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A Reliability Based Solution to an Ambulance Location Problem Using Fuzzy Set Theory

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Abstract

In this study, the reliability based double standard model (RBDSM) was proposed to address the travel time uncertainty associated with the determination of the coverage. This new covering model was applied to solve the ambulance location problem of the province of Sivas, Turkey. The reliability analyses were performed between all demands as a function of the fuzzy shortest travel time and the fuzzy target travel time. The results indicate that the fuzzy travel times may be appropriate to handle the uncertainty investigated here and have a good potential to be used for the ambulance location problems.

Keywords: Covering model, travel time uncertainty, fuzzy travel time, emergency.

INTRODUCTION

An emergency medical service (EMS) which aims to reduce mortality, disability and suffering in persons [1,2] requires locating a limited number of ambulances at appropriate points in order to provide a quick response to an emergency situation. Covering models are the most widespread location models for formulating the ambulance location problem in the literature [3-5]. The objective of the covering models is to provide adequate coverage to the demand points. A useful extension of the basic coverage models is the double standard model (DSM). This model seeks to maximize the double-covered demand points within two time limits [6], one to cover the whole demand area and another to cover part of it. The concept of coverage can be explained that a demand point is considered covered by an ambulance location point if the travel time between the two points is less or equal to a standard arrival time. However, in real-life applications, the difficulty in the accurate estimation of expected coverage arises from the uncertainty surrounding the travel time [7, 8]. Hence, the incorporation of the quantified uncertainty of the travel time into a covering model is important in order to provide an accurate estimation of the coverage. Travel time reliability is a useful measure for the quantification of the travel time uncertainty [9, 10]. This reliability measure is the probability that the destination can be reached in a time less than some threshold value and it concentrates on a particular link or a set of links representing any path within a transportation network [10]. Travel time reliability can be estimated by Monte Carlo simulation which is a common approach to reproduce stochastic variables preserving the specified distributional properties [11]. However, it is often difficult to obtain adequate statistical information for application of this method. In such case, fuzzy set theory [12, 13] can provide a useful tool for directly working with experts' knowledge [14, 15]. Fuzzy set theory has been successfully used in different areas for solving complex and uncertain problems [16-19]. Although, many covering models have been extended to the probabilistic models to capture the uncertainty in the coverage problem [20-22], the fuzzy set theory has received little attention in ambulance location problems.

In this study, the DSM was extended to a reliability based double standard model (RBDSM). The RBDSM evaluated the travel time reliability of each path between demand points in the province of Sivas, Turkey. The Monte Carlo simulation method was used to estimate the travel time reliability indices in the model. Travel time information between all of the demand points were approximated by using fuzzy membership functions, since the sufficient data about the EMS' operations were not available. Thus, this study also aimed to investigate the potential use of fuzzy set theory in the ambulance location studies.

MATERIALS AND METHODS

Reliability Based Double Standard Model

The RBDSM has the same objective function and constraints as the DSM except for the constraints related to the coverage requirements, as detailed follows: The DSM [6] defines two coverage standards as r_1 and r_2 with $r_1 < r_2$. In the DSM, the objective function is to maximize the demands covered twice within r_1, while all demands must be covered at least once within r_2. The definition of the coverage in the DSM is that the demand point is covered if it is within the target time or distance, otherwise it will not be covered. However, this deterministic definition of the coverage is unrealistic because of not taking into account the travel time uncertainty [7]. The RBDSM is proposed to address this problem: A demand point $i \in v$ is said to be covered by site $j \in w$ if and only if the travel time reliability between i and j is equal to or higher than the desired reliability levels β_1 and β_2 : $P_{ij}\ (t_{ij} - r_{ij} \leq 0) \geq \beta_1$ and β_2 with $\beta_2 < \beta_1$. Thus, every demand point is covered or not by the measures of the reliability. Where, t_{ij} and r_{ij} are the shortest travel time (STT) and target travel time (TTT) between i and j, respectively. $P_{ij}\ (t_{ij} - r_{ij} \leq 0)$ is the travel time reliability between i and j. The TTT is defined as the acceptable travel time for ambulance response to an emergency incident. Let $w_i^1 = \{ j \in w : P_{ij}(t_{ij} - r_{ij} \leq 0) \geq \beta_1 \}$ and $w_i^2 = \{ j \in w : P_{ij}(t_{ij} - r_{ij} \leq 0) \geq \beta_2 \}$ be the sets of potential location sites covering demand point i within β_1 and β_2, respectively. The binary variable x_i^k is equal to 1 if and only if the demand at point $i \in v$ is covered k times within β_1 reliability level. The objective function maximizes the demand covered twice within β_1:

$$Maximize \sum_{i \in v} d_i x_i^2 \qquad (1)$$

Subject to

$$\sum_{j \in w_i^2} y_j \geq 1 \qquad (i \in v) \qquad (2)$$

$$\sum_{i \in v} d_i x_i^1 \geq \alpha \sum_{i \in v} d_i \qquad (3)$$

$$\sum_{j \in w_i^1} y_j \geq x_i^1 + x_i^2 \qquad (i \in v) \qquad (4)$$

$$x_i^2 \leq x_i^1 \qquad (i \in v) \qquad (5)$$

$$\sum_{j \in w} y_j = p \qquad (6)$$

$$y_j \leq p_j \qquad (j \in w) \qquad (7)$$

$$x_i^1, x_i^2 \in \{0,1\} \qquad (i \in v) \qquad (8)$$

Where, d_i is the demand of point i the integer variable y_j denotes the number of ambulances located at $j \in w$, p is the number of available ambulances, p_j is the maximum number of ambulances at $j \in w$. Constraint (2) expresses that all demand is covered within β_2. Constraint (3) represents a proportion ά of the demand is covered. The left hand side of constraint (4) denotes the number of ambulances covering point i within β_1, while the right-hand side is equal to 1 if i is covered once within β_1, and equal to 2 if it is covered at least twice within β_1. Constraint (5) states that point i cannot be covered at least twice, if it is not covered at least once. In Constraint (7), p_j can be set as 2 since an optimal solution using this upper bound always exists.

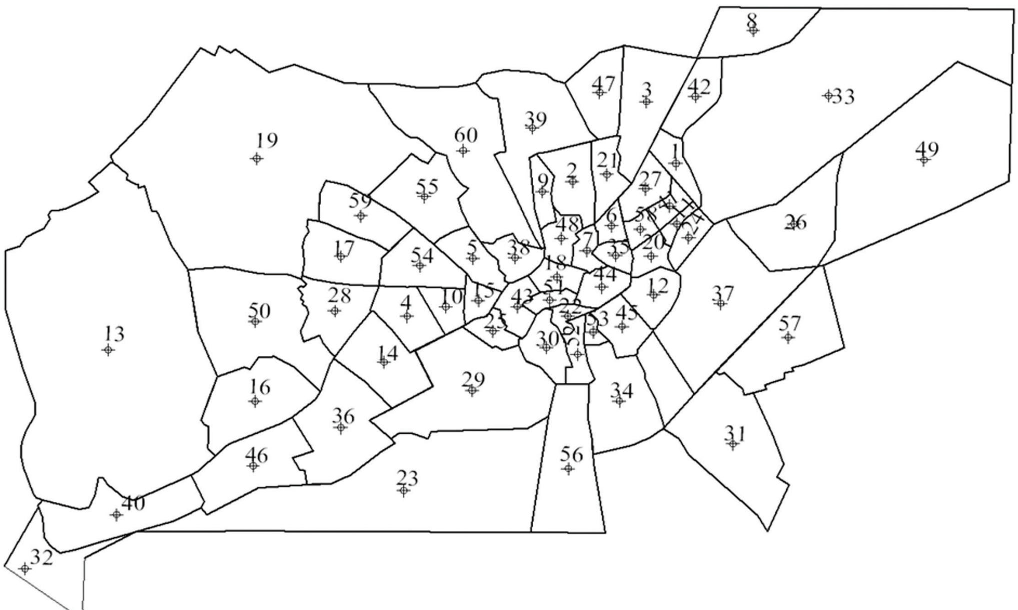

Figure 1. Demand points of the province of Sivas

Implementation of the RBDSM

In this study, the RBDSM was applied to the province of Sivas, Turkey. The province is located at the eastern part of the Central Anatolian region of Turkey, with a surface area of 35.712 km2 and a population of 283233 inhabitants. It is distributed among 60 zones (Figure 1). It was assumed that each zone represents a potential demand point, as well as a potential ambulance location site. The values of $\acute{\alpha}$, β_1 and β_2 were taken as 0.75, 0.80 and 0.70, respectively. The number of available ambulances in the province was limited to 6 and the upper bound on the number of ambulances per site was set to 1.

The STT and TTT were evaluated in linguistic variables of "very very low" (VVL), "very low" (VL), "low" (L), "medium" (M), "high" (H), "very high" (VH), "very very high" (VVH). These linguistic terms were expressed by triangular fuzzy numbers for each of the variables and fuzzy shortest travel time (FSTT) and fuzzy target travel time (FTTT) were identified as shown in Figure 2. The linguistic expressions about them between all demand points were obtained from the opinions of 10 ambulance drivers. Table 1 shows the linguistic terms of the drivers' opinions between some points, as an example. Their corresponding fuzzy numbers were then aggregated into one fuzzy number for each of the fuzzy variables. The mean operator [23] was used as the aggregation operator. After converting all of the aggregated fuzzy numbers into the probability distributions [24], the Monte Carlo simulation of the travel time reliability for each path was performed with 10000 trials. The inverse transformation sampling [25] was used to generate random variates for each of the input variables. Figure 3 shows the Monte Carlo simulation results for the path between the points 16 and 3, as an example. Then, a simple tabu search [26, 27] algorithm was implemented to solve the model and the results were presented in Table 2.

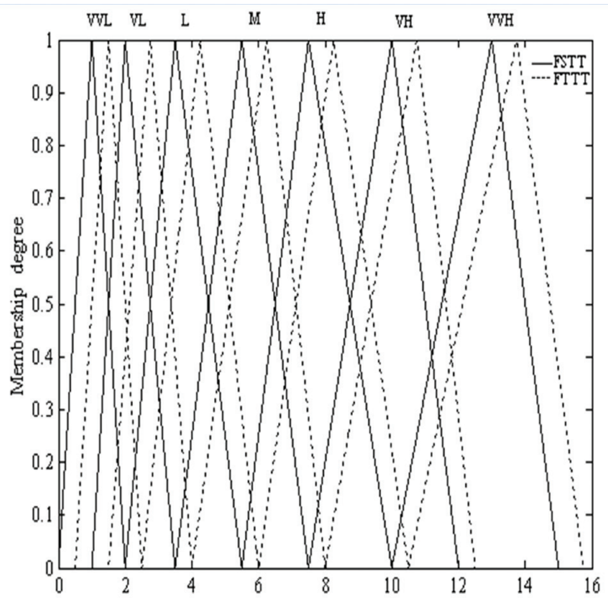

Figure 2. Fuzzy numbers represent linguistic variables

Table 1. Linguistic expressions of the ambulance drivers

Points	**Linguistic terms**
16-33	VH, M, M, VH, H, H, H, VH, VH, VH
16-3	H, H, M, L, H, L, H, L, H, M
4-12	L, M, M, M, L, L, L, M, L, L
50-28	L, L, VL, L, L, L, VL, L, VL, L
28-4	M, M, M, VL, VL, M, L, M, L, L

Table 2. The optimum solutions of the RBDSM

β1 (%)	Coverage		Optimum solution
	Demand	(%)	
80	283233	100	36-44-45-46-58-60
81	281254	99.30	36-44-45-46-58-59
82	266723	94.17	48-50-54-58-59-60
83	219764	77.59	36-45-53-55-58-60
84	100750	35.57	35-36-44-47-59-60
85	30313	10.70	41-44-45-56-58-59

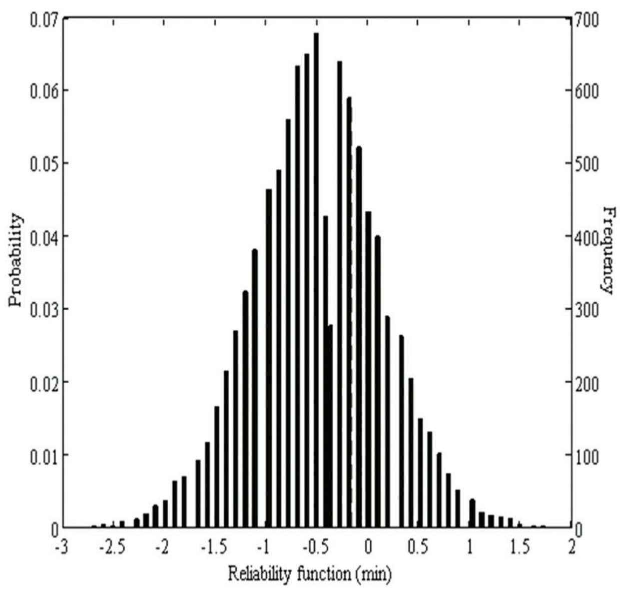

Figure 3. The results of the Monte Carlo simulation for the path between the points 16 and 3

RESULTS AND DISCUSSIONS

Table 2 shows the comparison of the optimum solutions in terms of the demands covered twice as a function of β_l. It was observed that the selected value of β_l (80 %) is lower to reach the coverage threshold (75%). On the other hand, when the desired reliability level is higher than 83%, the coverage ratio decreases abruptly as shown in Table 2. Comparing the obtained coverages for two different choices of β_l (83 and 85%) indicates that the difference between their percentages is 86.20%. This result shows that there is a rapid increase in the number of the unreliable paths between the demand points for a threshold reliability value above 83%. It can be noted that the most appropriate value of β_l for any given problem is not known a priori, in this case, β_l is determined parametrically as a function of α and p, as follows: the highest one that maximizes the coverage is selected.

The examination of the agreement between the simulated and observed distributions of the STT between the points 16 and 3, 16 and 33 shows that the fuzzy approach under consideration may work well for reproducing the STTs (Figure 4-7) The statistics of the simulated and observed data were given in Table 3, in which m, s, CV and CS denote the mean, the standard deviation, the coefficient of variation and the skewness, respectively. The mean difference was also tested using the paired t-test on H0: msimulated STT=mobserved STT and H1: msimulated STT≠mobserved STT. Table 3 shows that there is no statistical evidence that the simulated STT differs from the observed STT at the 95% confidence level.

CONCLUSIONS

In this study, the RBDSM addresses the travel time uncertainty in determining the coverage of the demand points. The results of the case study show that the RBDSM can be applied easily and successfully to a real-world problem. Despite the insufficient information, a fuzzy approach (fuzzy shortest travel time and fuzzy target travel time) can be useful to handle the travel time uncertainty in the ambulance location problems. It may provide a flexible and efficient alternative to estimate the travel times. These fuzzy travel times may also serve as a reference for future measures by EMS staff or other health providers in an emergency.

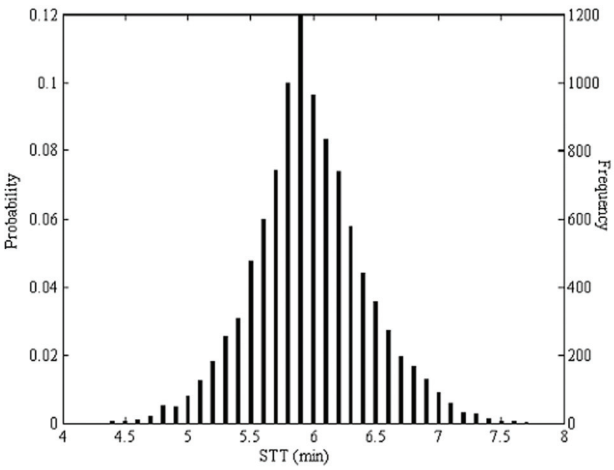

Figure 4. The simulated distribution of the STT between the points 16 and 3

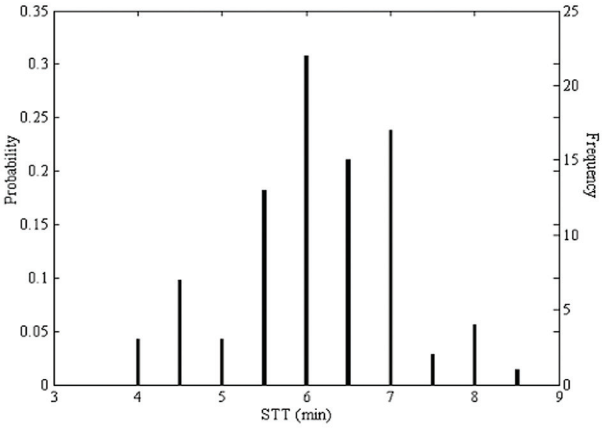

Figure 5. The observed distribution of the STT between the points 16 and 3

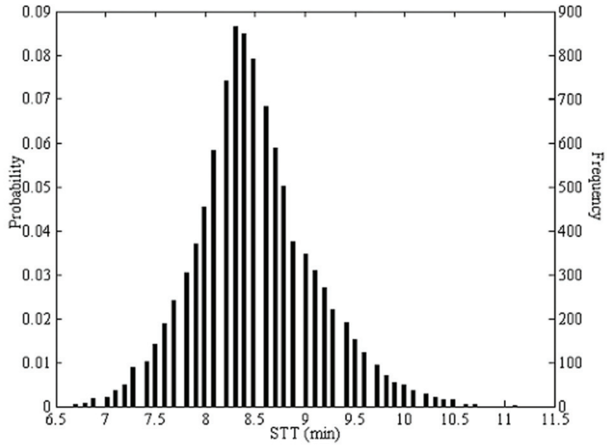

Figure 6. The simulated distribution of the STT between the points 16 and 33

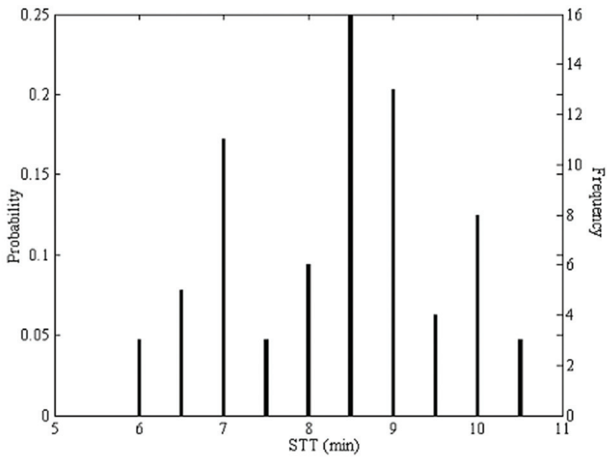

Figure 7. The observed distribution of the STT between the points 16 and 33

Table 3. The statistics of the simulated and observed STTs and the t-test results

Points	Data	μ	σ	CV	CS	t	tcritical
16-3	Observed	6.137	0.963	0.156	-0.160	1.634	1.662
	Simulated	5.968	0.450	0.075	0.172		
16-33	Observed	8.340	1.206	0.144	-0.171	1.066	1.666
	Simulated	8.492	0.592	0.069	0.409		

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