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Investigation of Symmetry Properties in Deformed Light Even-Even Nuclei

Savaş SÖNMEZOĞLU[1] Şeref OKUDUCU[2] Erhan ESER[1]

[1] Gaziosmanpaşa University, Faculty of Arts and Science, Department of Physics, 60250 Tokat, Turkey
[2*] Gazi University, Faculty of Arts and Science, Department of Physics, 06500 Ankara, Turkey

*Corresponding Author
e-mail: syssonmezoglu@gop.edu.tr

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Abstract

In this study we have investigated the systematic structures and $R_{4/2}$ energy ratio of the yrast bands for deformed light even-even nuclei from Z=20 to 34 by using of obtained simple and validity analytical relation in terms of K-quantum number which is the projection of the total angular momentum on the axis of symmetry of a nucleus, and also the present calculation results have been compared with the corresponding compiled results and are in agreement quite well with the compiled results.

Key words: Nuclear energy levels, symmetry, deformed light even-even nuclei.

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INTRODUCTION

The concept of symmetry, which is a fundamental physical quantity in gaining on understanding of the physical laws governing the behavior of nuclei especially in the applied nuclear physics, has played an increasingly important role in physics, in particular 20th century with the development of quantum mechanics and quantum field theory [1-11]. Although the lowest levels of even-even nuclei are well-known signatures of systematic structure and symmetry, of course, not new, there has been little study of their energy relationship over the entire nuclear chart. During the past few years, many scientists have been proposed to solve this problem in the framework of different models such as interacting boson model (IBM) [12-34] one of the most used to describe the symmetry properties, variable moment of inertia (VMI) model [35-37], and the others [38-46]. Most of these studies concern the ground band (yrast band) of the even-even nuclei from Z=38 to 82 and the ratio $R_{4/2}=E(4_1^+)/E(2_1^+)$ defined from $\sim 1.2-1.6$ near doubly magic nuclei to $\sim 2.0-2.2$ in vibrational nuclei, 2.5-3.0 in transitional species, and ~ 3.3 in well-deformed symmetric rotor nuclei. In addition, symmetry has usually a central role in collective motion of the nucleons which may be described as a vibrational motion about equilibrium position and a rotational motion that maintains the deformed shape of nucleus. The existence of collective energy level bands of rotational and vibrational types can now easily be identified from nuclear spectra data [47] of many deformed light even-even nuclei. Thus, as it is well known, deformed nuclei have rotational energy spectrum due to their collective motion [1, 2]. These energy spectra generally are defined by the total angular momentum I, parity π and the quantum number K which is the projection of the angular momentum on the intrinsic coordinate axis of the nuclei. Here, K which is the conserved quantum number for the nuclei having axial symmetry could take values $K = I, I-1, ..., -I$. Indeed, K which is defined the angular momentum of the nuclei depending on the attached coordinate system, has a definite and constant value for a specified intrinsic state of any deformed nuclei. Intrinsic energy E_k of slowly rotating nuclei depends on K quantum number and it forms the base of the rotational energy band, and the energy eigenvalues of such a band could be defined as,

$$E(I,K)=E_K+E_{rot}(I) \tag{1}$$

where $E_{rot}(I)$ is the rotational energy.

The aim of this paper is to obtain the systematic structures and $R_{4/2}$ energy ratio of the yrast bands from experimental data and its applications to deformed light even-even nuclei. Many deformed light even-even nuclei, especially with mass ranging from Z=20 to 34, have stable deformation in their ground-states [48]. Such nuclei may rotate due to interactions with an external incident particle or emitting the particle.

OUTLINE OF ANALYTIC FORMULA

Rotational energy of an axially symmetric deformed even-even nucleus is given as [2],

$$\mathrm{E}_{rot}(I,K)=\frac{\hbar^2}{2}\left[\frac{I(I+1)}{J_0}+\left(\frac{1}{J_3}-\frac{1}{J_0}\right)K^2\right] \tag{2}$$

where I and K are the total angular momentum and its projection on the axis of symmetry, respectively, of a nucleus; J_3 and J_0 are the principal moments of inertia about a symmetry axis and an arbitrary axis perpendicular to the symmetry axis, respectively. Authors of Ref. [2] have used the hydrodynamic moments of inertia restricting the deformed nuclear surface by a quadropole term only and so these nuclei are, on the average, symmetric: that is $J_3 = 0$. Therefore, Eq. (2) will be meaningful only if the value of K is taken identically zero. This yields what is often known as the ground state rotational band, and then we come to the following rotational energy equation:

$$\mathrm{E}_{rot} = \frac{\hbar^2}{2J_0} I\left(I+1\right), \qquad K=0 \tag{3}$$

The above expression is in good agreement with the observed low-lying energy levels of the even–even deformed nuclei, which is the values of angular momentum I, I = 0, 2, 4, 6,…. As mentioned above the energy level sequence in such a case is called as ground-state rotational band having positive-parity.

More generally, the rotational energy for the ground state bands of even-even nuclei, could be written as an infinite power series,

$$E_{rot}\left(I\right)= AI\left(I+1\right)+BI^2\left(I+1\right)^2+CI^3\left(I+1\right)^3+DI^4\left(I+1\right)^4+.... \tag{4}$$

where A is the well-known rotational constant parameter for sufficiently small values of I, and B, C, D,…, are the corresponding higher-order constant parameters [49, 50].

In the view of above-mentioned, it seems that the ground state energy bands of deformed even-even nucleus have zero quantum number ($K=0$), together with even parity and even nuclear angular momentum. Their energy eigenvalues could be defined by even number starting from zero, as 0^+, 2^+, 4^+, 6^+,…. We could obtain an energy ratio, $E(I^+)/E(2^+)$, to define the energy eigenvalues if we use the equation given by Ref. (35), such that,

$$R\left(I\right)=\frac{E\left(I^+\right)}{E\left(2^+\right)}=\left[\frac{I\left(I+1\right)}{6}\right]^{\frac{2}{3}} \tag{5}$$

In Eq. (5), $E(2^+)$ represents the energy of the first excited state and could be set as the unit energy for the even-even nuclei. Using Eq. (5) one can be written in the following form,

$$R(2^+):R(4^+):R(6^+):R(8^+):R(10^+):...=1:2\frac{1}{5}:3\frac{3}{5}:5\frac{1}{4}:7:... \tag{6}$$

However, the real values are a little different from the above ratios in Eq. (6) and these differences are getting smaller and smaller starting from $I=8$. The deviation of the rotational energy from $I(I+1)$ has resulted from the change as a dependence of the variable moment of inertia, and the dependence of the rotational energy to some other factors [1-3, 35-39]. Since we will only concentrate on the symmetry properties and $R_{4/2}$ energy ratio of the ground state (yrast) energy bands, in this study, the natures and other details of the energy bands of interest are not emphasized. On the other hand, we can remark that the energy eigenvalues of the ground states could be given by Eq. (4) more precisely for the various deformed light even-even nuclei and their values could be extracted from the experimental data [47]. If we represent $R(4^+)/R(2^+)= r$ in order to find out symmetry properties of the yrast energy bands instead of $R(4^+)/R(2^+)$, in consideration of the measured energy eigenvalues [47] we get an approximated equation,

$$R(2^+):R(4^+):R(6^+):R(8^+):...\approx 1:r:2r:3r:... \tag{7}$$

Instead of Eq. (6), if we consider the experimental data, we should emphasize once more here, that the last approximated equation gives more accurate results than the previous one Eq. (6). The last expression is clearly defines the "equal intervals" and illustrates the "symmetries" in the energy bands. If we rewrite Eq. (7) starting from the ratio $R(4^+)$ then we get the ratio of integers as,

$$R(4^+):R(6^+):R(8^+):...\approx r:2r:3r:... \tag{8}$$

An example of such a symmetry is obtained using the ratio $R(I)=E(I^+)/E(2^+)$ and starting from $I=4$, presented in Fig. 1 for the yrast bands in the some deformed even-even light nuclei.

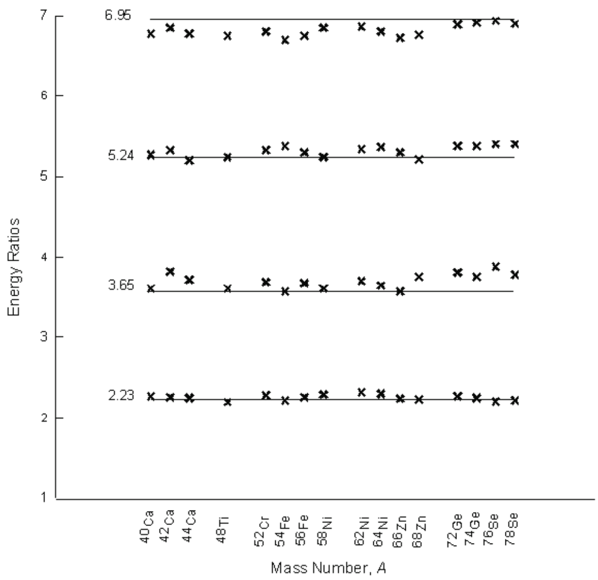

Figure 1. Mass dependence of energy ratios for the yrast (ground state) bands of deformed light even-even nuclei. The horizontal dashed lines indicate the values given by the I (I+1) rule.

We may choose an energy unit $E_1(I_1^\pi)-E_0(I_0^\pi)$ and consider the related ratios to define the mentioned symmetry for the excited bands of the even-even nuclei. Here, $E_0(I_0^\pi)$ is the energy of the lowest state having I_0 *spin* and π parity, and $E_1(I_1^\pi)$ is the energy of the first excited state above the $E_0(I_0^\pi)$ having suitable I and π quantum numbers. In the calculations of the energy ratio for excited bands, instead of Eq. 5, one could use, in general,

$$R_n=\frac{E_n\left(I_n^\pi\right)-E_0\left(I_0^\pi\right)}{E_1\left(I_1^\pi\right)-E_0\left(I_0^\pi\right)} \tag{9}$$

where $E_n\left(I_n^{\pi}\right)$ is the energy of the n'th excited state above the $E_0^n\left(I_0^{\pi}\right)$ having suitable I and π quantum numbers. In that case using the experimental data [47] we get,

$$R_2 : R_3 : R_4 : \ldots \approx r : 2r : 3r : \ldots : (n-1)r \tag{10}$$

where,

$$r = \frac{E_2\left(I_2^{\pi}\right) - E_0\left(I_0^{\pi}\right)}{E_1\left(I_1^{\pi}\right) - E_0\left(I_0^{\pi}\right)} \tag{11}$$

In Eq.(11), the values of energy ratio, r, have changed between 2.20 and 2.32 for yrast bands of deformed light even-even nuclei.

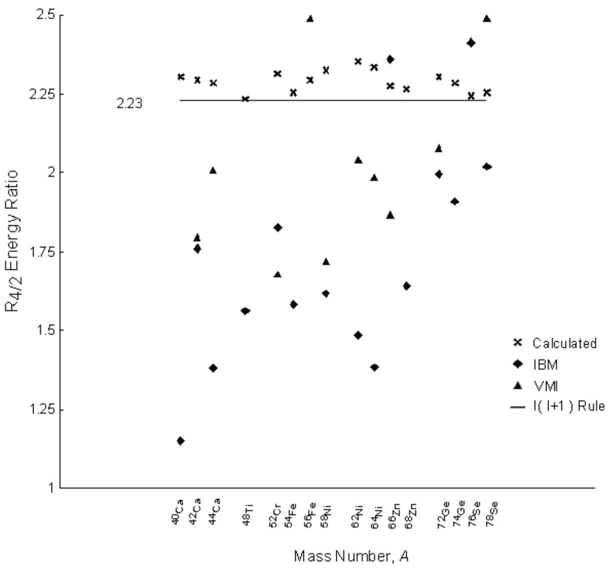

Figure 2. Mass dependence of energy ratios $R_{4/2} = E_4/E_2$, ratio of energy of the first 4^+ state to the energy of the first 2^+ state, for deformed light even-*A* nuclei. The horizontal line at the top indicates the values given by the I (I+1) rule. The values of energy ratios E_4/E_2 are calculated for the ground state bands, and compiled values of energy ratios E_4/E_2 are taken from interacting boson model (IBM) Ref. [33] and variable moment of inertia (VMI) model Ref. [35].

RESULTS AND DISCUSSION

In the present study we have seen that mass dependence of energy ratios for the ground state rotational bands in deformed light even-even nuclei approximately satisfies Eq. (11) in Fig. 1 and Fig. 2. From Fig. 1, we have seen that the obtained energy ratios calculated by Eq. (11) demonstrate a well consistency with Eq. (10) and also show the systematic structure of energy levels for the yrast band in deformed even-even nuclei from Z=20 to 34. Indeed, in Fig. 2, we have seen that the our calculated energy ratios, $2.20 \le R_{4/2} = E_4 / E_2 \le 2.32$, denote a well consistency with I(I+1) rule, values of calculated Eq. (5), respect to interacting boson model [33] and variable moment of inertia (VMI) model [35]. Thus, these results obtained by Eq. (11) almost represent the empirical data values for all energies quite well for the nuclei of interest. This situation illustrates almost equidistant rotation nature of the lower spectra of the nuclei considered. Therefore, the present study we shall consider the rotational excitation modes of almost equidistant form to calculate the systematic structure and $R_{4/2}$ ratio of energy levels.

CONCLUSION

In the view of presented study, we can conclude that the systematic can be used as means of predicting the whole low-energy collective level scheme of even-even nuclei of interest on the basis of a minimum of information (e.g., a few energy ratios). Finally, we remark that this property of the energy spectra is very important since the possibility of the use by the collective modes in the identification of the physical characteristic such as level density parameters in the applied nuclear physics.

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