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A Statistical Approach for Estimating The Probability of Occurrence of Earthquake on The Northern Anatolian Fault Zone

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Abstract

Probabilistic methods are useful for characterizing earthquake prediction because they are, for all practical purposes, random phenomena. Earthquake occurrence probabilities can be predicted by using different probability distributions. In this study eleven distributions are examined for determining the best one that represents earthquake data from Turkey during the last 104 years. In such a determination, Kolmogorov-Smirnov, Chi-square, Anderson-darling test statistics are used independently for treating Turkish earthquake data. It is concluded that the Weibull probability distribution is the most appropriate than any other distribution with the data at hand. This result is valid for the earthquake risk evaluation in the area between longitudes 40^{o} 50' – 41^{o} 50' N and latitudes 30^{o} 00' – 40^{o} 00' E which includes the Northern Anatolian Fault Zone (NAF) as the most active fault in Turkey.

Key words: Earthquake, prediction, probability, Turkey, Weibull Distribution.

INTRODUCTION

It has been one quarter century since Utsu [1,2], Rikitake [3] and [4] proposed probabilistic approaches for predicting the magnitude of the next earthquake on a specific fault. Poisson distribution is commonly applied to estimate the occurrence of earthquakes in seismic hazard studies [5,7]. A number of probability distributions have been proposed and used for computation of conditional probability of future earthquake occurrences, including the double exponential, Gaussian, Weibull, Log-normal [8], Gamma [9], and Pareto distributions [10]. The difficulty lies in determining the suitable distribution given the available earthquake data for a certain active fault zone. Nishenko and Buland [1987] obtained a reasonably good fit to the data using a log-normal distribution. McNally and Minster [11] argued that the Weibull distribution would be more appropriate [22]. Past studies on seismicity and seismic hazard in Turkey includes the use of geostatistical methods without consideration of probability distribution functions and regional seismic hazard estimations [12].

It is necessary to make several assumptions and establish definitions in modeling of earthquake occurrences on a fault system. According to the Poisson model, earthquake occurrences have no memory and they occur independent of each other. The random variable is defined as either the magnitude of the earthquake in modeling and/or as inter-event time between two earthquake incidences in the continuous distributions [13]. A fault system usually creates earthquakes of many different magnitudes, not just one magnitude and inter-event time. The magnitude of the earthquake is the most important parameter used in the modeling studies. Although the devastation caused by the earthquake is directly related to the magnitude of the earthquake when it occurs near urban areas, site conditions and quality of the buildings are also important parameters. For example, while an earthquake of magnitude of over 7.5 causes no devastation in Japan, 20.000 people died in the earthquake occurred in 1999 in Turkey [14]. In countries where 'sound' building codes either do not exist or do exist but are not followed earthquakes with magnitude larger than 5 can be devastating events. For this reason, earthquakes with magnitude equal to or larger than 5 are taken into consideration in this study.

In this study, the set of data presented in Table 1 is used, which consists of earthquakes whose magnitudes are equal to or larger than 5 between the years 1900 and 2004 in the area within longitudes 40o 50' – 41o 50' N and latitudes 30o 00' – 40o 00' E [23]. This area (see Figure 1) includes the north Anatolian fault zone, which is the most active earthquake zone in Turkey. It is obvious from table 1 that the number of the earthquakes with magnitude of 5 or over is 63 during the last 101 years. On the other hand, it is worthy to notice that 3 earthquakes have magnitudes more than 7. The temporal magnitude randomness is obvious and therefore it is important to try and search the most suitable probability distribution model for prediction in this region. The objective of this study is to determine which probability distribution best fits the earthquake data considered. Four probability distributions are commonly used in the literature Poisson, Semi-Markov, Gumbel and Weibull.

The comparisons are made by the help of Chi-Square, Kolmogorov-Smirnov and Anderson- Darling test statistics. For the parameter estimations of these distributions three estimation methods, namely, least squares, maximum likelihood and moment methods are considered. Earthquake prediction probabilities based on these three estimation methods have been calculated for different time periods.

Table 1 (Cont.) The earthquakes at the magnitude of 5 or over between the years 1900-2004 in the region between (40.50 - 41.50) north and (30.00 – 40.00) east coordinates

Date	Time	Latitude	Longitude	Depth (km)	Magnitude
21.09.1957	20:16	40,75	34,02	40	5.1
23.06.1967	10:06	40,85	33,65		5.1
22.07.1967	16:56	40,67	30,69		6.0
22.07.1967	17:14	40,70	30,80	6	5.2
22.07.1967	18:09	40,72	30,51	35	5.1
30.07.1967	01:31	40,72	30,52	18	5.4
22.12.1969	04:47	40,60	34,20	33	5.1
05.10.1977	05:34	41,02	33,57	10	5.3
14.08.1996	01:55	40,75	35,30	12	5.3
14.08.1996	02:59	40,78	35,31	3	5.3
11.01.1997	06:42	40,55	35,25	10	6.0
13.09.1999	11:55	40,75	30,08	10,4	5.6
11.11.1999	14:41	40,75	30,25	7,5	5.4
12.11.1999	16:57	40,81	31,19	10,4	6.2
12.11.1999	17:17	40,75	31,08	27,8	5.2
12.11.1999	17:29	40,70	31,47	11	5.0
16.11.1999	17:51	40,73	31,59	5	5.0
14.02.2000	06:56	41,02	31,76	10	5.0
06.06.2000	02:41	40,69	32,99	10	6.1
23.08.2000	13:41	40,68	30,72	15	5.8
26.08.2001	00:41	40,95	31,57	7	5.4
22.10.1905	03:42	41,00	31,00	27	5.2
21.08.1907		40,70	30,10	15	5.5
21.06.1908	03:55	40,60	35,90	0	5.2
25.06.1910	19:26	41,00	34,00	0	6.2
09.08.1918	00:39	40,89	33,41	10	5.8
29.08.1918	06:39	40,58	35,16	10	5.3
09.06.1919	07:13	41,16	33,20	10	5.7
09.06.1919	15:47	40,68	33,89	10	5.0
29.05.1923	11:34	41,00	30,00	25	5.5
16.08.1923	03:52	41,02	34,41	40	5.2
24.01.1928	07:36	40,99	30,86	10	5.3
21.09.1936	11:41	41,21	33,53	20	5.1
18.11.1936	15:50	41,25	33,33	10	5.2
21.11.1942	14:01	40,82	34,44	80	5.5
02.12.1942	19:04	41,04	34,88	20	5.4
11.12.1942	02:39	40,76	34,83	40	6.1
20.12.1942	14:03	40,70	36,80	16	7.0
20.06.1943	15:32	40,85	30,51	10	6.5
20.06.1943	16:47	40,84	30,73	10	5.5
26.11.1943	22:20	41,05	33,72	10	7.2
07.12.1943	01:19	41,00	35,60	0	5.6
02.01.1944	10:59	41,00	33,70	0	5.0
01.02.1944	06:08	40,70	31,27	10	5.0
02.02.1944	03:33	40,74	31,44	40	5.1
10.02.1944	12:05	41,00	32,30	10	5.3
05.04.1944	04:40	40,84	31,12	10	5.5
30.09.1944	04:13	41,11	34,87	10	5.5
18.10.1944	12:54	40,89	33,47	10	5.2
02.03.1945	10:39	41,20	33,40	10	5.6
21.01.1946	11:25	41,05	33,48	60	5.0
19.12.1947	17:31	40,71	32,82	10	5.1
13.05.1949	20:14	40,74	32,71	20	5.1
13.08.1951	18:33	40,88	32,87	10	6.9
14.08.1951	18:46	41,08	33,18	40	5.1
07.09.1953	03:59	41,09	33,01	40	6.4
19.08.1954	21:03	41,21	36,41	30	5.0
06.01.1956	14:52	41,00	30,20	10	5.0
26.05.1957	06:33	40,67	31,00	10	7.1
26.05.1957	08:54	40,60	30,74	40	5.4
26.05.1957	09:36	40,76	30,81	10	5.9
27.05.1957	11:01	40,73	30,95	50	5.8
01.06.1957	05:26	40,75	30,86	50	5.0

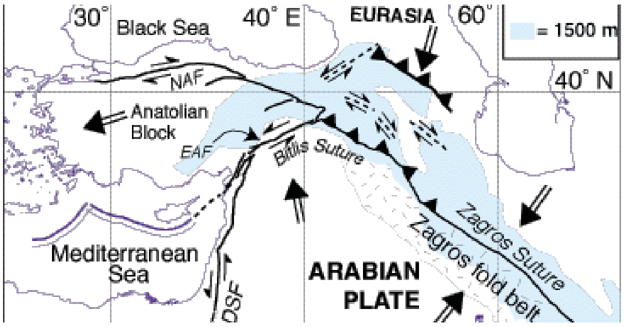

Figure1. The Area of investigation (NAF: 40.50 - 41.50 north and 30.00 – 40.00 east)
[http://www.iris.iris.edu/volume2000no1/page-14-16.htm]

STATISTICAL ANALYSES

In determination of the distribution which fits the earthquake data best, the test statistics, summarized briefly below, have been used. The results of the test statistics used in determination of the effective distribution are given in Table 2. It is seen that the test statistics have minimum values for Weibull Distribution. For this reason, it has been concluded that the distribution that best represents the data set is the Weibull Distribution. In terms of compliance, Pearson VI, lognormal and log-logistic follow the Weibull distribution.

Table 2 Goodness of fit results

Distributions	Chi-Square Value(df)	Kolmogorov Smirnov	Anderson Darling
Beta	16.5 (4)	0.2350	4.11
Erlang	28.4 (4)	0.3240	17
Exponential	28.4 (4)	0.3240	17
Gamma	1.43 (4)	0.0981	0.644
Log-Logistic	2.38 (4)	0.0821	0.53
Lognormal	1.06 (4)	0.0901	0.535
Pearson V	15.8 (4)	0.2120	3.42
Pearson VI	1.43 (4)	0.0821	0.386
Tringular	110 (4)	0.5970	67.8
Uniform	140 (4)	0.7070	101
Weibull	1.06 (4)	0.0697	0.277

In making predictions by probability distributions, the first step is the estimation of distribution parameters. The three methods mostly used in estimation of parameters are Least Squares Method (LSM), Maximum Likelihood Method (MLE) and Method of Moments (MOM)[15,16]. In this study, fitting of Weibull Distribution for calculation of earthquake prediction probabilities in the mentioned region is introduced and then the mathematical process in the estimation of MLE, LSM and MOM and as well as Weibull Distributions is explained.

Weibull Distribution

Weibull Distribution with two parameters is given below [17];

$$f(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}} \quad \beta>0,\eta>0 \tag{1}$$

In the equation (1); t is the time passing between the events, β is the shape parameter, and η is the scale parameter. Weibull Cumulative Distribution Function is as follows;

$$F(t)=1-e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{2}$$

The expectation of the earthquake in period **t** is calculated with the expected value, as shown in equation (3).

$$E(t)=\int_R t.f(t).dt$$

$$E(t)=\int_0^{\infty} t.\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}.dt=\eta\Gamma\left(1+\frac{1}{\beta}\right) \tag{3}$$

MLE, LSM and MOM are the most common estimation methods that are used in order to estimate the shape and scale parameters of Weibull Distribution [18,19,20].

DISCUSSION

Estimations of Weibull distribution parameters are obtained with the three methods mentioned above and the results are presented in Table 3.

Table 3 Estimated values of shape and scale parameters of Weibull Distribution obtained using three different calculation methods

Method	Parameters $\hat{\beta}$	Parameters $\hat{\eta}$
Least Squares Method (LSM)	0.514	329.26
Maximum Likelihood Method (MLE)	0.507	335
Method of Moments (MOM)	0.765	785.066

When Table 3 is studied, it is seen that parameter estimations obtained by LSM and MLE methods are close to each other. Especially, η, estimated using these two methods are close to each other but different from that obtained by MOM.

The probabilities are calculated using equation (2) for magnitudes equal to or greater than 5 in the area of investigation. The substitution of parameters estimated by LSM, MLE and MOM into equation (2) leads to the following probability statements.

$$F(t)_{LSM}=1-e^{-\left[\frac{t}{329.26}\right]^{0.514}} \tag{4}$$

$$F(t)_{MLE}=1-e^{-\left[\frac{t}{335}\right]^{0.507}} \tag{5}$$

and

$$F(t)_{MOM}=1-e^{-\left[\frac{t}{785.066}\right]^{0.765}} \tag{6}$$

Accordingly when t_i= 100, 365, 730, 1825, 3650, 5475, 7300 days are substituted in these equations then the probability results appear as in Table 4.

Table 4. Estimating the probability of occurrence with different prediction periods for the earthquakes at the magnitude of 5 or larger

Method	Probability	Prediction period (days)						
		100	365	730	1825	3650	5475	7300
LSM	*p*	0.418	0.652	0.778	0.910	0.968	0.986	0.993
MLE	*p*	0.418	0.648	0.773	0.906	0.965	0.984	0.992
MOM	*p*	0.187	0.427	0.612	0.851	0.961	0.988	0.996

The probabilities of an earthquake occurrence with a magnitude of 5 or higher are 0.418, 0.418 and 0.187 according to LSM, MLE, MOM methods, respectively within 100 days after an earthquake of magnitude 5 or larger has already occurred. These probabilities successively increase to 0.652, 0.648 and 0.427 in one year, to approximately 0.90 in 5 years and to 0.99 in 20 years. It can be deduced from the results that there is a very high probability of occurrence of an earthquake with a magnitude of 5 or higher within 20 years after an earthquake with a magnitude of 5 or higher has already occurred in the area of investigation. If Table 4 is studied in terms of parameter estimation methods, MOM gives lowest probability in shorter time periods when compared with LSM and MLE. However, when the time period increases, it is seen that all the three estimation methods give similar results. In order to decide on a proper method to make a better estimation on shape and scale parameters, a simulation study was carried out with samplings composed of 10, 30, 50, 100 and 120 units. At the end of this simulation study on 250 comparisons, 122 comparisons revealed that MLE was the best estimation method, while 104 of the 250 showed the LSM, and the other 24 of 250 showed the MOM as the best estimation method. It was observed that in the simulation studies with 10 units the MLE was the best estimation method while in studies with 30 and 50 units the MLE and LSM, in studies with 100 units the LSM, and in the simulation studies with 120 units the MLE performed best. In general, the MLE appeared to be the best estimation method [21]. For the size of the data set used in this study, it is seen that estimated values of the parameters obtained by using LSM and MLE methods are close each other, while those obtained with MOM show differences. If sampling set is large enough however, it is seen that the results obtained from the three methods are closer to each other.

The probability of an earthquake occurrence at a magnitude of 5 or higher for different time periods in the area of investigation is given in the following figure. It is seen in Fig. 2 that as the inter-event time increases, the probability of earthquake occurrence approaches to 1. For example, the probability of earthquake occurrence is 0.87 in 1400 days and 0.91 in 2000 days. The prediction of an earthquake can be defined as the time, place and magnitude parameters of a possible future earthquake and the prediction of uncertainties depend on these parameters. Because of the divergences that earthquakes show in terms of time, place and magnitude, prediction of the seismic hazard with probabilistic methods provides the best results. Statistical approaches based on probability and random processes are the most appropriate methods to use in the analyses of seismic hazard due to the divergences seen in time and location of earthquakes.

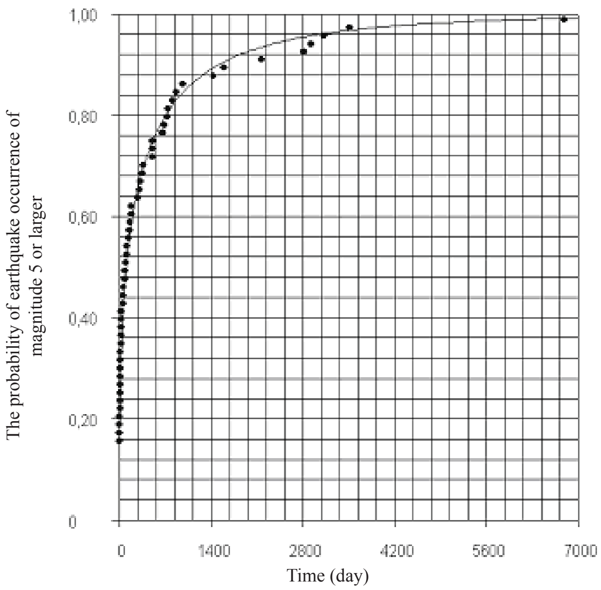

Figure 2. The probability of earthquake occurrence of magnitude 5 or larger in the area of investigation

CONCLUSION

It is known that the probability of occurrence of earthquakes in seismic zones with a certain size and tectonic regime can be estimated using Weibull probability distribution. Hagiwara (1974) and Rikitake (1974) used Weibull distribution to estimate probability of earthquake occurrence. The basic feature of the Weibull distribution is the time between events. Only the occurrence probabilities of the magnitudes that are equal to or greater than M_{min} (the smallest magnitude defined), can be computed with the Weibull distribution unlike other probability models (Gumbel, Poisson, Semi-Markov) used in earthquake probability analyses. Contrary to other approaches, the basic principle of the Weibull distribution is to take into consideration the distributions of the events in specific time intervals.

In this paper, the authors attempted to find a probability distribution that best represents the set of earthquake data from Turkey. KS, AD and Chi-Square statistical tests have been used in determining that the most representative probability model is the Weibull distribution. Also, the time intervals between the successive earthquake events in days are considered for the probability calculations instead of classifying the set of data according to the years as in other general studies. Shape and scale parameter estimations regarding the Weibull distribution are obtained by using LSM, MLE and MOM methods; and the probability of earthquakes of magnitude 5 or higher is calculated for different time periods by these methods. The study is carried out for a region that includes the most active fault zone in Turkey, the north Anatolian fault. The probability of an earthquake with a magnitude of 5 or larger in the mentioned region is 0.96 in 10 years; and the repetition period of an earthquake with a magnitude of 5 or higher is 651.8 days. The study area is the most densely populated region of Turkey in terms of both population and industry. In this region, investigation and control on compliance with building codes, new building construction directives, and building rehabilitation studies, which have been neglected until today, must be carried out and completed as soon as possible.

The estimations in this study depend just on the magnitudes of the earthquakes with a specific magnitude. For that reason, the results found in this study should be assessed as statistical findings that emerge from the estimation of earthquake probabilities. Statistical results obtained in the study are intended to help promote preventive precautions to minimize losses due to earthquakes. In conclusion, this study aims to act as a preliminary work that would be a base for further interdisciplinary studies to be carried out that are necessary to mitigate earthquake risk in the region.

APPENDIX

Maximum Likelihood Method

Likelihood function for Weibull Distribution having two parameters is as follows;

$$L(\eta,\beta)=\prod_{i=1}^{n} f(t_i;\theta)$$

$$L(\eta,\beta)=\prod_{i=1}^{n}\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)\beta} \tag{7}$$

If the natural logarithm of equation (7) is taken, the following formula is obtained;

$$InL(\eta,\beta)=\sum_{i=1}^{n} In\beta-\beta In\eta+(\beta-1)Int_i-\eta^{-\beta}t_i^{\beta}$$

$$InL(\eta,\beta)=nIn\beta-n\beta In\eta+(\beta-1)\sum_{i=1}^{n} Int_i-\eta^{-\beta}\sum_{i=1}^{n} t_i^{\beta}$$

The derivates of the likelihood function is taken according to the parameters and equaled to zero.

$$\frac{\partial InL(\eta,\beta)}{\partial\eta}=-n\beta\eta^{-1}+\beta\eta^{-(\beta+1)}\sum_{i=1}^{n} t_i^{\beta}=0 \tag{8}$$

$$\frac{\partial \ln L(\eta,\beta)}{\partial\beta}=n\beta^{-1}-n\ln\eta+\sum_{i=1}^{n}\ln t_i-\sum(\ln\frac{t_i}{\eta})t_i^{\beta}\eta^{-\beta} \tag{9}$$

When η is eliminated in equation (9), equation (10) is obtained.

$$\frac{\sum_{i=1}^{n}\ln t_i^{\hat\beta}\ln t_i}{\sum_{i=1}^{n} t_i^{\hat\beta}}-\frac{1}{\hat\beta}=\frac{\sum_{i=1}^{n}\ln t_i}{n} \tag{10}$$

Equation (10) can be numerically solved with Newton method and $\hat\beta$ is obtained from here. Then $\hat\beta$ obtained is used equation (11) and $\hat\eta$ is estimated.

$$\hat\eta=\left(\frac{\sum_{i=1}^{n} t_i^{\hat\beta}}{n}\right)^{1/\hat\beta} \tag{11}$$

Least Squares Method (LSM)

If we take the natural logarithm of the both sides of equation (2);

$$F(t)=1-e^{-\left(\frac{t}{\eta}\right)\beta}$$

$$-Ln(1-F(t))=\left(\frac{t}{\eta}\right)^{\beta}$$

is obtained. When the natural logarithm is taken again;

$$\underbrace{Ln\left\{-Ln\left(1-F(t)\right)\right\}}_{Y}=\underbrace{\beta Lnt}_{ax}-\underbrace{\beta Ln\eta}_{b} \tag{12}$$

is obtained. When Y = $Ln\left\{-Ln\left(1-F(t)\right)\right\}$ is defined with ax = âLnt and b = âLn; Y = ax+b; the Least Squares Method is obtained.

By the help of the

$$\overline{t}=\frac{1}{n}\sum_{i=1}^{n}\ln\left\{\ln\left[\frac{1}{\left(1-\frac{i}{n+1}\right)}\right]\right\} \tag{13}$$

and

$$\overline{Y}=\frac{1}{n}\sum_{i=1}^{n}\ln t \tag{14}$$

definitions, β and η are estimated with equations (15) and (16).

$$\hat\beta=\frac{\left\{n\sum_{i=1}^{n}\left(\ln t_i\right)\ln\left\{\ln\left[\frac{1}{\left(1-\frac{i}{n+1}\right)}\right]\right\}\right\}-\left\{\sum_{i=1}^{n}\ln\left(\ln\left[\frac{1}{1-\frac{i}{n+1}}\right]\right)\sum_{i=1}^{n}\ln t_i\right\}}{\left\{n\sum_{i=1}^{n}\left(\ln t_i\right)^2\right\}-\left\{n\sum_{i=1}^{n}\left(\ln t_i\right)\right\}^2} \tag{15}$$

$$\hat\eta=\exp(\overline{Y}-\frac{\overline{t}}{\hat\beta}) \tag{16}$$

Method of Moments (MOM)

To estimate the parameters of two-parameter Weibull Distribution using the Method of Moments parameters, the first and second moments. The *k*. Moment of the Weibull Distribution having to parameters around zero is given equation (17).

$$E(t^k)=\eta^k\Gamma(\frac{k}{\beta}+1) \tag{17}$$

The first moment around zero is obtained by writing *k*=1 and k= 2 in equation (17) [21].

$$E(t)=\eta\ \Gamma(\frac{1}{\beta}+1) \tag{18}$$

$$E(\tau^2) = \eta^2 \Gamma(\frac{2}{\beta} + 1) \tag{19}$$

When the square of the equation (18) is taken and divided equation (19), equation (20) only depended on $\hat{\beta}$ parameter is obtained.

$$\frac{\{\sum_{i=1}^{n} t_i\}^2}{n \sum_{i=1}^{n} t_i^2} = \frac{\{\Gamma(\frac{1}{\beta} + 1)\}^2}{\Gamma(\frac{2}{\beta} + 1)} \tag{20}$$

Equation (20) is solved by the help of the standard iterative methods and the $\hat{\beta}$ parameter is obtained. $\hat{\eta}$ parameter is found with equation (21) by substituting $\hat{\beta}$ in equation (18).

$$\hat{\eta} = \frac{\bar{t}}{\Gamma(\frac{1}{\hat{\beta}} + 1)} \tag{21}$$

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