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Application of Genetic Algorithm for Dimensional and Tolerance Synthesis

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ABSTRACT

Dimensional and tolerance synthesis is a key part of process planning activity that has a profound effect on the product cost and quality. This research presents a method for synthesis of dimensions, discrete tolerances, and machine process assignment for each machining operation of the given part(s) that minimizes the expected production cost. The minimization problem was formulated as a non-linear mixed integer-programming problem and solved using a Genetic Algorithm (GA), employing a non stationery penalty function. The GA coding employed generates feasible tolerances and machine process assignment for each of the machining operations of the part(s). Optimal machining dimensions of a part, given the associated tolerances, were determined using scaled sphere fitting method. Implementation of the proposed method on a specific example brought out the ability of the proposed method to obtain better quality solutions compared to conventional methods, and the potential to obtain a near optimal solution.

Key Words: process planning, Genetic Algorithm, Tolerance Synthesis.

INTRODUCTION

Process planning is the link between the product design stage and product manufacturing. The process planning activity includes interpretation of design data, selection and sequencing of operation to manufacture the part, dimensional and tolerance Synthesis, selection of machines and cutting tools, determination of cutting parameters, choice of jigs and fixtures and calculation of machining times and costs. Design information from part blue print and information on shop floor resources available for production are the inputs to these tasks. Dimensional and tolerance synthesis is a key activity in process planning that has a profound effect on product cost and quality. Hence, it is natural to optimize the dimensions and their tolerances towards a measure of cost and/or quality. Dimensional and tolerance synthesis in process planning deals with determining the exact machining dimensions and their associated tolerances to meet the blueprint specifications on dimensions and assembly requirements**.** Most of the work published in recent years is in the area of tolerancing with emphasis on both tolerance analysis, using computer simulation models, and in tolerance synthesis, using cost optimization techniques.

There are two basic processes in tolerance design: tolerance analysis, and tolerance synthesis. Tolerance analysis is concerned with the calculation of the final tolerance that is specified for a particular component. Tolerance synthesis involves the allocation of specified assembly tolerances among the component dimensions of an assembly to ensure a specified yield. Tolerance allocation is concerned with minimizing the total manufacturing cost through the allocations of the respective component tolerances.

Bennet and Gupta [1] synthesized tolerances by minimizing the sum of the residual tolerance after stack up in each design constraint. Szadkowski [2] used a graphical representation of the feasible machining processes at each stage of the process plan, and used Bellman's optimum principle to determine the economical path. Irani et al. [3] presented a comprehensive tolerance chart optimization model. Optimal tolerance allocation among the machining dimensions was achieved using linear programming; while, a mixed integer programming model was used to incorporate linear cost-tolerance functions and alternative process selection.

However, there was no discussion on how the working dimensions can be calculated, and the approach was also computationally intensive. Ngoi and Ong [4] used a relationship matrix and a special path-tracing technique to formulate the working dimensions into a system of linear equations. In a later paper, they integrated tolerancing into the model, and solved the model using a linear programming approach. Dimensional synthesis obtained using their approach was feasible but not optimal.

Lee and Johnson [5] developed a stochastic constraint-optimization model for optimal tolerance allotment with manufacturing cost as the objective function. The tolerance allocation was constrained by an initially specified minimal acceptable yield, spec-yield. The problem was solved using a genetic algorithm and a truncated Monte Carlo simulation. Zhang and Wang [6] proposed a simultaneous optimization model for design and machining tolerances with process selection, using simulated annealing. Anselmetti and Bourdet [7] developed a dimensioning and tolerancing scheme to include manufacturing constraints. Their algorithm was well adapted to consider unilateral conditions extracted from the functional dimensioning and from the machining requirements. Zhang and Wang [8] formulated the tolerance allocation problem as discrete and continuous optimization problem, and solved them using simulated annealing and sequential quadratic programming. Based on the results for numerous test problems they obtained corroborative evidence to show that SA performs better than SQP for problems with wide process limits and overlapping cost curves.

As seen from the preceding review, researchers have treated dimensional synthesis and tolerance synthesis in process planning as separate activities. Optimal machining dimensions were determined based on the design data and tolerance chart [4]. The vector of synthesized machining dimensions is called as the set point. Tolerance synthesis was carried out subsequently by optimizing an objective that is a function of tolerances [8]. Thus, optimal machining tolerances were synthesized separately from machining dimensions. Such an approach would be useful if the optimization formulation for tolerance synthesis is independent of set point, decoupled dimensional and tolerance synthesis problems. Cases in which the tolerance synthesis formulation was dependent on the set point by the way of the objective function [9], constraints [10], or both; the tolerance synthesis formulation was solved for a Fixed set point.

Usually, the set point made available from earlier dimensional synthesis is used as the Fixed set point and such methods are referred to as Fixed set point methods in this research.

The formulation approach of the Fixed set point method is approximate because it treats a coupled dimensional and tolerance synthesis problem as decoupled problems of dimensional synthesis and tolerance synthesis that are formulated and solved separately.

The resulting benefit is that the solution quality is good, because the decoupled problems of dimensional and tolerance synthesis could be solved pretty accurately using standard deterministic or stochastic optimization techniques. Nevertheless, the dimensions and tolerances synthesized are suboptimal due to an approximate formulation.

The objective of this research is to synthesize dimensions and tolerances by formulating and solving a single coupled optimization problem. Machine-Process assignments for each of the machining operations are considered, based on an earlier approach by Anand [11]. The rest of the research report is organized as follows. Section 2 deals with key issues in model development and presents a formal statement of the model. Section 3 presents the solution procedure and Section 4 concerns with model implementation on one specific examples and discussion of the results obtained from implementation. Finally, Section 5 concludes the report with a summary of findings and conclusions resulting from this research work.

MATERIALS AND METHODS

Blueprint drawing of a part specifies the nominal design dimensions of components, their tolerances and the assembly constraints on the components that constitute the part. The shop floor consists of machines that are capable of single or multiple processes, and capable of realizing discrete tolerances within the range dictated by the process. The goal was to determine for each operation of every part, the machining dimensions, tolerances, and the machine-process assignment. In doing so, one should satisfy the blue print requirements, satisfy part demand, and minimize the total production cost. The key model assumptions are as follows:

1. Machining operations and their sequence for part manufacture are available.

2. For all processes, the distribution of dimensions machined with a specific tolerance follows a normal distribution-- nominal dimension as the mean and one third of the tolerance as the standard deviation.

3. Estimates of parameters, A, B, for cost-tolerance and time-tolerance models are available. A typical cost-tolerance and time-tolerance model is of the form:

$$A + B/\delta \quad (1)$$

A and B are constants specific to each process-machine combination and is the tolerance. Diplaris and Sfantsikopulos [12] have pointed out shortcomings of δ such a modeling. However, the problem formulation is not restrictive on the type of time–tolerance and cost-tolerance relationships assumed.

4. The realizable tolerances from processes are discrete, and lie within the tolerance range specific for that process.

In practice, individual yields of the operations are less than 100% (i.e. number of acceptable parts), and hence, the total part yield for a given process plan is less than 100%. Consequently, more number of parts than the demand has to be produced to satisfy the demand. That is, if a part demand is d_p, and the part yield of the corresponding process plan is e_p; the expected number of parts to be produced is d_p/e_p The production cost (sum of manufacturing cost and material cost) based on expected demand, d_p/e_p is the expected production cost-- henceforth referred as EPC. This is a realistic yield-based cost estimate compared to the cost estimate based on demand, d_p. EPC was used as the objective to be minimized and could be expressed as in equation 2.

$$EPC = \sum_{part} \frac{(\text{Manufacturing cost per part} + \text{Material cost per part}) \times d_p}{e_p} \quad (2)$$

The proposed objective function is a ratio of two sub functions, total production cost and the part yield. Based on our model assumptions, tolerances allocated influence both the production cost and the part yield; while, nominal dimensions influence only the part yield. This is obvious from the fact that the cost-tolerance model is independent of the associated nominal dimension. In essence, both tolerances and nominal dimensions influence the objective function.

The former through direct functional relationship (to manufacturing cost and hence production cost); while, the latter indirectly through the yield term. And dimensions that maximize the yield for a given set of tolerances optimize the objective function for that set of tolerances.

Cavalier and Lehtihet [13] proposed a heuristic procedure called scaled sphere fitting for dimensional synthesis (machining dimensions) that optimizes part yield given the process variability of all the machining operations. Scott et al. [14] report that scaled sphere fitting method performs as well as simulation based search for optimizing yields of parts manufactured with normally distributed process variations. From model assumption 2, feasible tolerance allocated to a particular machine–process combination is the measure of process variability. So, nominal machining dimensions that maximize part yield can be synthesized using the scaled sphere fitting method. A brief outline of the method is given in the following.

Suppose if m design constraints of a given part are expressed in terms of the n constituent machining dimensions x_j , j =1 to n, as

$$c_{\min} \le AX \le c_{\max} \quad (3)$$

Where, $X = [x_1, x_2, ..., x_n]^T$ is the column vector of machining dimensions. **A** is an $m \times n$ matrix called the design constraint matrix, row i of which expresses the way in which the design dimension, is expressed in terms of the machining dimensions, X. Let a_i is row i of matrix **A,** and δ_j is the tolerance allocated to the dimension x_j. Now, maximizing overall yield given the tolerances and the design constraints can be accomplished by solving the following linear programming problem.

Maximize r

Subject to**:**

$$a_i\ \mathbf{z} + r\| a_i \| \le c_{\max}(i) \quad i = 1,2\ldots,\text{n} \quad (4)$$

$$-a_i\ \mathbf{z} + r\| a_i \| \le -c_{\min}(i) \quad i = 1,2\ldots,\text{n} \quad (5)$$

$$\mathbf{z} = [z_1, z_2, ..., z_n] \text{ unrestricted} \quad (6)$$

$$r >= 0 \quad (7)$$

Where, $z_j = x_j / \delta_j$ is the scaling factor (8)

Let **z*** be the optimal solution for the above linear programming problem. The vector of optimal machining dimensions, **x*** can be obtained from **z*** using equation 9. Hence, the dimensions that maximize the yield for a given set of tolerances on those dimensions are obtained.

The following notations are used in the model development:

p: Part index

i: Component index

k: Machining operation index

j: Process index

m: Machine index

Following are the list of parameters and functions used:

k_{pi} : Last machining operation of component i of part p.

$\delta_j^{\min}, \delta_j^{\max}$: Minimum and maximum *max* tolerances for process j.

T_p : Assembly tolerance for part.

d_p : Demand for part p in a given time interval.

L_m : Availability of machine m in the above time interval.

s_k^p : Variation in stock removal for operation k of part p.

τ_{pkj} : Binary matrix specifying stage-process relationship for part p. 1, if operation k can be performed using process j. 0, otherwise

φ_{pjm} : Binary matrix specifying machine-process relationship for part *p*. 1, if machine *m* can perform process *j*. 0, otherwise

η_p : Unit material cost for part *p*.

A_P : Design Constraint matrix for part p.

n_P : Number of machining stages for part *p*.

$c_P^{\min}$: Column vector of minimum values of design dimensions for part *p*.

$c_P^{\max}$: Column vector of maximum values of design dimensions for part *p*.

$\alpha_{jm}(\delta)$: Cost for machining to tolerance δ using process *j* on machine *m*.

$\beta_{jm}(\delta)$: Time required to achieve tolerance δ, for machine *m*, using process *j*.

e_P : Yield of part *p*.

f_P : Feasible region of operational dimensions on part *p*.

u_P : Vector of feasible operation dimensions for part *p*.

$\phi_P(u_P)$: Multivariate normal probability density function of operation dimensions for part *p*.

The decision variables in model are:

X_P : $X_P = [x_1^P, x_2^P, \ldots, x_{n_p}^P]^T$, column vector of nominal machining dimensions for part *p*. Where, x_k^p is the nominal dimension of operation *k* of part *p*.

δ_P : $\delta_P = [\delta_1^p, \delta_2^p, \ldots, \delta_{n_p}^p]^T$, column vector of tolerances associated with nominal dimensions for part *p*. Where, δ_k^p is the tolerance associated with nominal dimension *k* of part *p*.

y_{pkjm} : Process-Machine selection variable for part *p*.

=1, if machine *m* and process *j* are selected for operation *k*.

=0, otherwise

Minimize Expected production cost:

$$\sum_p \{(d_p / e_p)[\sum_k \sum_j \sum_m \alpha_{jm}(\delta_k^p)) y_{pkjm} + \eta_p]\}$$

Where, $e_p = \int \phi(u_p) du_p$ (9)

$$u_p \in f_p$$

$$f_p = \{u_p : c_p^{\min} \leq A_p u_p \leq c_p^{\max}\}$$

Subject to:

Design Constraint:

$$c_p^{\min} \leq A_p X_p \leq c_p^{\max} \quad (10)$$

Design tolerance constraints expressed in terms of the constituent nominal machining dimensions.

Assembly constraint:

$$\sum_i \delta_{k_{pi}}^p \leq T_p \ldots \forall p \quad (11)$$

The sum of tolerances assigned to the last stage of every component of making up the assembly should be less than the assembly tolerance of the part.

Stock removal constraint:

$$\delta_k^p + \delta_{k-1}^p \leq s_k^p \ldots \forall (k,p) \quad (12)$$

For every machining stage, the sum of the tolerance in the present stage and that in the previous stage of machining should be less than the allowable variation in stock removal for that stage.

Process capability constraint:

$$\delta_j^{\min} y_{pkjm} \leq \delta_k^p y_{pkjm} \leq \delta_j^{\max} y_{pkjm} \ldots \forall (p,k,j,m) \quad (13)$$

If a process is being chosen to machine a certain component dimension, then the tolerance of that dimension must be within the band specified for the chosen process.

Process and machine feasibility constraint:

$$\tau_{pkj} \varphi_{pjm} y_{pkjm} = y_{pkjm} \ldots \forall (p,k,j,m) \quad (14)$$

The process-machine combination chosen for a particular machining operation should be feasible.

Process and machine uniqueness constraint:

$$\sum_j \sum_m \tau_{pkj} \varphi_{pjm} y_{pkjm} = 1 \ldots \forall (k,p) \quad (15)$$

The process-machine combination chosen for a particular machining operation should be unique.

Machine loading constraint:

$$\sum_p \sum_k \sum_j (w_p / e_p) \beta_{jm}(\delta_k^p) y_{kjm} \leq L_m \ldots \forall m \quad (16)$$

For every machine, the total of the time taken by all the machining operations done on that machine should be less than the machine availability.

Non-negativity constraints:

$$\delta_k^p \geq 0 \ldots \forall k \quad (17)$$

$$x_k^p \geq 0 \ldots \forall k \quad (18)$$

The stated formulation is a non-linear mixed integer discrete optimization problem. The highly non-linear nature of the objective function and the constraints coupled with a variety of decision variables preclude any attempt to solve this problem by a deterministic non-linear optimization technique. Most of the reported solution methodologies in literature for tolerance synthesis formulations make use of stochastic optimization techniques like genetic algorithms or simulated annealing. Genetic algorithms are well suited for searching complex search spaces quickly by virtue of their evolution and have the potential to find near optimal solutions. More importantly, they do not pose any restriction on the nature of the objective function (continuous, differentiable, unimodal etc.) and are zero order methods, i.e. based only on objective function evaluation.

For each operation of a part, there is a set of feasible process-machine assignments. These feasible assignments are coded as integers and stored in an array. And similarly, the set of discrete tolerances for each process-machine combination is coded as an integer and stored in another array. The solution sub string for each part is represented as string of integers; one half containing process-machine assignment information for each operation and the other half containing the tolerance information for that particular operation based on the coding. Thus, for a part with *n* machining operations the solution string length is 2*n*. And if there are *p* parts the string length is 2*pn*. The complete solution string is composed of the sub strings for each part. While process capability, process-machine feasibility and process-machine uniqueness constraint are implicitly built into the GA string coding; rest of the constraints are handled using a non-stationery penalty function. Here, the penalty for infeasibility is based on the sum of constraint violations and the generation number in which the infeasible solution is encountered in the evolution.

In order to evaluate the fitness, EPC, of each string one has to estimate the set point and the yield corresponding to the tolerances coded in the string. Optimal set point is obtained using the scaled sphere fitting method. Part yield given the set point and tolerances was estimated using First order second Moment Method [15]. In a nutshell, first the Genetic algorithm generates feasible tolerance vectors with machine process assignment; set point and yield are computed next; fitness of the strings, EPC is then evaluated and the Genetic algorithm evolves. The overall schematic of the solution procedure is:

Step1: Start with a population of strings

Step2: Cull out Sub strings containing tolerance and assignment for each part

Step3: Determining tolerances and set point for each part (using scaled sphere fitting)

Step4: Estimate yield e for each part from set point using First order second moment method (FOSMM).

Step5: If solution is feasible Penalty=0 else Calculate penalty based on sum of violated constraints and generation number

Step6: Fitness value = Objective function value +Penalty

Step7: Consolidate sub strings and the fitness value

Step 8: Create new population using GA evolution and go to step1

The first order second moment method is a convenient approximation scheme for the expected part yield when the variability of operations is normally distributed. The design constraint system for part *p,* given by equation 12 can be written as in equations 21 and 22.

$$\mathrm{a}_i X_p - c_p^{\min}(i) \geq 0 \qquad i = 1,2,...,n_p \tag{20}$$

$$-\mathrm{a}_i X_p + c_p^{\max}(i) \geq 0 \qquad i = 1,2,...,n_p \tag{21}$$

Where, a_i, X_p and n_p have their earlier developed meanings. The above system of inequalities

can be written concisely as,

$$h_i(x) >= 0 \qquad i = 1,2,...,n_p \tag{22}$$

The yield can then be expressed as,

$$\mathrm{Yield} = 1 - \Pr\{\rho\} = 1 - pr\{\overset{2n_p}{\underset{i=1}{Y}} \rho_i\} \tag{23}$$

Where, Pr{ ρ } is the probability of a defective part, and ρ_i is the event that the constraint $\mathrm{h_i(x)} >= 0$ is violated i.e. $\mathrm{h_i(x)} < 0$. A conservative approximation for the yield expression is given by equation 24.

$$Yield = 1 - \sum_{i=1}^{2n_p} pr\{\rho_i\} + \sum_{i=2\, j<i}^{2n_p} \max(pr\{\rho_i \cap \rho_j\}) \tag{24}$$

RESULTS AND DISCUSSIONS

We implement the proposed model on one sample part for dimensional and tolerance synthesis. The inputs to the problem, as discussed earlier, are part drawings with dimensional constraints on the sizes, the machining stages, and sequence of operations through which the part is manufactured. The sample part shown in Figure 1 is a machined casting with 6 design dimensions (D_1 to D_6) and their tolerances. The design constraints can be represented as in equation 26.

$$\begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{pmatrix} = \begin{pmatrix} 3.00 \pm 0.05 \\ 15.00 \pm 0.06 \\ 10.00 \pm 0.05 \\ 5.00 \pm 0.04 \\ 29.00 \pm 0.10 \\ 28.00 \pm 0.10 \end{pmatrix} \tag{25}$$

The process plan involves 6 machining stages (x_1 to x_5) in the machining sequence shown in bottom part of figure 1. The first machining operation, x_1 makes use of two surfaces, 1(datum surface) and 2(machined surface). The datum surface (surface 1) is placed above the horizontal line and the machined surface (surface 2) is placed below. In a similar fashion the rest of the operations are depicted in the correct sequence. The chart obtained from such a representation of processes is called a relational chart.

Next, the design constraints are written down in terms of the machining dimensions through a path tracing approach. For example, design dimension D_3 is bounded by surfaces 2 and 4. To obtain the constituent machining dimensions, move from surface 2 to surface 4 in the relational chart. Using appropriate sign conventions for the directions can be written as in equation 26.

$$D_3 = x_2 - x_4 \tag{26}$$

Continuing, the design constraint matrix, design constraints in terms of the machining dimensions can be constructed as shown in equation 27.

Most of the parameters used in the formulation like φ_{pjm}, τ_{pkj}, d_p, L_m, etc are conventional data used in shop floor management. On the other hand, parameters A and B used in the time-tolerance and cost-tolerance models can be obtained from the past shop floor data. Other data such as tolerance range for processes, $\delta_j^{\min}, \delta_j^{\max}$ and stock removal value, s_k^p are readily obtainable from data books.

$$\begin{pmatrix} 2.95 \\ 14.96 \\ 9.95 \\ 4.96 \\ 28.90 \\ 28.00 \end{pmatrix} \le \begin{pmatrix} 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \le \begin{pmatrix} 3.05 \\ 15.06 \\ 10.05 \\ 5.04 \\ 29.10 \\ 28.10 \end{pmatrix}$$

(27)

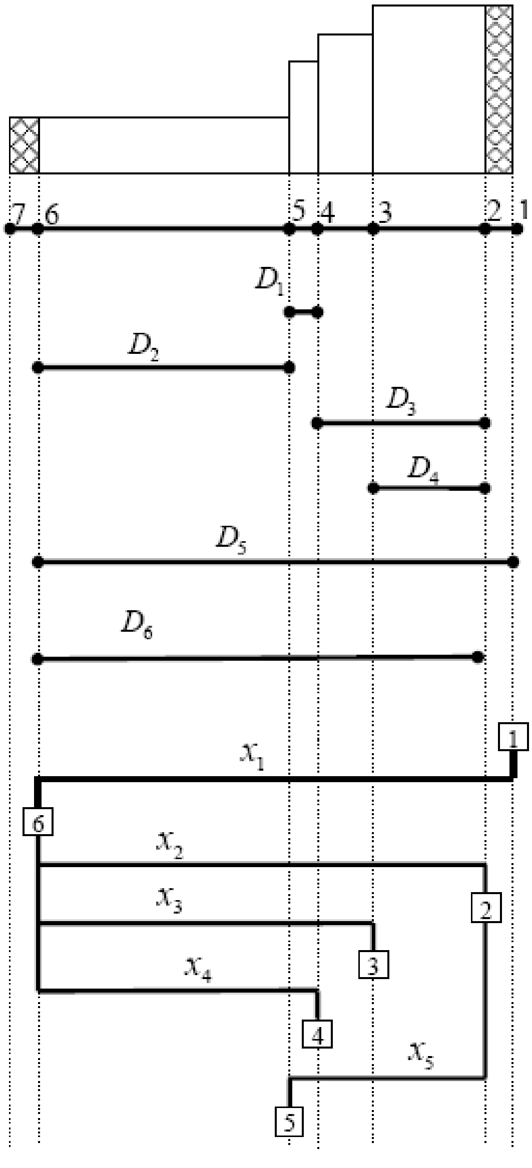
Number of processes available, number of Machines available and Part Demand are 4, 6 and 10 respectively. Discrete tolerances realizable from processes are as fallow:

Process 1: [0.001 0.002 0.003 0.004]

Process 2: [0.005 0.007 0.010]

Process 3: [0.01 0.03 0.05]

Process 4: [0.05 0.06 0.07 0.10]



Stage-Process relationship matrix is,

$$\tau = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Machine-Process relationship matrix is,

$$\varphi = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Machine Availability matrix is (in time units) is,

L = (20 50 50 40 50 100)

The operators and parameters for genetic algorithm are presented in table 1

Figure1. Machined-Casting

The GA procedure converged to a feasible solution consistently. The best solution returned by the GA over various trials is summarized in table 2 and the performance of the GA is shown in figure 2. The obtained results for the production cost and the estimated yield were compared with the fixed set point method, in which dimensions were synthesized independent of machining tolerances. The conventional method of synthesizing dimensions from tolerance chart using tolerance chains fared very poorly with less than 50% yield. As a result, this method was not included for comparison.

Table 1. Operators and Parameters of GA

Operator/Parameter	Value/Selection
No of generations	25
Population size	100
Constraint handling	Non-stationary penalty function
Crossover operator	Simple crossover
Percentage of crossovers	90
Mutation operator	Multi-non-uniform mutation
Percentage of mutations	15
Selection function	Normalized geometric ranking
Probability of best individual	0.08

Table 2. Best Solution: Proposed Method

x_i	Nominal value	Tolerance	Process	M/c
1	29.0082	0.06	3	6
2	28.0439	0.02	3	6
3	23.0239	0.02	3	6
4	18.0256	0.04	2	6
5	13.0039	0.02	3	6

The proposed method was able to return feasible solutions consistently within realistic computational time. Implementation on a PC with 2000MHz Pentium V processor took about 1000 CPU seconds.

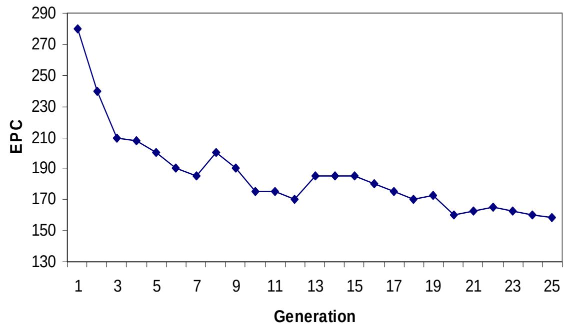


Fig. 2. Example of the performance of the proposed GA

An attempt was made to qualify the EPC of the obtained solution in terms of the lower bound for the EPC of the optimal solution. A conservative lower bound for the optimal EPC can be calculated as follows. For each machining dimension, find out the least cost process-machine combination and then allocate the maximum possible tolerance to that dimension. Thus, each dimension is manufactured most economically as possible, and if one assumes the corresponding yield as 100%, the obtained EPC is the conservative lower bound.

The important assumption in this lower bound estimation is that the least cost process machine combination for a particular machining dimension allows maximum tolerance allocation to that dimension compared to the others. This assumption is generally true due to the monotonicity of cost-tolerance relationship.

CONCLUSIONS

A procedure for simultaneous synthesis of dimensions and tolerances in process planning was outlined. Process yield was addressed and the shop floor restrictions were given adequate treatment. Discretization of the tolerances as dictated by the process machine combination makes the formulation realistic.

The model provides scope to synergize the effect of dimensions and their associated tolerances in minimizing the expected production cost. Comparatively better performance of the proposed method over fixed set point methods was seen in both cases. In example, higher EPC over fixed set point methods was obtained at the cost of slightly less yield. More importantly, the solution procedure was able to maintain consistent solution quality over repeated runs.

Fixed set point methods synthesize dimensions based on the feasible region dictated by the system of equations *min max* $\mathbf{c} \leq \mathbf{Ax} \leq \mathbf{c}$. Trials with test cases have shown that for certain feasible regions (e.g. a hypercube) the optimal set point obtained by scaled sphere fitting method for any random tolerance vector is the same.

Such optimal set points can be called as invariant set points, since they are invariant with respect to tolerance vector. And such invariant set points, if present, are obtainable from sphere fitting and tolerance chart methods. In such cases, the set points are no longer a function of tolerances and so the distinction between the proposed method and fixed set point methods fades away and simultaneous treatment of tolerances and dimensions is no longer necessary. Hence, it is useful to check for the presence of invariant set points in the feasible region before using the proposed method. This is can be done by generating random tolerance vectors and checking whether the set point remains the same. On the other hand, feasible regions where the set point is greatly influenced by the tolerance vector (e.g. highly skewed feasible regions) the distinction becomes significant necessitating a simultaneous treatment of tolerances and dimensions. In such cases, the results obtained by the two methods (proposed and fixed set point) are expected to be considerably different.

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