International Journal of Natural and Engineering Sciences

Uluslararası Doğa ve Mühendislik Bilimleri Dergisi

E-ISSN: 2146-0086, 11 (1): 04-09, 2017

A Gain Tuning Method for Active Vibration Control of 1 Degree-of-Freedom Mechanical Systems: A Proportional-Integral-Derivative Control Approach

Barış BIDIKLI

Department of Mechatronics Engineering, Faculty of Engineering and Architecture, İzmir Katip Çelebi University, İzmir, Turkey

Received: February 20, 2017

Email: baris.bidikli@ikc.edu.tr

Accepted: May 03, 2017

Abstract

Proportional-integral-derivative (PID) controller is one of the most popular controllers that are used for the active vibration control of mechanical systems. In the most of these studies it can be observed that control gain tuning is the main issue of the active vibration control. An inappropriate selection of control gains may cause inefficient control performance and be harmful for the controlled system. In PID control it is possible to select the control gains according to desired transient and steady state specifications when the system model is exactly known but in general exact knowledge of the system model is not provided. In this study a system model based gain tuning algorithm is proposed for the active vibration control of 1 degree-of-freedom mechanical systems. To realize this purpose, the transfer function of the system is estimated from the free vibration response of the system. The estimation process is realized by obtaining damping ration and undamped natural frequency of the system from the transient state specifications of free vibration response. Then an estimated transfer function based PID control gain adjustment method is proposed. Performance of the proposed method is demonstrated via numerical simulation studies.

Keyword: 1 degree-of-freedom systems, Mechanical systems, Active vibration control, PID control, Gain tuning.

INTRODUCTION

Since it affects consistency and repeatability negatively, vibration is one of the most important problems for the mechanical systems. Vibration is an unavoidable fact especially in mechanical systems that have flexibility in their structure. In flexible mechanical systems, eliminating the occurred vibration as quickly as possible after the system has reached to desired position is the main aim of the vibration control. In the active vibration control a closed-loop system model and an external actuator are needed to reach this aim. Proportional-integral-derivative (PID) controllers are widely preferred as a closed-loop controller in active vibration control systems.

In [1], a PID controller was designed to provide an active seismic control of a multi degree-of-freedom (dof) structural systems against earthquakes. In [2], PID controller was used as an outermost controller of a hybrid control technique applied to a vehicle active suspension system of a quarter car model using skyhook and adaptive neuro active force control. In [3], a PID based output feedback controller is utilized for the active vibration control of cantilever beam. In [4], vibration suppression of smart beams using the piezoelectric patch structure was presented. In the mentioned study, a state space model characterizing the dynamics of the physical system was developed from experimental results using PID approach for the purpose of control law design. In [5], an active vibration control system of a smart plate is realized by utilizing PID control strategy. Mentioned studies are a few of the numerous PID control studies in the literature. Moreover, realizing the performance demonstration by comparing the performance of the designed controller with a PID controller is an another prevalent approach used in the literature [6], [7].

Thanks to its simple structure, robustness and applicability, PID control has been successfully used in both industrial and academic applications. Nevertheless, adjusting the control gains, which is the most important aspect of the PID control process, remains as an open problem. An inappropriate selection of control gains decreases the efficiency of control. As a result of this, trial-and-error method and other methods that are not based on a valid basis may not be considered as proper solutions to cope with this issue. Since it has been observed that poor selection of the control gains affects the control process dramatically, proposing optimal tuning methods for PID control gains has become a hot topic in the control literature. Detailed surveys about this topic can be found in [8] and [9]. This topic maintains its validity in the last decade. In [10], a new robust PID tuning method for the optimal closed-loop performance with specified gain and phase margins based on nonlinear optimization was developed. A stochastic, multi-parameters, divergence optimization method for the auto-tuning of PID controllers according to a fractional-order reference method was presented in [11]. In [12], gain and phase margin specifications of the inner and outer loop based internal model control plus PID tuning procedure was proposed. In [13], a new model reduction method and an explicit PID tuning rule for the purpose of PID auto-tuning on the basis of a fractional order plus time delay model are presented. In [14], a Newmark method based PID control rule was proposed for the active vibration control of multi dof flexible systems. As it can be clearly seen from these studies partial or exact knowledge of the system model or obtaining a model that provides a good representation of the behavior of the system is a critical issue to propose an optimal gain tuning method for the PID control.

Proposing a system model based gain tuning method for the active vibration control of 1 dof mechanical systems is the main purpose of this study. Single link flexible robot manipulators are the most known examples of these type of mechanical systems. There are lots of available vibration control studies about single link flexible robot manipulators in the control literature [15-21]. Active suspension design of a quarter car can be considered as an another example of these type of systems. In general, mechanical structure of a quarter car is modeled as 2 dof mass-spring-damping system. This situation based on the structure that both of the vibration between tire and body and body and driver and/or passenger seats are eliminated via same external actuator. However in studies where only the design of the active suspension system between the tire and the body or between the body and the seat is considered, the system is designed as 1 dof mechanical system. Active suspension system design examples for a quarter car can be found in [2], [21-28]. According to the author's best knowledge there is no proposed gain tuning method for the active vibration control of these type of mechanical systems. 1 dof mechanical systems are modeled as mass-spring-damping systems and thanks to this structure they can be considered as linear systems and are represented via second order transfer functions [29]. To reach the main purpose of this study, a second order transfer function model that represents the mechanical system is obtained from its free vibration response by utilizing transient state specifications of a standard second order transfer function (*i.e.* percentage overshoot and settling time). It can be observed that transfer function of the overall closed-loop system is represented via the third order transfer function when obtained transfer function placed into it and a PID controller is applied to the system. Then, PID control gains are adjusted to express overall closed-loop system via its second order approximation according to the dominant pole approximation [30]. It is also considered that the final model provides the desired transient state specifications while choosing PID control gains. For this purpose transient state specifications namely as percentage overshoot and settling time that are the main issue of active vibration control are considered. Finally, steady state error can be checked via final value theorem. Since it is possible to interfere all of the coefficients of the denominator of a third order transfer function by using a PID controller all of these specifications can be tuned properly. After that point, a PID controller with determined control gains can be applied to the mechanical system to provide the active vibration control. The mentioned procedure is applied successfully to an example 1 dof mechanical system in this study. All steps are tried to be explained clearly and the numerical simulation studies are utilized for the performance demonstration.

MECHANICAL SYSTEM

In the first part of this section, structure and mathematical model of 1 dof mechanical systems are introduced. Then, a free vibration response and transient state specifications based transfer function estimation process is explained.

Structure of the system and its mathematical model

Structure of 1 dof mechanical systems is shown in Figure 1. As it can be seen from this figure the mentioned mechanical system is modeled as 1 dof mass-spring-damping system. In this figure mass of the body, spring and damping constants are denoted by m_b, k and c, respectively. Base excitation is represented by z(t). For this study it is assumed that the control input is applied to the system as a base excitation.

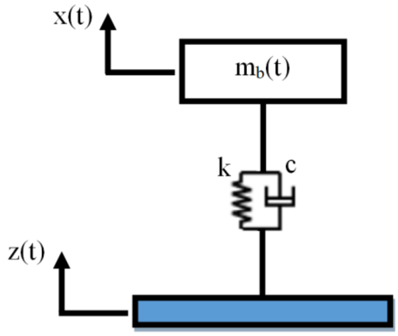

Figure 1. 1 dof mechanical systems

Eliminating the vibrations that occur at the end point position denoted by x(t) by applying an appropriate control input is the main objective of active vibration control. Mathematical model of the given system can be obtained via Euler-Lagrange equations and is expressed as

$$m_b\ddot{x}(t) + c\dot{x}(t) + kx(t) = c\dot{z}(t) + kz(t). \quad (1)$$

Transfer function that represents the relationship between the base excitation z(t) and the end point position x(t) is obtained as follows when the Laplace transform is applied to the mathematical model in (1)

$$G(s) = \frac{X(s)}{Z(s)} = \frac{cs + k}{m_b s^2 + cs + k} \quad (2)$$

where X(s) and Z(s) denote the Laplace transforms of x(t) and z(t), respectively. In the above equation transfer function of the mechanical system is represented by G(s) while s denotes the Laplace variable. At this point it should be stated that mathematical model and the transfer function of the system are given to show that these type of mechanical systems are represented by second order transfer functions. Neither mathematical model nor transfer function is not used for modeling and/or gain tuning process. Remaining parts of the study are based on the free vibration response of the system as it is given in the next subsection.

Transfer function estimation

Transfer function of 1 dof mechanical systems can be estimated by utilizing standard unity gain second order system definition. The structure of the standard unity gain second order transfer function is given as [30]

$$G_{so}(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{3}$$

where ω_n denotes the undamped natural frequency and ξ denotes the damping ratio. From (3) it is clear that systems that have a second order transfer function can approximately be represented via this structure as long as their undamped natural frequencies and damping ratios are known.

These values can be determined by utilizing transient state specifications namely as percentage overshoot and settling time [30]. The mentioned values can be obtained from the free vibration response of a mechanical system given in Figure 1. To obtain this response, a unit step input is applied to the system as a base excitation and the free vibration response of the end point is measured. Percentage overshoot is related with the peak value of the response. Its mathematical expression is given as

$$OV = \frac{x_{peak} - x_{ss}}{x_{ss}} 100 \tag{4}$$

where OV denotes the percentage overshoot, x_{peak} and x_{ss} denote the peak and steady state values of the end point position, respectively. From (4) it can be seen that percentage overshoot can be calculated by observing peak and steady state values of the free vibration response. After that point it is possible to reach the damping ratio of the system via the following formula

$$\xi = \frac{-\ln(OV/100)}{\sqrt{\pi^2 + \ln^2(OV/100)}}. \tag{5}$$

Settling time is defined as the time for the step response to reach and stay within 2% of the steady-state value. Undamped natural frequency can approximately be calculated via the settling time and the damping ratio according to the following formula [30]

$$\omega_n = \frac{4}{\xi t_s} \tag{6}$$

where the settling time is denoted by t_s. This point is the end of the transfer function estimation process. After that point estimated transfer function can be used in the closed-loop system to tune PID control gains.

TUNING OF PID CONTROL GAINS

In this section it is explained that how PID control gains can be selected according to the estimated transfer function. Firstly overall transfer function of closed-loop system is obtained. Then, the control gains are selected by considering the following issues:

- Reducing the transfer function of the overall closed-loop system by utilizing dominant pole approximation.
- Obtaining the desired percentage overshoot and settling time.
- Eliminating the steady state error.

All of these steps are tried to be explained in a more clear manner.

Laplace domain expression of the PID controller is given as [30]

$$Z(s) = K_p + \frac{K_i}{s} + K_d s \tag{7}$$

where Z(s) denotes the Laplace transform of the control input defined in (1) while constant proportional, integral and derivative gains are denoted by K_p, K_i and K_d, respectively. These gains are adjustable parameters of the PID controller. Increasing the efficiency and the performance of the control by adjusting them to the appropriate values is the main purpose of PID control. The structure of the closed-loop system is shown in Figure 2. In this figure r(t) denotes the reference input.

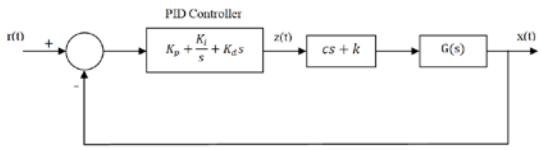

Figure 2. Closed-loop system

Transfer function of the closed-loop system is obtained as follows by utilizing Mason formula [30]

$$H(s) = \frac{\omega_n^2[cK_{ds}^3 + (kK_d + cK_p)s^2 + cK_i s + kK_i]}{(1 + \omega_n^2 cK_d)s^3 + [2\xi\omega_n + \omega_n^2(kK_d + cK_p)]s^2 + \omega_n^2(1 + cK_i)s + \omega_n^2 kK_i} \tag{8}$$

where H(s) denotes the closed-loop transfer function and (3) and (7) are utilized. From (8) it can be seen that transfer function of the closed-loop system is a third order transfer function. However, all of the coefficients of the denominator can be adjusted indirectly via PID control gains. Thanks to this structure PID control gains can be selected to reduce the transfer function to its second order approximation by utilizing dominant pole approximation and provide the desired transient state specifications and steady state error.

According to dominant pole approximation, stable systems (*i.e.* systems whose all poles in the left half plane) can be represented via lower order transfer functions if the real parts of some of their poles are more than 10 times lower than real parts of the remaining ones. For the third order systems this situation can be mathematically expressed as [31]

$$H(s) = \frac{\alpha\omega_n^2}{(s+\alpha)(s^2 + 2\xi\omega_n s + \omega_n^2)} \approx \begin{cases} \dfrac{\alpha}{(s+\alpha)}, & \alpha \ll \xi\omega_n \\ \dfrac{\omega_n^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)}, & \alpha \gg \xi\omega_n \end{cases} \tag{9}$$

where α is a positive constant. The third order transfer function can be expressed by its second order approximation by selecting PID control gains according to the second inequality in (9).

***Remark* 1.** As it can be seen from (9), the third order systems can also be represented via the first order transfer function. However, it is not possible to obtain an appropriate representation for flexible mechanical systems via the first order transfer functions. The general behavior in these type of systems is that vibration starts with an overshoot and reaches the desired level for a certain period of time [29]. It is not possible to represent the mentioned overshoot via a first order transfer function [30]. Since the control gains are tuned according to the estimated model, model's representation capability is the most crucial part of the study. The second order approximation is preferred by considering this issue.

Control gains can also be selected to provide the desired percentage overshoot and settling time for the second order system. At this point it should be stated that these specifications are crucial for the active vibration control and they can be adjusted for the second order systems via damping ratio and undamped natural frequency as [30]

$$OV = e^{-\left(\xi\pi\sqrt{1-\xi^2}\right)}100$$
$$t_s = \frac{4}{\xi\omega_n}. \tag{10}$$

Finally, it is possible to guarantee zero steady-state error by utilizing final value theorem. Final value theorem is given as

$$x(\infty) = \lim_{s\to 0} sX(s). \tag{11}$$

In the active vibration control, the main purpose is to keep the end point at a constant amplitude so step input with a specific amplitude is applied as the reference input [29]. From (8) and (11) it can be seen that end point position always converges to amplitude of the reference step input as long as K_i is not equal to zero. This situation guarantees zero steady state error.

***Remark* 2**. Although the procedure described in this section is given for the PID controller, it can be used to set the control gains of all controllers have the structure that can meet the specified requirements.

***Remark* 3**. A second order system is needed to determine the desired percentage overshoot and settling time and tuning the control gains according to these specifications. Expressing the third order transfer function of the closed-loop system is only possible with the dominant pole approximation. To guarantee this approximation using a controller whose structure is capable to adjust all coefficients of the denominator of the transfer function of the closed-loop system is a crucial issue.

SIMULATION RESULTS

Computer based simulations were realized to demonstrate the efficiency of the proposed method. For these simulations numerical values of 1 dof mechanical system were selected as m_b=240 kg, k=16000 N/m, c= 280 Ns/m. At this point it should be stated that, a mechanical system that is similar to a quarter car model is considered in this study. As a result of this it can be considered that this simulation is an example of active suspension system design between tire and the body of a quarter car model.

A unity gain step input was applied to the system and free vibration response was obtained to estimate the transfer function of the system. Free vibration response is shown in Figure 3.

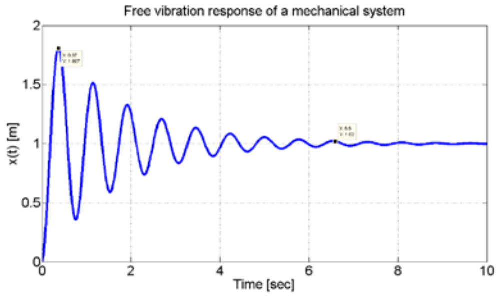

Figure 3. Free vibration response

As it is highlighted in Figure 3, system has 80.7% percentage overshoot and 6.6 s settling time. The damping ratio and the undamped natural frequency of the system can be obtained by utilizing these values as ξ=0.0681 and ω_n=8.899 rad/s, respectively. The estimated transfer function of this mechanical system G_{MS} is obtained as follows by utilizing the structure in (3)

$$G_{MS}(s) = \frac{79.2087}{s^2+1.2121s+79.2087}. \tag{12}$$

Step response of the transfer function given in (12) and the free vibration response of the system in are given together in Figure 4. From this figure it can be seen that the estimated transfer function represents the mechanical system successfully.

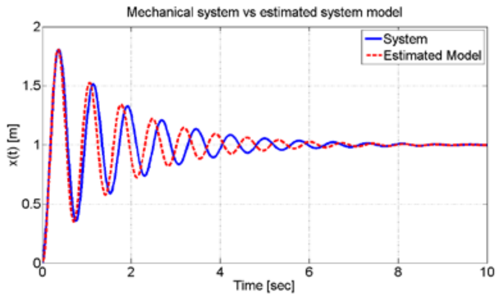

Figure 4. Step responses of the mechanical system (line) and the estimated system model (dashed)

For the closed-loop system the desired percentage overshoot and the desired settling time were determined as

$$OV \leq 10\%,$$
$$t_s \leq 2\ s. \tag{13}$$

Control gains were adjusted as K_p=0.1522, K_i=3.2982 and K_d=0.3189 to provide the desired specifications and other requirements. As a result of this adjustment, transfer function of the closed-loop system is obtained as

$$H(s) = \frac{27.68s^2+13.21s+286.3}{s^3+29s^2+100s+286.3}. \quad (14)$$

where (8) is utilized. Poles of the transfer function of closed-loop system are obtained as

$$p_1 = -25.5213,$$
$$p_{2,3} = -1.7394 \pm 2.8623i. \quad (15)$$

According to poles given in the above equation, dominant pole approximation is provided and the closed-loop system can be represented via a second order transfer function. When the PID controller with the determined gains was applied to the mechanical system the closed-loop response was obtained as shown in Figure 5. From this figure it can be seen that the percentage overshoot decreased to 7% while the vibration was eliminated in 2.04 s. According to these results it can be said that the control objective was met.

***Remark* 4.** At this point it should be highlighted that, PID controller with the control gains that were determined according to the estimated transfer function of the system was directly applied to the mechanical system given in Figure 1. During the control process, mathematical model of the system and/or estimated transfer function were not utilized. This process can be applied for the active vibration control of any experimental system having the structure given in Figure 1. Obtaining the free vibration response of the mechanical system to estimate its transfer function is the only necessity of this procedure.

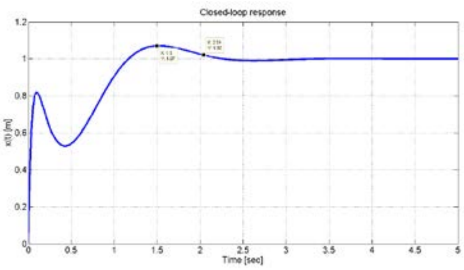

Figure 5. Closed-loop response

CONCLUSIONS

In this study a system model based control gain adjusting method is proposed for the active vibration control of 1 dof mechanical systems. To estimate the transfer function of the mechanical system, its free vibration response is examined. To obtain this response, a unity gain step input is applied to the system as a base excitation and the occurred vibration at the end point is measured. From this measurement transfer function can be estimated by utilizing transient state specifications of the standard second order systems.

Then, active vibration control of a mechanical system is provided. PID controller is used as a controller for the closed-loop system and its gains are tuned according to desired transient state specifications and the steady state error. Providing the dominant pole approximation to reduce the third order transfer function to its second order approximation is an another critical issue for this study. It must be considered during the control gain tuning process.

Finally PID controller whose gains were determined according to the given procedure is applied to the mechanical system. From the simulation results given in this study it is seen that the control gains selected by utilizing the estimation of a transfer function of the mechanical system provide the desired transient state specifications and steady state error for the active vibration control of the mechanical system.

FUTURE WORKS

This study is specified for the active vibration control of 1 dof mechanical systems. For the proposed method a system that can be represented via a second order transfer function is needed. However, it may not be provided for the higher order system. Although, it can be considered as a feasible solution for the active vibration control of 1 dof mechanical systems, it is not possible to generalize it for higher order systems. Proposing a method that can be applied for the active vibration control of higher order systems is considered as the main future work. Experimental verification is also aimed for both cases.

REFERENCES

[1] Guclu, R. 2006. Sliding mode and PID control of a structural system against eqarthquake. Mathematical and Computer Modelling. 44: 210-217.

[2] Priyandoko, G., Mailah, M., Jamaluddin, H. 2009. Vehicle active suspension system using skyhook adaptive neuro active force control. Mechanical Systems and Signal Processing. 23: 855-868.

[3] Khot, S. M., Yelve, N. P., Tomar, R., Sameer, D., Vittal, S. 2011. Active vibration control of cantilever beam by using PID based output feedback controller. J. of Vibration and Control. 18(3): 366-372.

[4] Rahman, N., Alam, M. N. 2012. Active vibration control of a piezoelectric beam using PID controller: Experimental study. Latin American J. of Solids and Structures. 9: 657-673.

[5] Simonovic, A. M., Jovanovic, M. M., Lukic, N. S., Zoric, N. D., Stupar S. N., Ilic, S. S. 2016. Experimental studies on active vibration control of smart plate using a modified PID controller with optimal orientation of piezoelectric actuator. J. of Vibration and Control. 22(11): 2619-2631.

[6] Yildirim, Ş. 2004. Vibration control of suspension system using a proposed neural network. J. of Sound and Vibration. 277: 1059-1069.

[7] Eski, İ., Yıldırım, Ş. 2009. Vibration control of vehicle active suspension system using a new robust neural network control system. Simulation Modelling Practice and Theory. 17: 778-793.

[8] Aström, K. J., Hagglund, T., Hang, C. C., Ho, W. K. 1993. Automatic tuning and adaptation for PID controllers - a survey. Control Engineering Practice. 1(4): 699-714.

[9] Cominos, P., Munro, N. 2002. PID controllers: recent tuning methods and design to specification. Control Theory and Applications. 149(1): 46-53.

[10] Li K. 2013. PID tuning for optimal closed-loop performance with specified gain and phase margins. IEEE Tr. on Control, Systems Technology. 21(3): 1024-1030.

[11] Alagoz, B. B., Ates, A., Yeroglu, C. 2013. Auto-tuning of PID controller according to fractional-order reference model approximation for DC rotor control. Mechatronics. 23: 789-797.

[12] Azar, A. T., Serrano, F. E. 2014. Robust IMC-PID tuning for cascade control systems with gain and phase margin specifications. Neural Computing & Applications. 25: 983-995.

[13] Jin, C. Y., Kyung, H. R., Sung, S. W., Lee, J., Lee, I. B. 2014. PID auto-tuning using new model reduction method and explicit PID tuning rule for a fractional order plus time delay model. J. of Process Control. 24: 113-128.

[14] Yavuz, Ş., Malgaca, L., Karagülle, H. 2016. Analysis of active vibration control of multi-degree-of-freedom flexible systems by Newmark method. Simulation, Modelling and Practice Theory. 69: 136-148.

[15] Sun, D., Mills, J. K., Shan, J., Tso, S. K. 2004. A PZT actuator control of single-link flexible manipulator based on linear velocity feedback and actuator placement. Mechatronics. 14: 381-401.

[16] Shan, J., Liu, H. T., Sun, D. 2005. Slewing and vibration control of a single-link flexible manipulator by positive position feedback (PPF). Mechatronics. 15: 487-503.

[17] Jnifene, A., Andrews, W. 2005. Experimental study on active vibration control of a single-link flexible manipulator using tools of fuzzy logic and neural networks. IEEE Tr. on Instrumentation and Measurements. 54(3): 1200-1208.

[18] Dubay, R., Hassan, M., Li, C., Charest, M. 2014. Finite element based model predictive control for active vibration suppression of a one-link flexible manipulator. ISA Transactions. 53: 1909-1919.

[19] Malgaca, L., Yavuz, Ş., Akdağ, M., Karagülle, H. 2016. Residual vibration control of a single-link flexible curved manipulator. Simulation Modelling Practice and Theory. 67: 155-170.

[20] Yavuz, Ş., Malgaca, L., Karagülle, H. 2016. Vibration control of a single-link flexible composite manipulator. Composite Structures. 140: 684-691.

[21] Ghasemi, A. H. 2017. Slewing and vibration control of a single-link flexible manipulator using filtered feedback linearization. J. of Intelligent Material Systems and Structures. In press. DOI: https://doi.org/ 10.1177/1045389X17704067.

[22] Hac, A., Youn, I. 1992. Optimal semi-active suspension with preview based on a quarter car model. J. of Vibration and Acoustics. 114(1): 84-92.

[23] Yoshimura, T., Kume, A., Kurimoto, M., Hino, J. 2001. Construction of an active suspension system of a quarter car model using the concept of sliding mode control. J. of Sound and Vibration. 239(2): 187-199.

[24] Yao, G. Z., Yap, F. F., Chen, G., Li, W. H., Yeo, S. H. 2002. MR damper and its application for semi-active control of vehicle suspension system. Mechatronics. 12(7): 963-973.

[25] Verros, G., Natsiavas, S., Papdimitriou, C. 2005. Design optimization of quarter-car models with passive and semi-active suspensions under random road excitation. J. of Vibration and Control. 11(5): 581-606.

[26] Türkay, S., Akçay, H. 2005. A study of random vibration characteristics of the quarter-car model. J. of Sound and Vibration. 282(1-2): 111-124.

[27] Belgacem, W., Berry, A., Masson, P. 2012. Active vibration control on a quarter-car for cancellation of road noise disturbance. J. of Sound and Vibration. 331(14): 3240-3254.

[28] Singh, D., Aggarwal, M. L. 2017. Passenger seat vibration control of a semi active quarter car system with hybrid Fuzzy-PID approach. International J. of Dynamics and Control. 5(2): 287-296.

[29] Preumont, A. 2011. Vibration control of active structures: an introduction. Springer & Business Media.

[30] Ogata, K. 2010. Modern control engineering 5th edition. Upper Saddle River, NJ, USA: Prentice Hall.

[31]Cheever, E. 2015. The dominant pole approximation. url: lpsa.swarthmore.edu/PZXferStepBode/DomPole.html