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Multiple Signal Classification Algorithms for Evaluation of Coherent and Non-Coherent Signals

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Abstract

Multiple Signal Classification (MUSIC) algorithm is a tool to estimate DOA of incoherent signals. Spatial smoothing of MUSIC has been introduced to estimate DOA of coherent and completely correlated signals. Previous researches simulated and evaluated the performance of conventional and Spatially Smoothed versions of MUSIC algorithm for special cases. In this investigation in addition considering AWGN the effect of multipath fading in different SNRs, number of sensors, number of signal samples (snapshots), distance between array elements and number of sub-arrays for MUSIC and smoothed version has been considered. Simulations have been done for two cases, two completely coherent paths as case one and three paths with two completely coherent paths and one incoherent as second one. Simulation results show high performance of MUSIC algorithm to estimate incoherent signals. Coherent signals will be estimated via spatially MUSIC algorithm as well as incoherent signals in MUSIC algorithm.

Keywords: Music Algorithms, Fading, Coherent signals.

INTRODUCTION

The MUSIC algorithm that presented first time by [1-3] has been used more in recent years. MUSIC algorithm can only be used to estimate incoherent signal sources. For coherent signals, the performance of MUSIC will be degraded and it is not an efficient tool. Recent studies shows MUSIC can approximately provide a solution of multipath fading, Also multipath fading causes similar receiving signals in chip length in some systems like CDMA [4-6]. $a(\theta_m)$ is the array response (or steering) vector, corresponding to the DOA of the m^{th} signal, and is defined as :

$$a(\theta_m) = \left[\exp(-j(n-1)2\pi(\frac{d}{\lambda})\sin(\theta_m)) \right]^T, 1 \le n \le N \quad (1)$$

Where T is the transpose operator, and λ is the wavelength of the incident signals. The combination of all possible steering vectors forms the array manifold matrix A. The MUSIC algorithm starts by applying temporal averaging over K snapshots (or samples) taken from the leads to forming a spatial correlation (or covariance) matrix R defined as:

$$R = \frac{1}{K}\sum_{t=1}^{K} x(t)x^H(t) \quad (2)$$

Where ^{H} denotes the Hermitian operator. Substituting x (t) from (1) into (2) results in:

$$R = \frac{1}{K}\sum_{t=1}^{K} A(\theta)s(t)s(t)^H A(\theta)^H + n(t)n(t)^H, \quad (3)$$

$$R = A\, R_{SS} A^H + \sigma_n^2 I \quad (4)$$

Where R_{SS} the signal covariance matrix is σ_n^2 is the noise variance and I is an identity matrix of size $N \times N$.

In the MUSIC algorithm, the eigenvectors E_n of matrix R that correspond to the smallest Eigen-values from the noise subspace. The eigenvectors E_n and the steering vectors that make up matrix A are used to form the MUSIC angular spectrum which is given by:

$$P(\theta) = \frac{A(\theta)^H A(\theta)}{A(\theta)^H E_n E_n^H A(\theta)} \quad (5)$$

The orthogonality between the eigenvectors gives rise to peaks in the MUSIC angular spectrum. These peaks correspond to the directions of arrival of the signals impinging on the sensor array. It is well known that with an N-element sensor array, MUSIC can detect up to N-1 uncorrelated signals [7-9]. This paper is structured as follows. Section II, has a brief overview on MUSIC and Spatially smoothed MUSIC and their models. Section III describes Input data and simulation assumption and simulation results of MUSIC and spatially smoothed version in two coherent signals and other including three signals that one of them is incoherent. In addition simulation results as figures, some table can be found that shows percentage of successful iterations and average as well as variance of DOA estimation of two algorithms. Section IV concludes this investigation.

SPATIALLY SMOOTHED MUSIC ALGORITHM

The signal covariance matrix R_{SS} is a full-rank matrix (i.e., non-singular) as long as the incident signals on the sensor array are uncorrelated, which is the key to the MUSIC Eigen-values decomposition. However, if the incident signals become highly correlated, which is a realistic assumption in practical radio environments, matrix R_{SS} will lose its non-singularity property and, consequently, the performance of MUSIC will degrade severely. In this case, spatial smoothing (SS) must be used to remove the correlation between the incident signals by dividing the main sensor array into forward/backward overlapping sub-arrays and introducing phase shifts between these sub-arrays. The vector of the received signals at the k^{th} forward sub-array is expressed as:

$$x_k^F(t) = A\,D^{(k-1)}s(t) + n_K(t) \tag{6}$$

Where (k-1) denotes the k^{th} power of the diagonal matrix D given by:

$$D = diag\left\{ e^{-j\frac{2\pi}{\lambda}\sin\theta_1}, \ldots, e^{-j\frac{2\pi}{\lambda}\sin\theta_m} \right\}. \tag{7}$$

The spatial correlation matrix R of the sensor array is then defined as the sample mean of the covariance matrices of the forward sub-arrays:

$$R = \frac{1}{L}\sum_{k=0}^{L-1} R_k^F \tag{8}$$

Here L is the number of overlapping sub-arrays. When applying FSS, the N-elements sensor array can detect up to N/2 correlated signals [7].

SIMULATION RESULTS

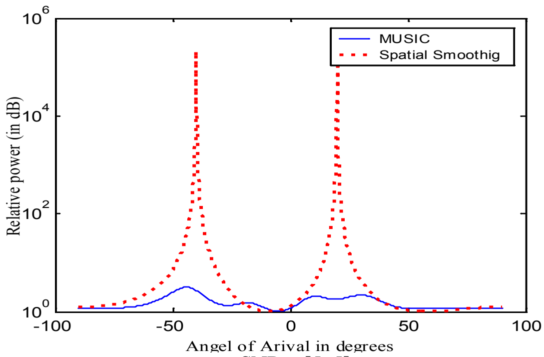
At first two coherent signals with different amplitude, impinge on the array from two paths with different powers has been simulated. If delay time between two signals less than a chip length then two completely coherent signals will be made. In this state the incident signal covariance matrix is not full-rank matrix, hence the performance of MUSIC algorithm will degrade severely and it cannot estimate the DOA. For this case spatially smoothed algorithm will be suggested. It is assumed that the signals and noise are stationary and ergodic complex-valued random processes with zero mean. In addition, the noise and signals are uncorrelated. In all simulations have been considered:

1- The number of samples (snapshots) is 1000.

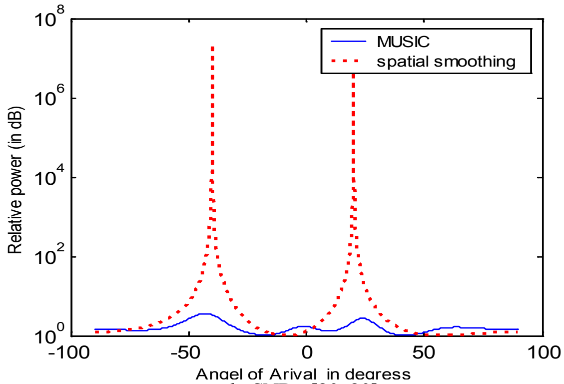
2- Two uncorrelated signals with same and different power incident at angels $-40^\circ, 20^\circ$ for coherent ones and 50° for incoherent one.

3- Appropriated iterations are 500 or 1000 times.

Figures 1 to 4 shows the simulation results of MUSIC and spatially smoothed version for two completely coherent users. In these figures effect of variations such as number of array elements and their spaces, number of snapshots and SNR are presented.

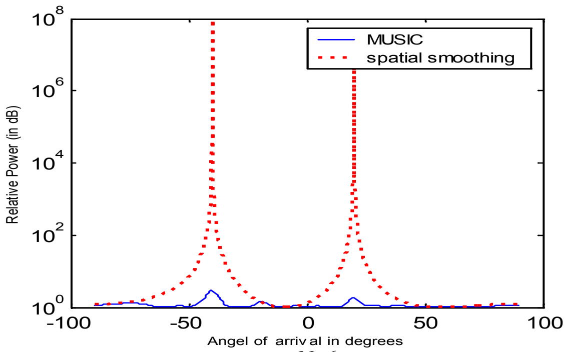


a. SNR = [5, 5]

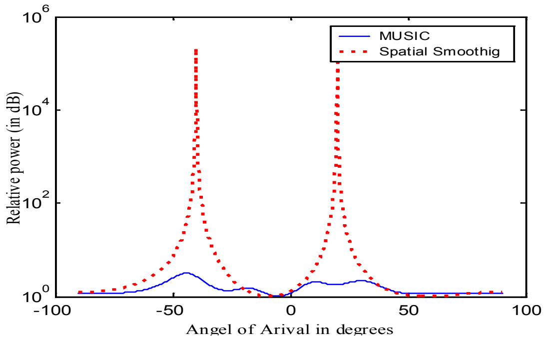


b. SNR= [20, 20]

Fig.1 DOA Estimation with Two Coherent Signals Belong to $-40^\circ, 20^\circ$, 100 Snapshots, N= 6 and L=4

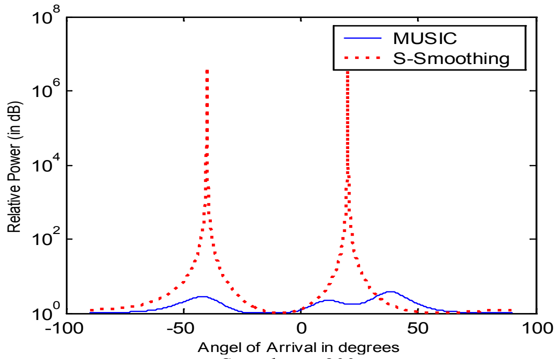


a. N=6

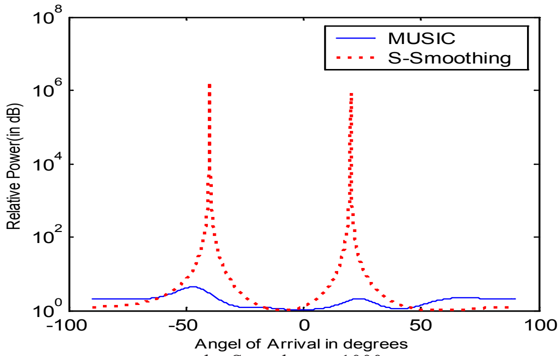


b. N=16

Fig.2 DOA Estimation with Two Coherent Signals from $-40^\circ, 20^\circ$, 100 Snapshots, SNR= [5, 5] and L=4

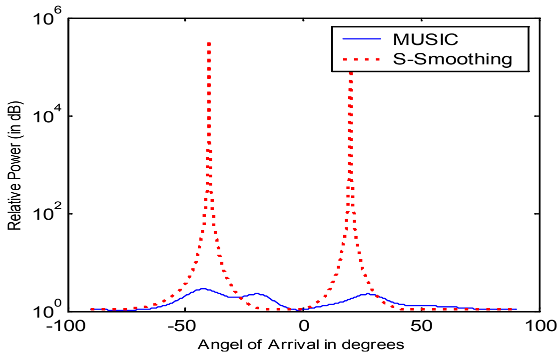


a. Snapshots= 200

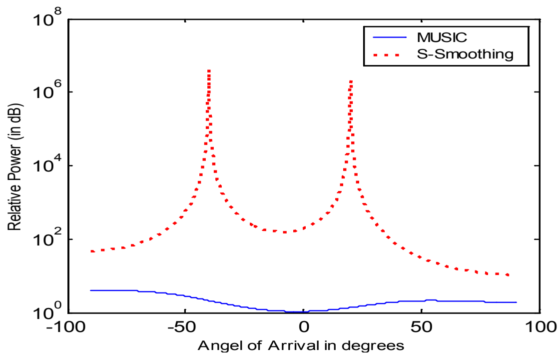


b. Snapshots = 1000

Fig.3 DOA Estimation with Two Coherent signals from $-40^{\circ}, 20^{\circ}$, N=6, SNR= [5, 5] and L=4.

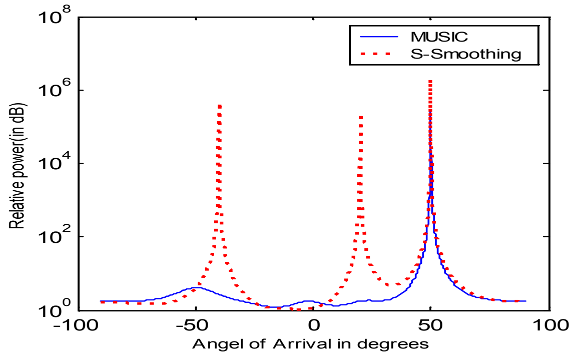


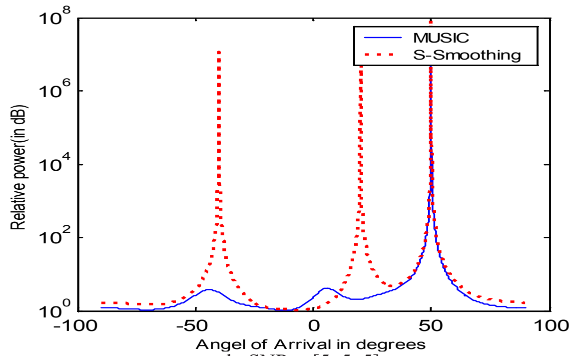
a. L = 5 , Element spacing= $\lambda/2$



b. L = 4 , Element spacing= $\lambda/4$

Fig 4. DOA Estimation with Two Coherent Signals from $-40^{\circ}, 20^{\circ}$, N=6, SNR= [5, 5] and 100 Snapshots.





b. SNR = [5, 5, 5]

Fig.5 DOA Estimation with Two Coherent Signals from $-40^{\circ}, 20^{\circ}$ and One Incoherent Signal Belongs to 50°, N=6, L=5 and 100 Snapshots.

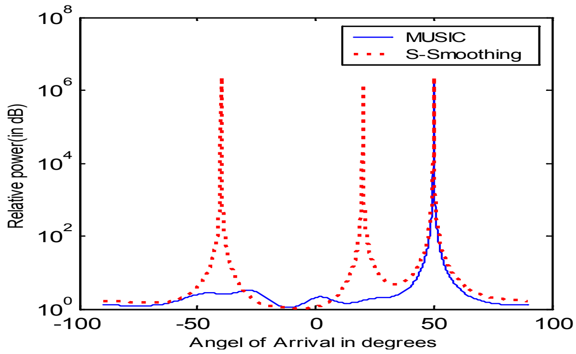


Fig.6. DOA Estimation with Two Coherent Signals Belongs to $-40^{\circ}, 20^{\circ}$ and One Incoherent Signal Belongs to 50°, N=16, SNR= [5, 5, 5], L=5 and 100 Snapshots.

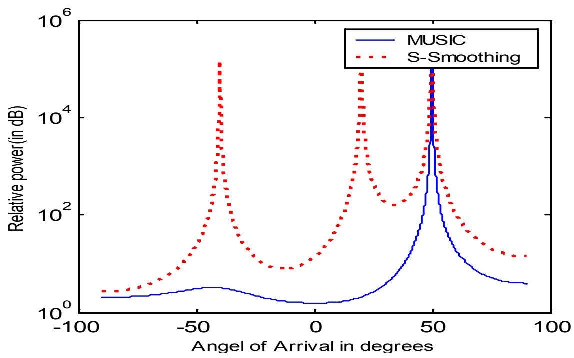


Fig 7. DOA estimation with two coherent signals from $-40^{\circ}, 20^{\circ}$ and one incoherent signal from 50°, N=6, SNR= [5, 5, 5], element spacing = $\lambda/4$, L=5 and 100 Snapshots.

To evaluate the performance of MUSIC and Spatially Smoothed version, a three signal structure has been simulated. Two of them are coherent and one of them is incoherent. Delay time between these two coherent signals is more than a chip length. As depicted in figures 5 to 7, performance metric, i.e. relative power, depends on different parameters same as snapshot and SNR.

Tables 1 to 4 shows the effect of SNR, number of array elements, number of snapshots and element spacing for spatially smoothed MUSIC, respectively. Performance average of estimated DOA and variance of angel. Due to estimate of coherent signals, these simulation results are not reported here.

Table 1. Effect of SNR in Spatially smoothed MUSIC Table2. Effect of snapshots in Spatially smoothed MUSIC

SNR	*Percentage Of Successful Iterations*	*DOA Estimation average*	*Variance of Angels*
10,10,10	100	-39.9934 19.9988 49.9940	0.0006 0.0002 0.0014
5,5,5	100	-40.0056 19,9924 49.9876	0.0011 0.0044 0.0012
15,15,15	100	-39.9986 20.0016 50.0002	0.1383×10^{-3} 0.1978×10^{-3} 0.0200×10^{-3}
20,20,20	100	-40.0000 20.0000 50.0000	0.0000 0.0000 0.0000
5,10,20	**100**	**-39.9948** **20.0070** **49.9998**	**0.0009** **0.0018** **0.0000**

Table 2. Effect of snapshots in Spatially smoothed

Snapshots	*Percentage Of Successful Iterations*	*DOA estimated average*	*Variance of Angels*
100	100	-40.0056 19,9924 49.9876	0.0011 0.0044 0.0012
200	100	-39.9934 19.9924 49.9878	0.0007 0.0029 0.0012
500	100	-39.9956 19.9962 50.0080	0.5818×10^{-3} 0.4064×10^{-3} 0.8176×10^{-3}
1000	**100**	**-39.9996** **19.9976** **49.9998**	$\mathbf{0.0399} \times 10^{-3}$ $\mathbf{0.2347} \times 10^{-3}$ $\mathbf{0.020} \times 10^{-3}$

Table 3. Effect of Number of Array Elements N in Spatially smoothed MUSIC

N	*Percentage Of Successful Iterations*	*DOA Estimation average*	*Variance of Angels*
6	100	-40.0056 19,9924 49.9876	0.0011 0.0044 0.0012
8	100	-39.9906 20.0030 49.9998	0.9335×10^{-3} 0.6122×10^{-3} 0.3807×10^{-3}
16	100	-39.9984 20.0000 49.9994	0.1578×10^{-3} 0.0401×10^{-3} 0.0598×10^{-3}

Table 4. Effect of Element spacing in Spatially MUSIC

Space Element	*Percentage Of Successful Iterations*	*DOA Estimation average*	*Variance of Aangels*
$\lambda/2$	100	-40.0056 19,9924 49.9876	0.0011 0.0044 0.0012
$\lambda/4$	99.8	-40.0281 19.7846 49.8651	0.0974 0.1161 0.0164
$\lambda/8$	**3.4**	**-44.3412** **38.4529** **53.1647**	**0.0876** **1.8751** **0.1687**

CONCLUSION

In this research, conventional MUSIC algorithm and spatially smoothed version have been simulated. Two and three users in exist of noise and fading. Moreover efficiency of algorithms in distinguish of signals, are evaluated. According to simulation results considering noise and multipath fading, MUSIC method cannot be used to distinguish correlated signals. DOA of signal sources can preferably be estimated via spatially smoothed version. Therefore subspace based algorithms like MUSIC, ML and ROOT-MUSIC are out of use. Spatial smoothing method must be used to remove the correlation between the incident signals by dividing the main sensor array in to forward/ backward overlapping sub-arrays and introducing phase shifts between these sub-arrays. It causes decrease of length of array, and with calculation and sample mean of covariance matrix of each sub-array, estimation is being possible. The effects of various investigated parameters in spatial smoothing algorithm are shown. Some of important results of these results of this research are as follow:

1- MUSIC algorithm is not appropriate method in the case of two incoherent sources.

2- Increasing the number of sensors, number of snapshots, sub-arrays and signal to interference ratio is a reason to improving the performance of algorithms

3- Estimation of DOA in signal to interference ratio of 20 dB without any error.

4- A good estimation of DOA in the case of fully coherent signals and considering fading will be achieved applying spatial smoothing algorithm.

5- Optimum distance between adjacent array's elements is $\lambda/2$.

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