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Novel Radar Signal with Chaotic Behaviour for Better Decide on the Presence of Targets

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Abstract

This paper is a preliminary investigation of a class of radar signals generated by recursively defined functions. Signals are of fundamental importance in radar, since a radar system uses transmitted and returned signals from the environment to decide on the presence of targets, as well as their range and speed. The issue of an appropriate choice of signal is complex, and application dependent. This paper is a brief simulation of a new class of signals based chaos theory. Such signals may be of importance in radar for a number of reasons. The first is that they are generated from a deterministic map, but can be made to appear as noise. This would be useful from an electronic protection point of view. Secondly, since these signals can be generated from a single dynamical system, with different control parameters and initial conditions, it may be possible to reduce the need for a comprehensive library of signals in a radar system. The generation of such signals, as a discrete time dynamical system, will be outlined. We investigate the stability of such signals, using the Lyapunov spectrum. The radar ambiguity function is used to decide whether this signal is practical use in radar.

Keywords: Lyapunov spectrum, Chaos theory, RCS, Chaotic signals, Ambiguity Function

INTRODUCTION

This paper is a preliminary investigation of a class of radar signals generated by recursively defined functions. Signals are of fundamental importance in radar, since a radar system uses transmitted and returned signals from the environment to decide on the presence of targets, as well as their range, bearing and speed. The issue of an appropriate choice of signal is complex, and application dependent. There are many types of signals, including linear frequency modulation (LFM), pseudorandom codes, step frequency continuous waves, step frequency pulse trains, single frequency pulse trains and random noise. This paper is a brief examination of a new generation and new class of signals, known as chaotic signals. Such signals may be of importance in radar for a number of reasons. The first is that they are generated from a deterministic map, but can be made to appear as noise. This would be useful from an electronic protection point of view. Secondly, since these signals can be generated from a single dynamical system, with different control parameters and initial conditions, it may be possible to reduce the need for a comprehensive library of signals in a radar system. The generation of such signals, as a discrete time dynamical system (DDS), will be outlined. We investigate the stability of such signals, using the Lyapunov spectrum. The purpose of this paper is to consider a class of recursively defined signals that exhibit complex dynamics in phase space. This work arose out of an interesting radar signal in [1]. This signal exhibits sensitive dependence on initial conditions, and has an unusual self replicating feature in phase space. The main issue to be addressed in this paper is whether such signals are of practical use in radar. Signals are of paramount importance in radar. Radar systems use different types of signals for specific applications in varying situational contexts [2]. There are hence many different classes of radar signals. [2] Describes four classes of radar signals, based upon their characteristics and ambiguity functions. Linear frequency modulation, single frequency pulse trains, step frequency continuous waves, step frequency pulse trains, pseudorandom codes and random noise are examples pointed out in [1]. A relatively new class of signals are those which have chaotic dynamics [1, 3, 4]. Such a class of signals exhibits a phenomenon known as sensitive dependence on initial conditions. These signals may be useful in radar in the way code division multiple access (CDMA) is useful in digital telephony [4]. The advantage of CDMA is that the transmitted signal appears as noise to all but the intended recipient. It is possible that chaotic signals could be used to mask the signal within environmental noise and interference, so that targets may not be aware of the presence of scanning radar. Hence these signals may be of use as an electronic protection measure for the radar platform. Chaos theory investigates strange behavior found in nonlinear deterministic dynamical systems. Such unusual behavior in dynamical systems was first discovered by Poincare, who was attempting to show rigorously that the solar system, as modeled by Newton's Laws of Motion, is dynamically stable [5]. He discovered that small differences in initial conditions can produce drastically different final solutions. Another early important study of dynamical systems exhibiting strange behavior is [6], who developed a system of coupled nonlinear differential equations to model weather patterns, and also observed this strange sensitivity to initial conditions. Lorenz coined the phrase "the butterfly effect" to illustrate this sensitivity. The latter implies a butterfly flapping its wings in one part of the world can have an effect on the weather in another distant part of the world [6,7]. The term chaos first appeared in a dynamical systems context in [8].A chaotic dynamical system is a nonlinear dynamical system whose output has sensitive dependence on initial conditions. Chaos theory is the analysis of the behavior of such systems [3,7]. As such, chaos theory is not really a theory of chaos, but is more concerned with understanding the complex behavior of nonlinear dynamical systems. We will introduce briefly the study of such systems, and in particular, will be interested in determining under what conditions such a system becomes chaotic. A class of radar signals will be introduced .We will investigate one signal in this class, and apply the ambiguity function to see whether they are of practical use in radar [7].

RADAR SIGNALS AND DYNAMICAL SYSTEMS

Let S be the class of all real-valued radar signals, including both those defined on a discrete and continuous time domain. In order to motivate the work to follow, we consider a subclass of S that can be defined through a dynamical system. Assume $x(t) \in S$, is continuous and differentiable within a domain. From a physical point of view, the derivative of this signal, $dx(t)/dt$, is the rate of change of it in a propagating medium. We may hence analysis the signal via this rate of change, so assume that $\varphi(t) = dx(t)/dt$. In addition to this, we may assume that in some cases, $\varphi(t)$ can be expressed as a nonlinear composition of the signal $x(t)$. Thus, there may exist a function ψ such that $\varphi(t) = \psi(x(t))$. Hence, equivalently, $dx(t)/dt = \psi(x(t))$. To show this class of signals is nonempty, consider the signal $x(t) = \sin(t)$, for $t \in D = (0, \pi/2)$. Its derivative is $dx(t)/dt = \varphi(t) = \cos(t)$. Choose $\psi(t) = \sqrt{1-t^2}$, Then $\psi(x(t)) = \varphi(t)$ on D. Hence, we can define the class of signals $C = \{x(t) : \text{there exists a } \psi : dx(t)/dt = \psi(x(t))\}$. As in [9], we can consider discretised time, since signals are often digitized so they can be processed by digital computers. Hence we can further restrict attention to the subclass $C_D \subset S$ defined by the set $\{x(n) : \text{there exists a } \psi : x(n+1) = \psi(x(n))\}$. The function ψ defined in the set C_D will be referred to as a generator of a signal. We specialize the discussion to one dimensional discrete maps, since we will be exclusively studying signals from the class C_D. A dynamical system is a physical system which is described by a deterministic set of rules that change with time. Suppose the variables $\{x(n) : n \in \mathbb{N}\}$ describe the states of the system at each discrete time point n. Then we assume there is a generator function ψ, defined on the range of $x(n)$, such that:

$$x(n+1) = \psi(x(n)) \tag{1}$$

The generator ψ in (Eq.1) determines the evolution of the system, and in general will be nonlinear. We refer to the system's state space as the space M where the functions $x(n)$ take values. In the context of the discrete signals considered here, the state space will be subsets of the real line. As an example, the well-known Logistic Map has generator $\psi(x) = \lambda x(1-x)$, and the corresponding map, defined through (1) has initial value $x(0) \in (0, 1)$. We will show that this map becomes unstable as λ changes, using its Lyapunov spectrum. It is not difficult to write down the general solution to (Eq.1). Let $\psi^k(x)$ be the kth composition of ψ with itself. By a simple recursion, it is not difficult to show that is the solution to (Eq.1).

$$x(n) = \psi^n(x(0)) \tag{2}$$

The path in the state space that a dynamical system, defined through (Eq.2), traces out is called its trajectory or orbit. A dissipative dynamical system is characterized by convergence of trajectories in its state space. An attractor is defined to be a set of points to which all neighboring trajectories converge in phase space. The attractor set consists of all limit points of the discrete map defined by (Eq.1). A point attractor is a single point to which trajectories converge, also known as a stable fixed point. A dynamical system may have a set of points which are visited periodically. Such points are referred to as a stable limit cycle, with periodic orbits. An attractor is also an invariant set, meaning that when a trajectory starts in it, it remains in the set forever. The largest subset of an attractor set, consisting of the largest set of points to which all nearby orbits converge, is called the basin of attraction. In a nonlinear dynamical system, where orbits in an attractor move apart with increasing time, the system is said to possess a strange attractor. Such dynamical systems are referred to as being chaotic. Chaos is often defined to be the periodic long run behavior in a deterministic dynamical system that exhibits sensitive dependence on initial conditions. There are three so-called signatures of chaos:

a) Aperiodicity in limiting behavior, meaning that the system does not converge to a single point as discrete time increases without bound;

b) It is a deterministic system, with no stochastic component, but is nonlinear;

c) The system exhibits sensitive dependence on initial conditions, meaning arbitrarily close trajectories will diverge apart exponentially fast.

LYAPUNOV SPECTRUM

Lyapunov exponents of a dynamical system provide a quantitative measure of its sensitivity to initial conditions. The Lyapunov spectrum of a map is a plot of its Lyapunov exponents. As pointed out in [9], they give the average rate of convergence or divergence of the system along the principal axes in phase space. The existence of at least one positive Lyapunov exponent is a necessary condition for a dynamical system to be chaotic [10]. Hence, given a dynamical system, if we can establish that there is at least one positive Lyapunov exponent, then we can be certain the system will exhibit chaotic dynamics. For a dynamical system, such as that generated by (Eq.1), where we know the generator ψ, it has been shown that the complete Lyapunov spectrum can be easily computed [11]. This can be done by considering the perturbation of a point of the system, and applying a linear stability analysis [8].We focus on the calculation of Lyapunov exponents in the current context of one-dimensional discrete maps. The ideas to follow can be found in [12, 13]. Consider the dynamical system (Eq.1), with initial condition $x(0)$. further details can be found in [12]. It is clear from (Eq.3) that μ depends on the starting point $x(0)$.

For a given attractor, μ is invariant in the basin of attraction [15]. Simulation tests of a small perturbation of this starting point, defined by $x(0) + \delta(0)$, where the initial separation $\delta(0)$ is assumed to be very small. Suppose $\delta(n)$ is the separation after n iterations of the system. If $|\delta(n)| \approx |\delta(0)|\, e^{n\mu}$, then μ is called a Lyapunov exponent. These can be found, for a trajectory starting at $x(0)$, from the limit

$$\mu = \lim_{m\to\infty}\left(\frac{1}{m}\sum_{j=1}^{m-1}\log|\psi'(x(j))|\right. \tag{3}$$

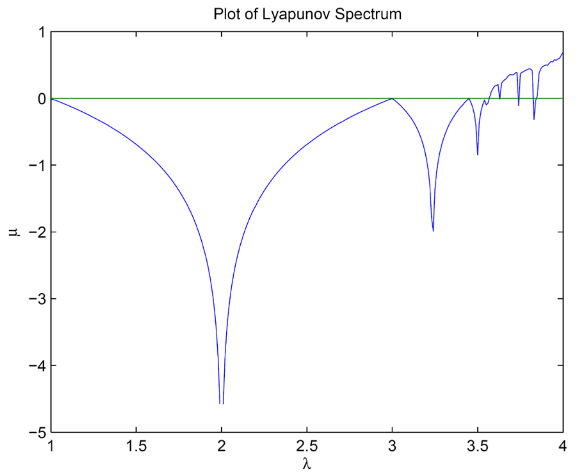

Figure 1. A plot of the Lyapunov spectrum for the logistic map $x(n+1) = \lambda x(n)(1 - x(n))$, with $x(0) = 0.1$.

If a Lyapunov exponent is negative for a particular orbit, then the orbit either has a stable fixed point, or a stable limit cycle. In the case where it is positive, the orbit is in a strange attractor, and the trajectory will be chaotic. Fig.1 is a plot of the Lyapunov spectrum for the Logistic map introduced previously, with $x(0) = 0.1$. As can be observed, the map has stable dynamics until at approximately $\lambda = 3.6$, where it becomes chaotic, followed by periods of stability and then chaos. In the current context of radar signals in the class C_D, we can calculate the Lyapunov spectrum, and use it to decide which parameter values generate a chaotic signal.

RADAR AMBIGUITY FUNCTION

An important tool in the design and evaluation of radar signals is the radar ambiguity function [2, 7, 14, 15, 16]. As pointed out in [15], it represents the time response of a signal filter matched to a specified signal of finite energy, when the signal is received with a time delay τ and a Doppler shift φ relative to the nominal values expected by the filter. We denote this function as $\chi(\tau, \varphi)$. There are number of variations in the definition taken for $\chi(\tau, \varphi)$. We base ours closely on that in [15,16]. For a discretised complex-valued signal $x(n)$, of length N, the ambiguity function is defined to be:

$$\chi(\tau, \varphi) = \frac{1}{N} \sum_{n=1}^{N} x(n) x^*(n+\tau) e^{-i\varphi n} \tag{4}$$

Where the star denotes complex conjugate [1] contains a detailed discussion of the desirable features of radar ambiguity functions. The following discussion is based closely on this source. One perspective in the radar community is that an ideal radar signal is one which produces an ambiguity function that is a spike at the origin, and zero everywhere else. The reason for this is that it can be shown in order to optimally detect a target, it is necessary to maximize $\chi(0, 0)$. Additionally, in order to minimize the probability of false detections of targets, it is necessary to minimize $\chi(\tau, \varphi)$ with $(\tau, \varphi) \neq (0, 0)$. As pointed out in [16],[17], the ambiguity function was not introduced to radar signal analysis via the matched filter, but as a normed difference between a signal and a copy of it that differs in time delay and Doppler shift [14]. To illustrate this, suppose we have a discrete radar signal $x(n)$, which we assume is a member of the Hilbert space of complex valued signals of finite energy, discrete time modulo N, with inner product $< x1, x2 > = \sum_{j=0}^{N} x_1(j) x_2^*(j)$. The norm induced by this inner product is $||x|| = \sqrt{< x, x >}$. We can consider the return from a radar signal x as a time delay and Doppler shifted version of the original, and so define an operator $D(\tau, \varphi)x(j) = e^{2\pi i \varphi j/N} x(j + \tau)$. By applying properties of inner products, it can be shown that :

$$||x - D(\tau, \varphi)x||^2 = 2||x||^2 - 2\,\Re e(\chi(\tau, \varphi)) \tag{5}$$

Where $\Re e$ is the real part of a complex number Expression (Eq.5) is the squared normed difference between the original signal x and its time delayed and Doppler shifted version $D(\tau, \varphi)x$.

To be able to differentiate between the two signals, we require the normed difference (Eq.5) to be maximized, except in the case where $\tau = \varphi = 0$. In the latter case, the two signals are the same. Note that $||x||^2$, which is the signal energy, is constant for a given signal. Hence, to maximize (Eq.5), we need to minimize $\Re e(\chi(\tau, \varphi))$. This can be achieved if we minimize the absolute value of the ambiguity function (Eq.4). Hence, in order to optimally differentiate a signal from its time delay and Doppler shifted version, we need an ambiguity function that is like a "thumbtack", namely a spike at the origin and almost zero everywhere else. It is, however, impossible to produce a radar signal with such an ambiguity function. The main reason for this is that it can be shown that the volume under $|\chi(\tau, \varphi)|$ is a non-zero constant, whose square is the energy in the transmitted signal, and cannot be confined to a single spike at the origin. If the absolute value of the ambiguity function has a large volume located near the origin, producing a wide peak, then the ability of the radar to resolve targets will be limited in that region. False detections may occur if there are large spikes in the ambiguity function's absolute value away from the origin. This may also cause the masking of secondary targets. Fig. 2 and 3 are plots of the absolute value of ambiguity functions of two standard radar signals. The plots in Fig. 2 are for a standard single frequency pulse, while that for Fig.3 are for a standard linear FM pulse [15,16,17 for more details]. In each Figure, two subplots are used. The first one shows the ambiguity function as a colored contour map, with colors illustrating the

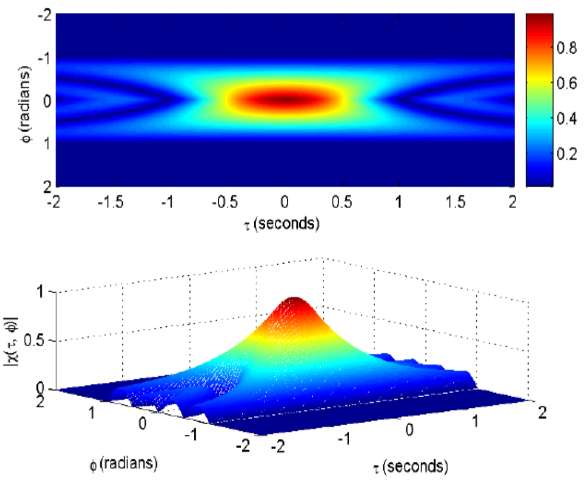

Figure 2. Ambiguity function plots for a single frequency pulse. The delay unit is seconds, the Doppler unit is radians and the absolute value of the ambiguity function scale is linear. The pulse duration is 1 second.

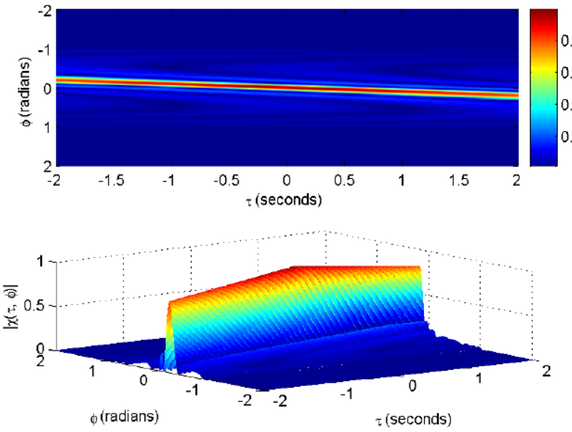

Figure 3. Ambiguity function plots for a linear FM pulse, with the same scale units as for Fig.2, including a linear scale for the absolute value of the ambiguity function. As for the example in Fig.2, the pulse duration is 1 second.[17]

magnitude of the function at each time delay (τ) and Doppler level (φ). The second plot shows the ambiguity function as a surface in space. Both signals have pulse duration of 1 second. The plots of Fig.2 show that the ambiguity function has a large ridge along the axis $\varphi = 0$. As pointed out in [2], this means that the corresponding signal will provide high resolution in Doppler shift, but not in time delay.

One signal design feature that can be deduced from the ambiguity function plot of Fig.2 is that a shorter pulse will provide better range resolution than a longer one. The ambiguity function plots in Fig.3 show a large ridge at an angle to the τ and φ axes. It is pointed out in [2] that the signals corresponding to such ambiguity functions will have some difficulty in resolving targets. Specifically, it is possible the signal will resolve all targets well, except those with a Doppler and time delay product which matches the angle of the ridge. We will use the ambiguity function in the following section, to decide whether our produce member of the class C_D would be of potential applicability in a radar context.

NEW RADAR WAVEFORM

The chaotic signal is that with generator $\psi(x,\lambda) = \sin(2\pi x) + \cos(2\pi\lambda x)$. As for the first, we show its sensitivity to initial conditions. Fig.4 is a plot of the evolutions of two orbits of this signal, with one slighly perturbed, and corresponding pointwise differences of two evolutions of (Eq. 2). The first begins with $x(0) = 0.01$, while the second is perturbed by 10^{-12}, so that it evolves from $y(0) = 0.01 + 10^{-12}$. In both cases we choose $\lambda = 4$. As can be observed from Fig.4, the two corresponding trajectories evolve together for a very short period, then diverge apart. Next we examine the Lyapunov spectrum for $\psi(x,\lambda) = \sin(2\pi x) + \cos(2\pi\lambda x)$ signal. Fig.5 is the Lyapunov spectrum, with λ ranging from 0.1 to 5. We have chosen the initial value $x(0) = 0.01$, for each λ, and $m = 1000$ as before. The spectrum shows this signal becomes increasingly more chaotic as λ increases. There is a very significant upward trend in Fig.5, and initial periods where there is stability. Fig.6 is a plot of the signal, as a function of λ. In this case, λ varies from 0.1 to 2, in steps of 0.001. The initial condition is $x(0) = 0.01$, for each λ, and $10{,}000$ iterations have been used to produce each value.

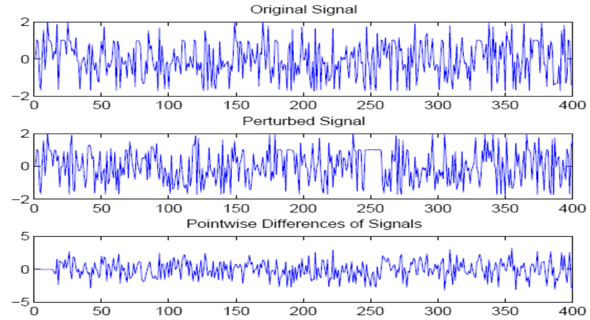

Fig.4. Differences of two signals generated with $\psi(x,\lambda) = \sin(2\pi x) + \cos(2\pi\lambda x)$, with a perturbation of 10^{-12} in initial starting values. The top plot is of the original signal, the second is a slightly perturbed version, while the third is of the corresponding pointwise differences.

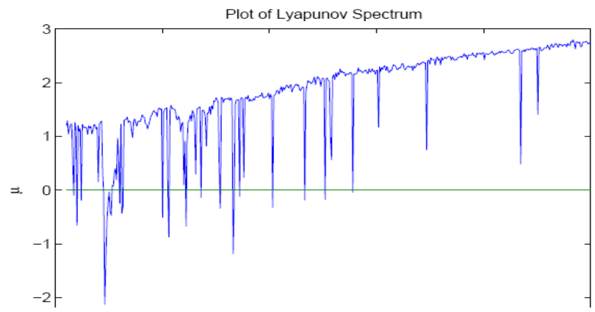

Fig.5. The Lyapunov spectrum for the signal with generator $\psi(x,\lambda) = \sin(2\pi x) + \cos(2\pi\lambda x)$.

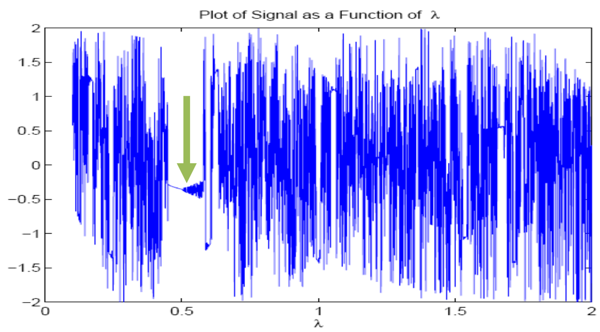

Fig.6. A plot of the result of 10,000 iterations of the signal with generator as a function of λ.

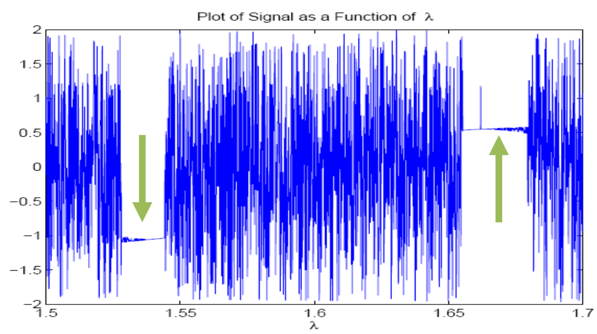

Fig.7. A magnification of the plot of Fig.6, with λ in the range from 1.5 to 1.7, in steps of 0.0001. The plot shows the generation of two new conical structures, indicating the transition from chaos to order. The first occurs at approximately $\lambda = 1.525$, while the second is at roughly $\lambda = 1.65$.

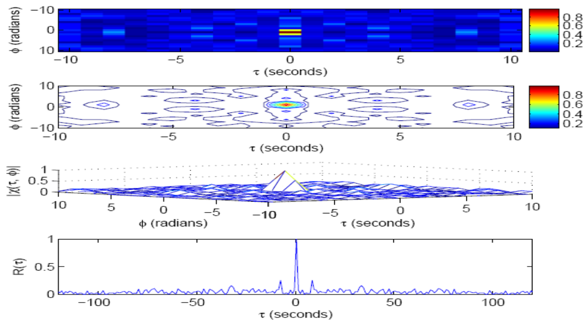

Fig.8. Ambiguity function plots for the signal with generator $\psi(x,\lambda) = \sin(2\pi x) + \cos(2\pi\lambda x)$, as a function of λ.

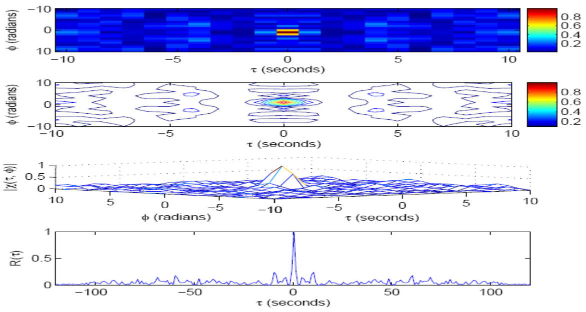

Fig.9. Ambiguity function and autocorrelation plots of the signal with generator $\psi(x,\lambda) = \lambda\sin(2\pi x)$. [17]

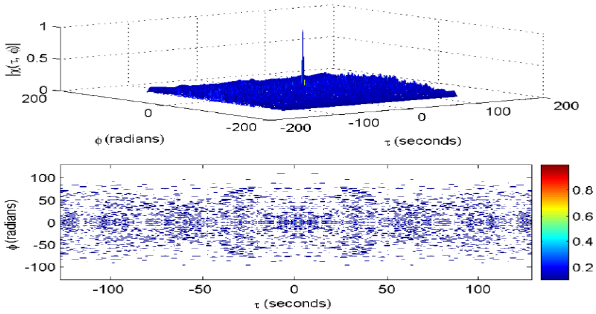

Fig.10. A plot similar to that of Fig.11, except for the signal of $\psi(x,\lambda) = \lambda\sin(2\pi x)$.. As before, the first plot shows the absolute value of the ambiguity function as a surface, while the second is a color-coded contour plot. The ambiguity function is also in a linear scale, as previously.[17]

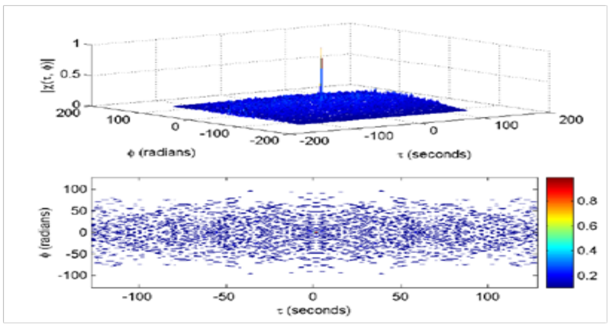

Fig.11. Ambiguity function plots for the signal with generator $\psi(x,\lambda) = \lambda\sin(2\pi x)$., except the plots are over larger time delay and Doppler shift intervals. The top plot is of the absolute value of the ambiguity function, again as a surface, while the second plot shows the contours and corresponding ambiguity function values using a color bar. The ambiguity function is in a linear scale.[17]

The plot shows some very unusual behavior. The Lyapunov spectrum over this range of λ shows the signal is chaotic, with short periods of stability. A close examination of Fig.6 shows that these periods of stability bifurcate to chaos in a conical structure. Near $\lambda = 0.5$ is a clear illustration of this. There is a short period where the signal has become stable, then bifurcates into a cone preceding chaos. What is not so clear from the plot, but can be found on magnification, is that there are many of these cone-like transitions. One small one can be seen roughly after the significant cone, near$\lambda = 0.5$, becomes chaotic. At around $\lambda = 1.2$ another can be observed. Fig.7 is a magnification of the plot in Fig.6, over the region where $1.5 \leq \lambda \leq 1.7$. Increasing the number of iterations used, for each λ, improves the resolution of these plots and shows these transitions more clearly.

Fig.8 is a complementary plot of those in Fig.9, for this second signal under investigation. In this case we take $\lambda = 4$. As for the plots in Fig.9, we see that the absolute value of the ambiguity function is very similar to that of the first example. There is a difference in the contours as can be seen by comparing both second plots in Fig.9 and Fig.8. As before, this signal is a member of the class of irregular/noise like signals. Its utility is exactly the same as that of the $\psi(x,\lambda) = \lambda\sin(2\pi x)$,[17]. Fig.10 is a complementary plot of that in Fig.11, showing the absolute value of the ambiguity function on a larger time delay and Doppler shift grid.

As in the ambiguity and autocorrelation plots in Fig.9 the first two subplots of Fig.8 are color-contour illustrations of the size of the absolute value of the ambiguity function in (τ, φ) space and the third plot is a surface plot, while the fourth plot is of the normalized autocorrelation function. The ambiguity function is in a linear scale.

We describe the plots in Fig.9, from the top down. The first plot shows the absolute value of the ambiguity function's concentration in terms of a color spectrum. The second plot shows a colored contour version of the first. The third plot shows the absolute value of the ambiguity function as a surface in space. Finally, the fourth plot shows the normalized autocorrelation function. Since the corresponding signal is in discrete time, the time delay (τ) axis values are in discrete time units. Better graphical resolution is achieved by extending this axis (see Fig.11). The Doppler axis is in units of radians, while the time delay is measured in seconds. The absolute value of the ambiguity function is in a linear scale. The autocorrelation plot is shown over a larger spectrum of values of τ than used in the previous three subplots, and also is in a linear scale.

CONCLUSIONS

This note introduced a class of recursively defined signals, motivated from [8]. Their stability was analyzed using a Lyapunov spectrum. These signals are members of a class of irregular/noise like signals described in [2],[17]. Based upon the shape of the absolute value of their ambiguity functions, we can conclude that they should have an ability to resolve targets in both time delay and Doppler shift. The disadvantage of such signals is that they have relatively high range and Doppler side lobes in the ambiguity diagram, which will limit their ability to discriminate small targets against clutter, and small targets in the vicinity of larger targets.

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