \text{when } x \in [4, 5), \\ 0 & \text{otherwise.} \end{cases}$$

Clearly

$$G^\alpha = \begin{cases} \left[2\alpha, \frac{(15-10\alpha)}{3} \right], & \text{when } \alpha \in (0, 0.3], \\ \left[2\alpha, \frac{(24-10\alpha)}{3} \right], & \text{when } \alpha \in (0.3, 1]. \end{cases} \tag{2.11}$$

There is

$$NS(G) = \int_0^{0.3} \alpha \text{Log}_2 \left[1 + \frac{(15-16\alpha)}{3} \right]^2 d\alpha + \int_{0.3}^1 \alpha \text{Log}_2 [1 + (24 - 24\alpha)]^2 d\alpha = 2.6317$$

and

$$i(G) = 1 - \frac{NS(G)}{\text{Log}_2(21)} = 0.4008.$$

Defuzzification: An Overview Of Proposed Methods

The term “defuzzification” in the sense we use it in this paper (i.e. as a replacement of a given fuzzy set by a representative crisp set) was first employed in Recasens et al. (1999). Although this paper corrected in some sense the terminology and attracted attention to the problem of representing fuzzy sets by crisp sets, this problem was already addressed in a special way more than ten years earlier by Dubois and Prade (1987) in their search for the interval-valued mean of a fuzzy interval. Given a fuzzy interval A with its α -cut $A^\alpha = [a_L^\alpha, a_R^\alpha]$ for all $\alpha \in [0, 1]$, the interval-valued mean of A is, according to Dubois and Prade, the crisp interval $N(A) = [n_L, n_R]$

Where

$$n_L = \int_{[0,1]} a_L^\alpha d\alpha$$

and

$$n_R = \int_{[0,1]} a_R^\alpha d\alpha$$

Since $N(A)$ is based on viewing A as a random set, it is sometimes referred to as expected interval of A (Heilpern 1992) or a probabilistic mean interval (Bodjanova 2005). It is significant that the interval $N(A)$ was also obtained by Ralescu(2000,2002) Saneifard (2010) by computing the average level of A via the Aumann(1965) interval, and by Grzegorzewski (2002) as the best crisp approximation of A in terms of the minimum Euclidean distance between $N(A)$ and A . Recognizing that the defuzzification via the Aumann integral is a kind of averaging procedure, Roventa and Spiricu(2003) try to identify reasonable requirements for averaging operations in this context for prospective defuzzification procedures on this basis. Roman-Flores and Chalco-Cano (2006) further discuss the requirements and argue that a continuity-type requirement is also needed. Chanas(2001) shows for the interval $N(A)$ that

$$n_R - n_L = \int_X A(x) dx$$

and asks the question: is $N(A)$ well placed in relation to A among other crisp intervals with the same length is $N(A)$. He then calculates the crisp interval $H(A) = [h_L, h_R]$ such that $h_R - h_L = n_R - n_L$ and the Hamming distance between A and $H(A)$ is minimized. An alternative crisp interval, $B(A)$, for representing a given fuzzy interval A was introduced by Carlsson and Fuller (2001) and Saneifard (2011) and further investigated by Fuller and Majlender(2003). This interval,

$$B(A) = [b_L, b_R]$$

where

$$b_L = 2 \int_{[0,1]} \alpha_L^\alpha \alpha d\alpha$$

and

$$b_R = 2 \int_{[0,1]} \alpha_R^\alpha \alpha d\alpha,$$

is based on possibilistic interpretation of A and it is called a possibilistic mean interval of A . Its left and right endpoints are called, respectively, the possibility-weighted averages of the minima and maxima of the α -cuts of A .

In a paper that well surveys the literature on crisp interval and point representations of fuzzy intervals. Bodjanova(2005) introduces, in addition to the intervals $N(A)$ and $B(A)$, the concept of a median interval of A , $M(A) = [m_L, m_R]$, where

$$\int_a^{m_L} A(x)dx = \int_{m_L}^b A(x)dx, \quad \int_c^{m_R} A(x)dx = \int_{m_R}^d A(x)dx$$

and a, b, c, d are the real numbers employed in the canonical form of A expressed by equation (2.1). In addition, she defines the concept of a central of A , $C(A) = [c_L, c_R]$, as $C(A) = N(A) \cap B(A) \cap M(A)$, or, alternatively,

$$C_L(A) = \max\{n_L, b_L, m_L\}, C_R(A) = \min\{n_R, b_R, m_R\}.$$

In her more recent paper, [12] focuses an approximations of fuzzy sets by their specific α -cuts. She investigates percentile characterizations of α -cuts in connection with three properties of each given fuzzy set: its height, width and cardinality. One paper whose title contains the term “defuzzification[13] is actually dealing with neither defuzzification nor disambiguation, but rather with the problem of standardization discussed in our previous paper [15]. A method is introduced in this paper for converting a general fuzzy set on X to the symmetric triangular fuzzy number that is nearest to the given fuzzy set according to a specific metric distance.

Ambiguity-Preserving Defuzzification

In the context of the retranslation problem, defuzzification (as viewed in this paper) is a very special way of standardization in the sense introduced in our previous paper [15]. The aim of defuzzification in this context is to eliminate linguistic uncertainty (fuzziness) while preserving information-based uncertainty (ambiguity). That is, given a fuzzy set G , we want to find a crisp set F (a defuzzification of G) such that the ambiguities of G and F are equal. That is, we require that $NS(F) = NS(G)$, where NS is defined by equation(2.5). When G is fuzzy interval, this equation has the form

$$f(\alpha) \text{Log}_2[1 + L_m(F)]^2 = \int_0^1 f(\alpha) \text{Log}_2[1 + L_m(G^\alpha)]^2 d\alpha \tag{4.12}$$

where the right-hand side is a constant. While the length of the sought crisp interval F is determined by solving this equation for $L_m(F)$, the location remains undetermined. To determine by it, we need to employ some additional criteria. The most

important among these criteria is the criterion of validity. We define the degree of validity of F with respect to G , as the degree to which G is contained in F . Formally, (2.8). The crisp interval F can be viewed as a fully valid approximation of G if only if F contains G and, hence, $v(F|G) = 1$. However, requiring full validity of F with respect to G would require that F be the support of G , and this implies that $L_m(F) > L_m(G)$. In order to satisfy equation(4.12), clearly, we need to sacrifice some validity, but it is desirable to sacrifice as little of it as possible. This results in a two-stage defuzzification procedure:

1. Determine $L_m(F)$ by solving equation (4.12).
2. Determine the location of crisp interval F whose length is $L_m(F)$ for which $v(F|G)$ is maximized.

This procedure can be implemented in slightly different ways depending on G . Let us examine some of them. The proposed defuzzification procedure is particularly easy to implement when the canonical representation of the given fuzzy interval, expressed by equation (2.1), is such that function α_L^α is strictly increasing and function α_R^α is strictly decreasing. In this case, there exists an α -cut of G , G_*^α , such that, $L_m(G_*^\alpha) = L_m(F)$, where $L_m(F)$ is obtained by solving equation (4.12). Moreover, among all crisp intervals whose length is $L_m(F)$, G_*^α is the only one with maximal validity.

In the discussed case (when α_L^α and α_R^α are strictly monotone) A_*^α can be determined more directly by solving the α equation

$$f(\alpha) \text{Log}_2[1 + L_m(A_*^\alpha)]^2 = \int_0^1 f(\alpha) \text{Log}_2[1 + L_m(A^\alpha)]^2 d\alpha \tag{4.13}$$

for α . This is illustrated in the following example.

Example 4.2. Let us consider a fuzzy interval A discussed in

$$A(X) = \begin{cases} 2x & \text{when } x \in [0,0.5], \\ 4(x - x^2) & \text{when } x \in [0.5,1], \\ 0 & \text{otherwise,} \end{cases}$$

and

$$A^\alpha = [0.5\alpha, 0.5(1 + \sqrt{1 - \alpha}) - 0.5\alpha].$$

To obtain the α -cut that has the same ambiguity, we first

$$\int_0^1 \alpha \log_2[1 + 0.5(1 + \sqrt{1 - \alpha}) - 0.5\alpha]^2 d\alpha = 0.642279.$$

We solve the equation

$$\alpha \log_2[1 + 0.5(1 + \sqrt{1 - \alpha}) - 0.5\alpha]^2 = 23.5833,$$

for α . The solution (obtained by Mathematica) is = 0.549562. The α -cut of A for this value of α, A_*^α , is then taken as the defuzzification of A .

$$F(A) = A_*^\alpha = [0.274781, 0.835573].$$

When functions α_L^α and α_R^α in the canonical representation of a given fuzzy interval A are not strictly monotone, no α - cut of A may exist that satisfies equation (4.13). This is due to discontinuities in the α - cut representation. In this case, the defuzzified interval F with the length $L_m(F)$ determined by equation (4.12) and maximum validity can be found at one of the levels of discontinuity and may not be unique. These issues are further discussed in the

$$A^\alpha = \begin{cases} [2.5\alpha, 13.3 - 1.7\alpha] & \text{when } \alpha \in [0.0, 0.2], \\ [0.8\alpha, 13.3 - 1.7\alpha] & \text{when } \alpha \in [0.2, 0.6], \\ [1.25\alpha + 2.5, 6 - \alpha] & \text{when } \alpha \in [0.6, 0.8], \\ [\alpha + 3.5, 6 - \alpha] & \text{when } \alpha \in [0.8, 1]. \end{cases}$$

Using equation (2.5), we determine that $(A) = 2.15621$. Solving equation (4.13) for each of the four ranges of the α -cuts of A gives a solution that is outside the respective range and, hence, is not valid. Solving now equation (4.12), we find that $L_m(F) = 3.45742$. This value must fit into one of the three α -cut discontinuities, at $\alpha = 0.2$, $\alpha = 0.6$, or $\alpha = 0.8$. It fits only in the discontinuity at $\alpha = 0.2$. At this level, the left-hand plateau is the smaller one and has the range $[3, 3.25]$. In order to maximize validity, the left-end point of f_R , is then equal to $f_L + L_m(F)$. That is, $f_R = f_L + 3.45742$. Hence all crisp intervals in the range from $[3, 6.45742]$ to $[3.25, 6.70742]$ are acceptable defuzzifications: they all preserve the ambiguity $NS(A)$ and maximize the validity $v(F|A)$. A choice of a unique defuzzification from this range may be based on additional criteria. If no additional criteria are employed, it is reasonable to take as a defuzzification the average of the whole range, F_{avg} . In this example,

$$F_{avg} = \frac{([3, 6.4574] + [3.25, 6.7074])}{2} = [3.15, 6.58].$$

Applications

In this section, the researchers introduce some of applications of the retranslation approximation of fuzzy numbers. This indices can be applied for comparison of fuzzy numbers namely fuzzy correlations in fuzzy environments and expert's systems.

Correlation Coefficient Between Fuzzy Numbers

In many applications the correlation between fuzzy numbers is of interest. Several authors have proposed different measures of correlation between membership functions, intuitionistic fuzzy sets and correlation [16,17]. Hung and Wu [18] defined a correlation by means of expected interval. They defined the correlation coefficient between fuzzy numbers A and B as follows:

$$\rho(A, B) = \frac{E_*(A)(E_*(B) + E^*(A)E^*(B))}{\sqrt{E_*^2(A) + E^{*2}(A)}\sqrt{E_*^2(B) + E^{*2}(B)}}, \quad (5.14)$$

where

$$[E_*(A), E^*(A)] = \left[\int_0^1 L_A(\alpha) d\alpha, \int_0^1 R_A(\alpha) d\alpha \right]. \quad (5.15)$$

This correlation coefficient shows not only the degree of relationship between the fuzzy numbers but also whether these fuzzy numbers are positively or negatively related. The researchers extend (5.14) by interval $H(A) = [H_*(A), H^*(A)]$ instead of (5.15), then,

$$\rho_D(A, B) = \frac{D_*(A)D_*(B) + D^*(A)D^*(B)}{\sqrt{D_*^2(A) + D^{*2}(A)}\sqrt{D_*^2(B) + D^{*2}(B)}} \quad (5.16)$$

is called the weighted correlation coefficient between two fuzzy numbers A and B . This correlation coefficient lies in

$[-1, 1]$ and gives us more information compared to correlation coefficient in [17,19] and some others, which lie within $[0, 1]$. It has all mentioned properties from correlation coefficient that introduced in [19], the researchers review properties $\rho_D(A, B)$ as follows:

Proposition 5.1. For any fuzzy numbers A and $B \in F$ we have:

- (1) $\rho_D(A, B) = \rho_D(B, A) \rho_D(A, B) = \rho_D(B, A)$,
- (2) if $A = B \Rightarrow \rho_D(A, B) = 1$ $A = B \Rightarrow \rho_D(A, B) = 1$,
- (3) if $A = HB$ for some $c > 0 \Rightarrow \rho_D(A, B) = 1$,
- (4) $|\rho_D(A, B)| \leq 1$.

Proof: The proof is obvious.

Example 5.4. Let

$$A_\alpha = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha]$$

$$\text{and } B_\alpha = [b_1 + (b_2 - b_1)\alpha, b_4 - (b_4 - b_3)\alpha]$$

be two trapezoidal fuzzy numbers and $f(\alpha) = (k + 1)\alpha^k$

$$\rho_D(A, B) = \frac{((k+1)a_2 + a_1)((k+1)b_2 + b_1) + ((k+1)a_3 + a_4)((k+1)b_3 + b_4)}{\sqrt{((k+1)a_2 + a_1)^2 + ((k+1)a_3 + a_4)^2} \sqrt{((k+1)b_2 + b_1)^2 + ((k+1)b_3 + b_4)^2}}$$

The above example shows that the parametric function interacts on the correlation coefficient between two fuzzy numbers such that for large values of k this article has:

$$\lim_{k \rightarrow \infty} \rho_D(A, B) = \frac{a_2 b_2 + a_3 b_3}{\sqrt{a_2^2 + a_3^2} \sqrt{b_2^2 + b_3^2}}$$

Moreover if A and B be two triangular fuzzy numbers, whenever $k \rightarrow \infty$, we have

$$\rho_D(A, B) = \frac{a_2 b_2}{|a_2 b_2|}, \quad a_2, b_2 \neq 0.$$

Example 5.5. Yen [20] used to six grade levels (6th-grade to 11th-grade) as student's mathematical learning progress. He assigned the linguistic values $G_6, G_7, G_8, G_9, G_{10}$ and G_{11} to these grade levels, respectively, and transferred these linguistic values to corresponding reasonable normal fuzzy numbers $\tilde{G}_6, \tilde{G}_7, \tilde{G}_8, \tilde{G}_9, \tilde{G}_{10}$ and \tilde{G}_{11} with triangular membership functions as follows:

$$\tilde{G}_6(x) = \begin{cases} 1 - 5x & 0.0 \leq x \leq 0.2, \\ 0 & 0.2 \leq x \leq 1. \end{cases}$$

$$\tilde{G}_7(x) = \begin{cases} 5x & 0.0 \leq x \leq 0.2, \\ 2 - 5x & 0.2 \leq x \leq 0.4, \\ 0 & 0.4 \leq x \leq 1. \end{cases}$$

$$\tilde{G}_8(x) = \begin{cases} 0 & 0.0 \leq x \leq 0.2, \\ 5x - 1 & 0.2 \leq x \leq 0.4, \\ 3 - 5x & 0.4 \leq x \leq 0.6, \\ 0 & 0.6 \leq x \leq 1. \end{cases}$$

$$\tilde{G}_9(x) = \begin{cases} 0 & 0.0 \leq x \leq 0.4, \\ 5x - 2 & 0.4 \leq x \leq 0.6, \\ 4 - 5x & 0.6 \leq x \leq 0.8, \\ 0 & 0.8 \leq x \leq 1. \end{cases}$$

$$\tilde{G}_{10}(x) = \begin{cases} 0 & 0.0 \leq x \leq 0.6, \\ 5x - 3 & 0.6 \leq x \leq 0.8, \\ 5 - 5x & 0.8 \leq x \leq 1. \end{cases}$$

$$\tilde{G}_{11}(x) = \begin{cases} 0 & 0.0 \leq x \leq 0.8, \\ 5x - 4 & 0.8 \leq x \leq 1. \end{cases}$$

The researchers use the weighted correlation coefficient to compute $\rho_D(\tilde{G}_6, \tilde{G}_i)$, $i = 7, 8, 9, 10, 11$. The result of comparison is summarized in Table 1. This example shows that $\rho_D(\tilde{G}_6, \tilde{G}_i)$ is decreasing in i , intuitively. But the methods of [21,22,23] have not decreasing behavior. For our method weighted function f interact on correlation value between two fuzzy numbers. Hence, there is reasonable advantage in using the proposed formula (5.16).

CONSLUSION

The correlation coefficients computed in this paper, which lie in $[-1, 1]$, give us more information than that the correlation coefficients computed by [21,22,23]. The correlation coefficients computed by us which show us not only the degree of the relationship between the intuitionistic fuzzy sets, but also the fact that these two sets are positive or negative related, which are better than the correlation coefficients from other methods, they review only the strength of the relation.

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