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Direct Smoothing Method for Detection of Fault Patterns

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Abstract

Process knowledge can be utilized to detect occurrences of faults that leave a fault signature or a pattern in the process data being monitored. Here, I assume that the process knowledge is in the form of a specified function for the fault pattern but the parameters of this function are unknown. Furthermore, unknown function parameters may be constant or varying over time. Hence, an efficient method is required for estimation of the fault pattern parameters for use in the fault detection algorithms. Although, the least squares is a well known and proven method for estimation, on-line parameter estimation calculations may get cumbersome as the number of observations increase. Therefore, I propose the use of direct smoothing method, which is recursive and based on the discounted least squares criterion. Other than the computational efficiency, direct smoothing is also robust in estimation fault patterns with varying parameters.

Key words: Quality control, fault detection, statistical process control, control charts, process monitoring, generalized likelihood ratio, cuscore statistic.

INTRODUCTION

Various faults may be experienced in production processes and these are generally attributable to errors of machines, materials, methods, and labor. Rapid detection of such faults, which are often called assignable causes of variation, is an important issue in quality control since they may significantly affect product quality characteristics. Nevertheless, detection of faults is generally not a simple task and requires application of statistical tools to discriminate inherent variation (noise) from the variation due to faults in the process. It is assumed that occurrence of faults would result in a pattern in the observations, where the observations are measurements of key product/process variables. Patterns due to presence of faults are called fault patterns (signatures) and these are generally in the form of mean changes of the observations. It is the noise in the observations that make the patterns hard to detect without statistical tools.

Statistical Process Control (SPC) tools have been developed to cope with fault detection problems. In SPC, a process functioning under normal operating conditions is assumed to be in-control and a process functioning under the influence of a fault is assumed to be out-of-control. Control charts are the major SPC tools used for detection of faults. Generally, a probability distribution is used to model an in-control process and it is assumed that a fault in the process will shift a parameter of the distribution to some value different from the in-control value, i.e. a fault pattern. The shift in the distribution parameter is often considered in terms of a mean change and this is often a step change pattern. In fact, a step function may be used as a global model for many fault patterns. Most of the traditional control charts, such as Shewhart, Cumulative Sum (CUSUM), and Exponentially Weighted Moving Average (EWMA) are affective in detecting such step type fault patterns in the mean. For details, interested readers are referred to [1-4].

In some practical cases, knowledge on the patterns of observations caused by a known fault may be available. Hereafter, I will call such known faults *specific faults* and their known patterns *specific fault patterns*. For example, measured surface texture of a metal sheet machined in a milling operation may have a sinusoidal fault pattern due to tool wear type fault. Specific fault patterns in the form of linear, exponential, or sinusoidal patterns and their combinations are common in production processes and these patterns should be identified and eliminated rapidly in order to improve the quality characteristics of the products. Such specific fault patterns are illustrated in Figure 1.

For a specific fault pattern, one may not need to use the traditional control charts designed for global fault patterns. By sensitizing a control chart to the specific fault pattern, presence of specific faults can be detected faster than the traditional control charts. One approach that facilitates knowledge of specific fault patterns for affective detection of specific faults is to use the concept of Fisher's efficient score statistic. The cumulative score (Cuscore) chart of Box and Ramirez [5] is based on this statistic (see also [6]). Cuscore charts sequentially accumulate the product of model residuals with an appropriate detector developed from the specific fault pattern. As the specific fault pattern appears, residuals will contain a component due to this pattern and that will resonate with the detector. However, note that the start time of a fault is unknown in practice and a control chart should have a mechanism to estimate the change time or the start time of the specific fault pattern. To solve this problem,

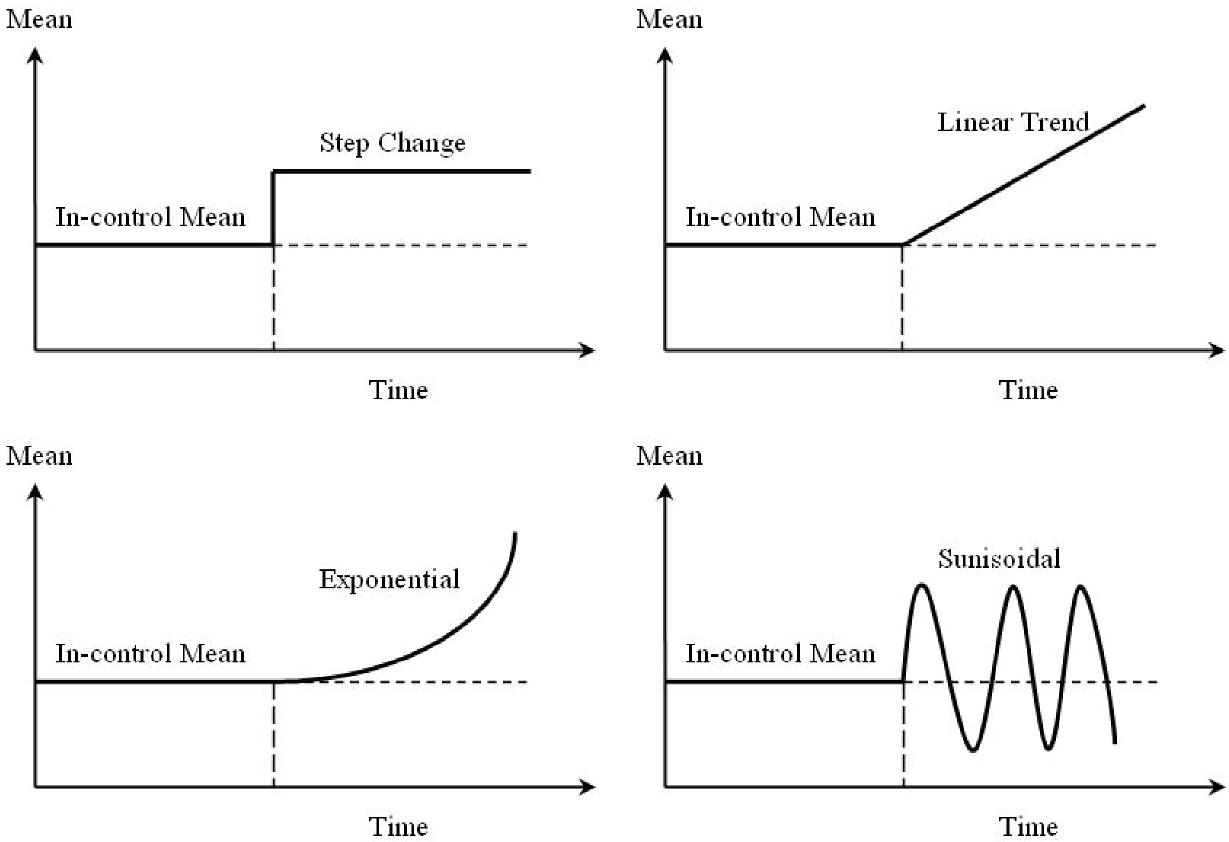

Figure 1. Examples of Specific Fault Patterns As a Change in the Mean

Luceno [7] suggested that a reinitialization of the Cuscore chart may be needed for some forms of fault patterns. This is used for updating the estimate of the start time of the fault pattern and hence preventing the Cuscore statistic from getting larger when the process is in-control. Although Cuscore charts are computationally efficient, Runger and Testik [8] showed through simulations that initial and steady-state performances of these charts may differ significantly and reinitialization may make the detection performance much worse for some fault patterns.

As an alternative to Cuscore charts for detecting specific fault patterns, a control chart can also be developed using a generalized likelihood ratio (GLR) statistic. This approach is used in Runger and Testik [8]. GLR charts are based on the log-likelihood ratio statistic computed by using the maximum likelihood estimate of the fault pattern's start time. Although, GLR charts are computationally intensive, their initial and steady-state performances are better than the Cuscore charts.

It was mentioned that a specific fault detection method should have a mechanism to estimate the unknown start time of the fault pattern. One other important problem in detecting specific fault patterns is the variability of the patterns. A parameterized mathematical function is often used to model specific fault patterns. Parameters of this function would characterize the variations of the pattern. Although in some cases defining a specific fault pattern is possible with known parameters, in other cases, these parameters may be unknown or varying as considered in this research. For example, a sinusoidal pattern caused by a tool wear in a milling operation may be varying in amplitude as a result of differences in the hardness of machined parts. To estimate the unknown parameters of the pattern, some methods for parameter estimation would be necessary for the effective operation of a control chart. In the GLR chart of Runger and Testik [8], least squares estimators (which are also the maximum likelihood estimators in the case of a normal distribution characterizing the process) are used for this purpose. However, least squares criterion is computationally intensive for online parameter estimation.

For specific fault patterns with unknown parameters, direct methods for updating estimates of unknown parameters are available. Direct smoothing is a recursive forecasting procedure, which can be used to predict unknown parameters of the specific fault pattern as new data are available. This procedure uses the errors from the specific fault pattern's forecasts to smooth the pattern parameter estimates. It is based on the discounted (weighted) least squares criterion, where decreasing weights by age are applied in the minimization of the sum of squares of the prediction errors [9].

In the following, direct smoothing procedure is proposed to be used as an on-line estimator for the parameters of specific fault patterns and how to incorporate this procedure into the GLR control charts is shown. Performances of GLR charts for detecting occurrences of specific fault patterns, which have known functional forms but unknown parameters, are studied by using both least squares and direct smoothing methods. Comparisons are provided. A robust study of the direct smoothing approach is also performed and the results are discussed.

MATERIALS AND METHODS

Process Model

Consider a process with the quality characteristic Y being monitored over time t. Suppose that under normal operating conditions, the model representing the sequential observations $y(t)$ is

$$y(t) = \mu + a(t), \ \text{t=1,2,...}$$

where μ is the mean of Y, $a(t)$ is the normally distributed white-noise sequence with mean zero and variance σ^2. Without lose of generality, let $\mu = 0$ for simplicity. Suppose further that this process is perturbed by a specific fault from time to time and let the mathematical function representing a specific fault pattern is known. For the specific fault pattern starting at an unknown time τ, let the sequential observations be represented by the model

$$y(t) = f(t,\theta,\tau) + a(t) \quad \text{for } t \geq \tau$$

where $f(.)$ is the function representing specific fault pattern, τ is the pattern start time of the fault, and θ is a $1\times k$ parameter vector corresponding to k additive components of the function representing the specific fault pattern, such that

$$f(t,\theta,\tau) = \sum_{i=1}^{k} \theta(t,i) z(t-\tau,i) \quad \text{for } t \geq \tau$$

Here, $\theta(t,i),\ i=1,...,k$ is the parameter of the i^{th} component of the parameter vector θ at time t and $z(t-\tau,i),\ i=1,...,k$ is the assessment of the i^{th} time dependent mathematical function of the specific fault pattern at time t, which starts at τ. For example, a fault pattern that has a trend component and an oscillatory component may be represented by

$$f(t,\theta,\tau) = \theta(t,1) + \theta(t,2)(t-\tau) + \theta(t,3)\sin(\omega(t-\tau)) + \theta(t,4)\cos(\omega(t-\tau))$$

GLR Control Chart

An important concept in mathematical statistics is the log-likelihood ratio

$$l(\theta,\tau) = \ln \frac{p_{\mu+f(T,\theta,\tau)}(y(1),...,y(T))}{p_{\mu}(y(1),...,y(T))} \quad \text{.........................(1)}$$

where, p is the parameterized probability density and T is the current time or the last observation sequence number. The key statistical property of the log-likelihood ratio is a sign change when the parameters of the probability densities given in equation are different. Hence, this concept is used widely in change detection algorithms. GLR algorithm is based on the log-likelihood ratio, where the specific fault pattern's start time τ that maximizes the log likelihood ratio over the data history is used. Then the GLR statistic is

$$\max_{1\leq t\leq T} \ln \frac{p_{\mu+f(T,\theta,\tau)}(y(t),...,y(T))}{p_{\mu}(y(t),...,y(T))}$$

Since the observations $y(t)$ are independent, the GLR statistic at time T is

$$g_T = \max_{1\leq t\leq T} \sum_{j=t}^{T} \ln \frac{p_{\mu+f(T,\grave{e},t)}(y(j))}{p_{\mu}(y(j))} \quad \text{.............................. 2}$$

For normal (gaussian) distributed observations $y(t)$, the GLR statistic in equation can be written as

$$g_T = \max_{1\leq t\leq T}\left[\sum_{j=t}^{T} f(j,\theta,t)\sigma^{-1}(y(j)-\mu) - \frac{1}{2}\sum_{j=t}^{T} f(j,\theta,t)\sigma^{-1} f(j,\theta,t)\right] \text{... 3}$$

Using the GLR statistic in equation, a control chart for detecting a specific fault pattern can be constructed. This is achieved by determining a control limit CL and each time checking for $g_T \geq CL$, which triggers an alarm for the presence of the specific fault pattern being monitored.

GLR algorithm is not recursive, which makes it computationally intensive. However, today's personal computers allow it to be efficiently used in practical applications. Furthermore, a moving window with size w of observations can be used to decrease computational requirements. The GLR statistic using a moving window of observations with window size w is,

$$g_t = \max_{T-w\leq t\leq T}\left[\sum_{j=t}^{T} f(j,\theta,t)\sigma^{-1}(y(j)-\mu) - \frac{1}{2}\sum_{j=t}^{T} f(j,\theta,t)\sigma^{-1} f(j,\theta,t)\right] \text{...... 4}$$

It is suggested that a window size of 30 is a good choice in change detection (see [8]). For further details of the GLR algorithm and sequential change point detection readers are also referred to Lai [10].

If the parameter vector θ is known, its value can be used in the specific fault pattern function $f(.)$ of the GLR statistic given in equations and . If the parameters are unknown, GLR is a function of τ and θ, which are independent and should be doubly maximized.

Figure 2 is a schematic of the GLR algorithm for the unknown parameters case, where $\hat{\theta}_{T-w}^{T}$ is the maximum likelihood estimate (MLE) calculated from the data in the interval [T-w,T]. Note that the algorithm first maximizes the likelihood of estimates for θ for each of the possible start times τ in the data window and the log likelihood ratios are calculated for each of these possible τ values using the corresponding MLEs of the θ. Then the second maximization is for the MLE of the τ, which is the maximum of the log likelihood ratios in the window.

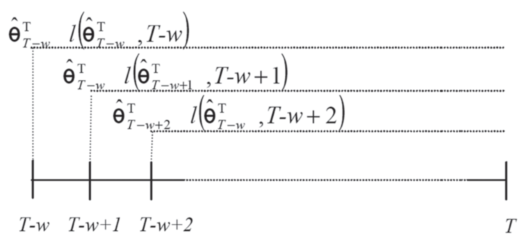

Figure 2. Representation of the GLR algorithm for unknown parameters case

Note that as the size of the window $[t, T]$ t=w,...,T used in the MLE calculations of the parameters gets smaller, parameter estimates may become poor, especially when the noise is high. Furthermore, GLR algorithm becomes computationally intensive due to these maximizations.

In order to improve the parameter estimates, use of a lower bound θ_f for the estimates is suggested and it is shown that GLR performance for unknown parameters case did not differ considerably from the known parameters case [8]. Known parameters case for a specific fault pattern can be used as a control setting for comparing estimation methods for unknown parameters cases.

It is also important not to assume that θ would assume a constant value over time. The values for the θ may be varying over time due to various factors such as dependence on the process state variables, operator-to-operator differences, etc. For example, a machine operated by different operators or differences in the tool wear for the same type of machines may result in varying parameters. Therefore, a constant value for θ over a data interval may not be a good way to represent the time behavior of a pattern.

In the following, a computationally efficient forecasting procedure, namely direct smoothing, is described and incorporated into the GLR statistic calculations. This is suggested as an alternative to the MLE procedure, which demands further computations. When the θ of the fault pattern is unknown or varying, one can replace $f(t,\theta,\tau)$ in equation or with the predictions $f(t,\hat{\theta},\tau)$ obtained using the direct smoothing procedure and let the GLR algorithm estimate the start time τ of the fault pattern that maximizes the likelihood

function over all possible time values *t*.

Direct Smoothing (DS) Estimation

A brief overview of the direct smoothing method is given in the following. Interested readers are referred to [9,11-13] for details and examples. Consider the least squares estimation criterion using the observations at $t=1,...,T$,

$$SSE=\sum_{t=1}^{T}\left(y_t-f(t,\hat{\theta},\tau)\right)^2$$

where SSE stands for the sum of squared of errors. For convenience, let $\mathbf{y}$ be a $T\times 1$ column vector of the successive values of the observations y_t for $t=1,...,T$, $\hat{\theta}_T$ be a $k\times 1$ column vector of parameter estimates computed using the observations at $t=1,...,T$, Z be a $T\times k$ matrix of the values of the components $z_t(i)$ of the function representing specific fault pattern where $t=1,...,T$ and i=1,...,k . It is well known that the least squares estimates of the parameters that minimize *SSE* are the solutions to the least squares normal equations

$$Z'Z\,\hat{\theta}_T=Z'\mathbf{y}$$

and the estimates are

$$\hat{\theta}_T=(Z'Z)^{-1}Z'\mathbf{y}$$

However, note that the GLR algorithm employs the least squares estimates computed from the data points including each possible specific fault start time *t* up to the current data point *T*. When $T<\tau$ the expected value of the estimates $\hat{\theta}$ is

$$\mathrm{E}(\hat{\theta})=\mathbf{0}$$

In other words, the estimated function $f(t,\hat{\theta},\tau)$ representing the specific fault pattern is expected to be zero when the pattern is not present in the data. Moreover, when the pattern occurs at time τ , the observations before the pattern start time τ will cause some bias in the parameter estimates for the possible change times $t<\tau$. Because each observation gets equal weights in the least squares procedure, number of data points required to decrease this bias in the estimates will be a linear function of $\tau-t$ for $\tau>t$. Alternatively this bias may be reduced if we assign weights to the observations that decrease by the age of the observations, hence utilizing the recent observations more than the distant past observations.

Least squares parameter estimation criterion can be modified by assigning weights to the errors in the SSE calculations such that

$$SSE=\sum_{t=1}^{T}\omega_t^2\left(y_t-f(t,\hat{\theta},\tau)\right)^2$$

where ω_t is the square root of the weight given to the t^{th} error. This parameter estimation criterion, which minimizes the weighted SSE is called the weighted least squares. Let the weights be discounted exponentially as we go back away from the current time T so that older observations receive proportionally less weight, i.e.

$$\omega_{T-t}^2=\beta^t,\ \ \text{t=0,...,T-1}$$

where β is the discount factor chosen to be $0<\beta<1$. Then the special case of the weighted least squares criterion with the exponentially discounted weights is called the discounted least squares criterion. The discounted least squares normal equations

$$C_T\hat{\theta}_T=c_T \quad\text{.......(5)}$$

can be solved to obtain the discounted least squares estimates of the parameters such that

$$\hat{\theta}_T=C_T^{-1}c_T \quad\text{.......(6)}$$

if C_T^{-1} exist. Here $C_T=(WZ)'(WZ)$, W is a $T\times T$ diagonal matrix of the weights with the t^{th} diagonal element ω_t and $c_T=Z'W^2\mathbf{y}$.

Note that the result of discounted weighting is a window of observations contributing considerably to the parameter estimation. For example, if $\beta=0.8$, the weight given to the data point y_{T-30} is approximately 0.001, indicating that this observation is almost not considered in the parameter estimation.

For some special cases of functions representing specific fault patterns, the discounted least squares estimates of parameters can be calculated recursively. This recursive parameter update method is called direct smoothing and it has computational advantages. In the following, direct smoothing is briefly described without giving the theoretical details.

Assume that the function components $z_t(i)$ have the following time dependencies,

$$z_{t+1}(i) = \ell_{i1} z_t(1) + \ell_{i2} z_t(2) + ... + \ell_k z_t(k), \quad i = 1,...,k$$

that is the value of a $z_{t+1}(i)$ is a linear combination of all components evaluated at time t. Let L be a $k \times k$ matrix of linear combination coefficients $\ell_{i,j}$ i=1...k, j=1...k . Then we can write

$$\mathbf{z}_{t+1} = L\mathbf{z}_t \quad \text{..........(7)}$$

Note that, given the matrix L and $\mathbf{z}_0$, one may calculate $\mathbf{z}_{t+1}$ by

$$\mathbf{z}_{t+1} = L^t \mathbf{z}_0$$

Therefore, L is named the transition matrix. Equation holds only for mathematical functions $z_t(i)$ that are polynomial, exponential, or trigonometric. As a matter of

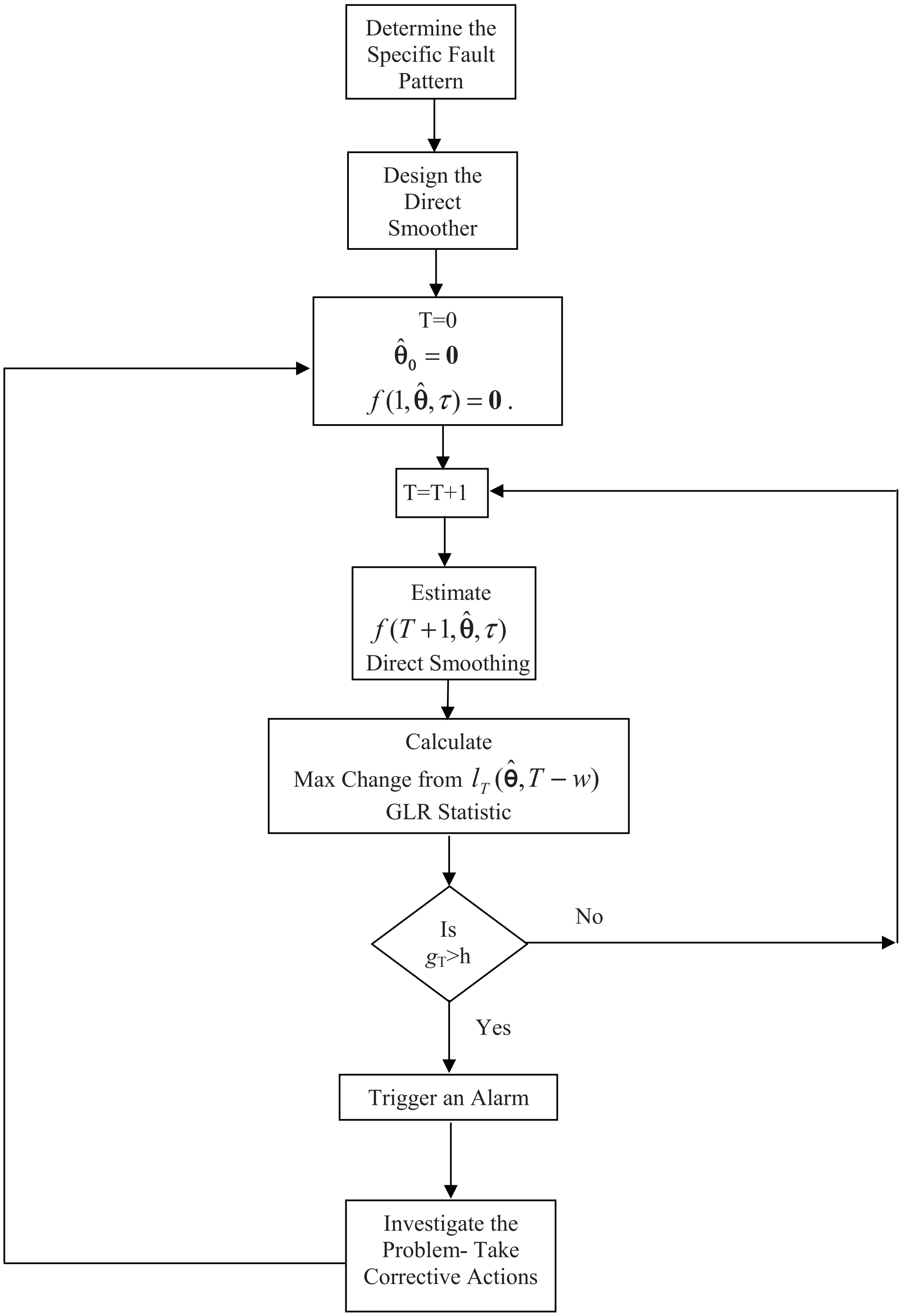

Figure 3. Flow Chart for the GLR Control Chart with Direct Smoothing.

fact, these mathematical functions and their combinations can represent a large number of signals. Let the matrix C_T be

$$C_T = \sum_{t=1}^{T} \beta^{T-t} z_t z_t'$$

If $z_t(i)$ do not decay too rapidly, there exists a limit such that

$$C \equiv \lim_{T \to \infty} C_T$$

C exists for all trigonometric and polynomial functions of $z_t(i)$, and for $z_t(i) = e^{-\alpha t}$ where $\beta < e^{-2\alpha}$. Using these results and after some algebra, the direct smoothing recursive parameter update equation is

$$\hat{\theta}_T = L'\hat{\theta}_{T-1} + s(y_T - z'_{T+1}\hat{\theta}_{T-1})$$

where $s = C^{-1} z_T$ is called the smoothing vector [11-12].

Consequently, the one-step-ahead specific fault pattern predictions can be calculated by

$$f(T+1, \hat{\theta}, \tau) = z'_{T+1}\hat{\theta}_T$$

The discount factor has the relation $\beta = \beta^{*1/k}$ where β^* is an appropriate discount factor for a constant process. For example, smoothing of a linear trend model with $\beta = 0.9$ is equivalent to smoothing of a constant model with $\beta^* = 0.81$.

A flow chart for implementing DS-GLR control charts is shown in Figure 3 [also in 9]. In order to apply the DS, some initial values are needed. Since the common procedure in statistical process control is to assume that the process is in-control when it restarts, it is reasonable to use $\hat{\theta}_0(i) = 0$ i=1,...,k and $f(1, \hat{\theta}, \tau) = 0$. Storing the DS estimates of the signal $f(T, \hat{\theta}, \tau)$ along with the corresponding data points of the sliding window, one can easily implement the GLR control chart for the unknown signal parameters case. It should be noted that rather than estimating unknown parameter *w* times with the availability of a new data point, only one parameter is estimated. This would reduce the computational requirements for large window sizes.

RESULTS

Analysis of GLR Performance with DS Estimation

For evaluating the performance of the control charts, the average run-length (ARL) criterion is used. First, a run-length (RL) can be defined as the number of time units until an alarm is triggered and consequently the ARL is the expected value of the run length. Because there is always a probability that a control chart will trigger a false alarm when the signal is not present in the data, ARL can be classified further into two: ARL_0 ARL_0 is the expected time between the false alarms and ARL_1 is the expected delay for triggering an alarm when there is an assignable cause. For equal ARL_0 values, a control chart with a smaller ARL_1 has a better performance over the one with a longer ARL_1. Nevertheless, there is always a trade-off between a longer ARL_0 and a shorter ARL_1.

Two different specific fault pattern functions are considered in the simulations: A linear trend with a slope of θ=0.1 and a sine wave with a periodicity of $(t-0.5)\pi/2$ and an amplitude of θ=1.

For the linear trend function,

$$f(t,\theta,\tau) = \begin{cases} 0 & \text{for } t < \tau \\ \theta \times (t-\tau) & \text{for } t \geq \tau \end{cases}$$

the following values are used in DS calculations,

$$\beta = 0.9,\ \mathbf{z_0} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ L = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},\ C = \begin{bmatrix} 10 & -90 \\ -90 & 1710 \end{bmatrix},$$

$$s = \begin{bmatrix} 0.1900 \\ 0.0100 \end{bmatrix},\ \hat{\theta}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{ and } f(1,\hat{\theta},\tau) = 0$$

For the sine wave function,

$$f(t,\theta,\tau) = \begin{cases} 0 & \text{for } t < \tau \\ \theta \sin\left((t-\tau-0.5)\frac{\pi}{2}\right) = \theta_1 \sin\left(\frac{\pi t}{2}\right) + \theta_2 \cos\left(\frac{\pi t}{2}\right) & \text{for } t \geq \tau \end{cases},$$

where

$$\theta_1 = \theta \cos\left(\frac{\pi}{2}(-\tau - 0.5)\right) \text{ and } \theta_2 = \theta \sin\left(\frac{\pi}{2}(-\tau - 0.5)\right),$$

the following values are used in DS calculations,

$$\beta = 0.9,\ \mathbf{z_0} = \begin{bmatrix} \sin(0) \\ \cos(0) \end{bmatrix},\ L = \begin{bmatrix} \cos\left(\frac{\pi}{2}\right) & \sin\left(\frac{\pi}{2}\right) \\ -\sin\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right) \end{bmatrix},$$

$$C = \begin{bmatrix} 4.74 & 0 \\ 0 & 5.26 \end{bmatrix},$$

Table 1. GLR Charts' Performance for a Linear Trend with slope 0.1.

	Pattern starts at	Known Parameters	Unknown Parameters (MLE)	Unknown Parameters ($\theta_j = 0.05$)	Unknown Parameters (DS)
		h= 3.05	*h*= 5.2	*h*= 4.5	*h*=2.45
ARL(0)		300.38 (4.10)	294.55 (4.11)	306.75 (4.30)	306.58 (4.18)
ARL(1)	$\tau = 0$	11.94 (0.04)	13.24 (0.05)	12.21 (0.05)	12.69 (0.04)
ARL(1)	$\tau = 50$	11.40 (0.05)	13.03 (0.05)	11.98 (0.05)	12.60 (0.05)

Table 2. GLR Charts' Performance for a sin[$(t-0.5)\pi/2$].

		Known Parameters	Unknown Parameters (MLE)	Unknown Parameters $(\theta_j = 0.5)$	Unknown Parameters (DS)
	Pattern starts at	*h*= 4.5	*h*= 5.8	*h*= 5.4	*h*=2.4
ARL(0)		323.83 (4.52)	311.90 (4.32)	322.14 (4.50)	338.76 (4.55)
ARL(1)	$\tau = 0$	18.56 (0.15)	20.35 (0.20)	19.74 (0.18)	18.89 (0.14)
ARL(1)	$\tau = 50$	17.06 (0.16)	19.08 (0.18)	18.34 (0.18)	17.77 (0.14)

$$s = \begin{bmatrix} 0.0000 \\ 0.1900 \end{bmatrix},\ \hat{\theta}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \text{and}\ f(1,\hat{\theta},\tau) = 0$$

Interested readers are referred to [11] for the details of the computations of the above-mentioned values.

The in-control observations are independent and identically distributed normal with mean $\mu = 0$ and variance $\sigma^2 = 1$. Signals were started at $\tau = 0$ and $\tau = 50$. Simulating a control chart with a signal starting at $\tau = 0$ is used for the initial performance evaluation and a signal starting at $\tau = 50$ is used for steady-state performance evaluation.

As suggested in Runger and Testik [8] a window size of 30 was used for the GLR control charts. Both of the cases mentioned earlier in text, known signal form but unknown parameter case and known signal form but varying parameter cases were studied. Simulations were performed using MATLAB ® 6.1 and 5000 replications were done for each of the cases. The results are shown in Tables 1 and 2. Standard errors of the results are given in parenthesis.

The linear trend results are given in Table 1. It can be seen that for equal (approximately) ARL_0 performances, ARL_1 performances with DS estimation are in between the ones of MLE estimates with a lower bound and the MLE estimates without a lower bound. As expected, the best performance is achieved when the true pattern parameters are known, which I provided as the control case.

From Table 2 for the sine wave results, DS estimation is better than both the MLE estimates and the MLE estimates with a lower bound. Note that one other advantage of the DS approach is the decrease in computational requirements.

Robustness of the GLR

Because the parameters of a signal may be time varying in practice, parameter estimation algorithms should be adaptive. To investigate the GLR charts and the parameter estimation algorithms further, a robustness study was conducted. In this section, I show the robustness of the direct smoothing parameter estimation algorithm in estimation of time-varying parameters. In the simulations, I generated a parameter value at each time *t* when the fault pattern was present. This robustness study was conducted by generating the specific fault pattern's parameters randomly from a probability distribution at each time *t*. Two probability distributions were used, a uniform distribution and a truncated normal distribution. For the linear trend cases, the distributions of the parameters are θ~ Uniform (0.05, 0.15) and

Table 3. Robustness of GLR Charts with DS and for a Linear Trend Type Pattern

h=2.45	Pattern starts at	Constant Parameters θ = 0.1	Uniform Distribution	Truncated Normal Distribution
ARL(0)		306.58 (4.18)	306.58 (4.18)	306.58 (4.18)
ARL(1)	$\tau = 0$	12.69 (0.04)	12.75 (0.04)	12.87 (0.04)
ARL(1)	$\tau = 50$	12.60 (0.05)	12.56 (0.05)	12.54 (0.05)

Table 4. Robustness of GLR Charts with DS and for a Sine Wave Type Pattern

h=2.4	Pattern starts at	Constant Parameters θ = 0.1	Uniform Distribution	Truncated Normal Distribution
ARL(0)		338.76 (4.55)	338.76 (4.55)	338.76 (4.55)
ARL(1)	$\tau = 0$	18.89 (0.14)	19.69 (0.14)	18.79 (0.14)
ARL(1)	$\tau = 50$	17.77 (0.14)	18.06 (0.15)	18.09 (0.15)

θ~ Doubly Truncated Normal (0.1, 0.00625) where truncations are at 0.05 and 0.15. For the sine wave cases, the distributions of the parameters are θ~ Uniform (0.5, 1.5) and θ~ Doubly Truncated Normal (1, 0.0625) where truncations are at 0.5 and 1.5. Note that the means of the distributions are at 0.1 for the linear trend cases and at 1.0 for the sine wave cases. In Tables 3 and 4, the results are provided. The standard errors of the ARLs are given in parenthesis.

In Tables 3 and 4, it can be seen that DS approach is robust in the sense that randomness in the parameter values do not affect the ARL_1 performances much (from comparisons of the ARL_1 performances for the constant parameter case with the uniform and truncated normal distribution cases). This property is important since changes in parameter values may be encountered in practice due to many factors and a control chart, which is not affected much in such circumstances, is preferable.

CONCLUSIONS

Fault detection is an important problem in quality control since occurrence of faults may significantly affect product quality characteristics. In order to detect occurrence of faults, statistical process control methods such as control charts can be utilized. In this research, I assumed that some knowledge on a fault and its affect on the quality characteristic were available from previous experience. Affect of faults on the quality characteristic is also considered to be in the form of a change in the mean of the monitored quality characteristics. For detection of such faults, I used a generalized likelihood ratio control chart but utilized the direct smoothing method for estimating the parameters of the fault's pattern.

Through simulations of a linear trend and a sine wave type fault patterns, it is shown that performances of these charts can be improved. Another advantage of the proposed approach is the reduction in the computational requirements; direct smoothing method is computationally less demanding.

In practice, parameters of a signal may be time varying. Parameter estimation algorithms that are adaptive might be useful. To investigate the GLR charts and the direct smoothing algorithm (which is adaptive) under the time varying parameter case, a robustness study was conducted. Simulations of the direct smoothing approach indicate that it is also robust when the parameters vary over time.

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