International Journal of Natural and Engineering Sciences 10 (2): 48-54, 2016
ISSN: 1307-1149, E-ISSN: 2146-0086, www.nobel.gen.tr

Computer Simulation Analysis of the Dynamic Response of two Coupled Masses

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Received: July 30, 2016
Accepted: December 06, 2016

Abstract

Computer simulation program was designed to analyze the dynamic response of two coupled masses supported by two springs and coupled with another spring by parallel massless bars. The equations of motion derived using Lagrange's principle and solved by the computer program for different cases with selected values of variables. The mode shapes, the modal vectors, the time response and the Power Spectral Density (PSD) estimation presented graphically for free and forced vibration. The effect of coupling on the time response examined theoretically and numerically with damping and without damping for different cases. The results indicate that the amplitude of motion of the two masses decreases as the coupling coefficient increases for free and forced vibration. Also, there is a particular value of the frequency at which the vibration of the mass is reduced to zero which forms the basis of the dynamic vibration of the absorber systems.

Keywords: Computer simulation, Dynamic response, Coupled masses, PSD, Lagrange principle

Nomenclature

L	The Lagrange function (N)
PSD	The power spectral density estimation
F1,F2	External forces act on the masses (N)
m1,m2	Masses (kg)
k1,k2	Springs coefficients (N/m)
kc	Two bars coupling spring coefficient
L	Length of the beam (m)
Qnc	Non-conservative forces (N)
R	Amplitude ratio (dimensionless)
T	Kinetic energy (N)
V	Potential energy (N)

Latin symbols

$\theta_1(t)$ and $\theta_2(t)$	Independent coordinates used to specify the motion of the system
α	The coupling coefficient
$\dot{\theta}$	The angular velocity of the coordinates
$\ddot{\theta}$	The angular acceleration
ω	Frequency
φ	The phase angle

Subscripts

c	Coupling
nc	Non-conservative

INTRODUCTION

Brennan M. J. et al. [1] presented a consistent and concise analysis of the free and forced vibration of a mass supported by a parallel combination of a spring and an elastically supported damper. The results are presented in a compact form and the physical behavior of the system is emphasized. The usefulness of the additional spring in series with the damper is investigated, and optimum damping values for the system subject to different types of excitation are determined and compared.

Wang L. and Cheng S. [2] discussed relative equilibria (or steady motions) and their stability for the dynamics of the system of two spring-connected masses in a central gravitational field. The system can be regarded as a simplified model for the Tethered Satellite System (TSS), where the tether is modeled by a (linear or nonlinear) spring. Numerical computations show some interesting non great-circle relative equilibria for the spring-connected system. It is shown that for practical configurations, the system at radial relative equilibria is stable if some conditions are satisfied.

Ram Y. M. and Caldwell J. [3] considered a linear, n degree-of-freedom, free–free, vibratory system and supposed that the system under consideration is a multiply connected system, in the sense that each of its concentrated masses may be connected through a linear spring to each of its other masses. An algorithm, which reconstructs the mass and stiffness matrices of the system from the data, is presented. The multiplicity of the solutions and the sensitivity of the problem to perturbation are investigated.

Satto H. and Wada H. [4] presented an analysis of the forced vibrations of a rigid mass connected to an elastic half-space by an elastic rod or a spring and subjected to a harmonic disturbing force in the direction of the rod or the spring axis. The response curves of a rigid mass are obtained, for both a uniform normal stress distribution and a uniform normal displacement at the interface between a rod and a half-space. The effect of stiffness, mass, and damping of an elastic half-space on the response curves of the system are shown.

Kaveh A. and Nikbakht M. [5] developed a methodology for an efficient calculation of the Eigen values for symmetric mass–spring systems in order to reduce the size of the Eigen problem involved. This is achieved using group-theoretical method, whereby the model of a symmetric mass–spring system is decomposed into appropriate sub models. The results are compared to those of the existing methods based on graph theory and linear algebra.

Dobry R. and Gazetas G. [6] presented a method to compute the effective dynamic stiffness and dashpots of arbitrarily shaped, rigid surface machine foundations placed on reasonably homogeneous and deep soil deposits. The method is based on a comprehensive compilation of a number of analytical results. The proposed method is applicable to a variety of area foundation shapes, ranging from circular to strip and including rectangles of any aspect ratio as well as odd shapes differing substantially from rectangle or circle. The results confirm that both frequency and foundation shape may significantly affect the stiffness and dashpot.

Shanshan Yao et al. [7] realized experimentally a mass–spring system with negative effective mass, and its transmission property is examined in the low-frequency range. The negative effective mass is confirmed by experiments through the transmission properties of a finite periodic system composed of such basic units. In the negative mass range, low transmissions of the system are observed and it is well predicted by the theory. In addition, zero effective mass is discussed and experimentally investigated, which gives rise to no phase shifts in the system.

Suggs C. W. et al. [8] developed a damped spring-mass system closely approximating to the dynamic characteristics of a seated man to vertical modes of vibration as the basis on which a standardized vehicle seat testing procedure can be built. Analysis of the problem by means of mechanical impedance techniques indicated that a two-degree-of-freedom system was sufficient to simulate the major dynamic characteristics of man in the frequencies below 10 Hz where seat vibration is most severe.

Ghayesh M. et al. [9] investigated the nonlinear coupled longitudinal-transverse vibrations and stability of an axially moving beam, subjected to a distributed harmonic external force, which is supported by an intermediate spring. The resulting equations are solved via two different techniques: the pseudo-arc length continuation method and direct time integration. The frequency-response curves of the system and the bifurcation diagrams of Poincaré maps are analyzed.

Miranda E. and Thomsen J. [10] predicted a simple model for vibration induced sliding of mass, and provides quantitative experimental evidence for the validity of the model. A mathematical model is set up to describe vibration induced sliding for a base-excited cantilever beam with a spring-loaded point mass. The experiments provide evidence that the simplified mathematical model retains those features of the real system that are necessary for making useful predictions of transient and stationary first-mode response.

Kirk C.L. and Wiedemann S.M. [11] presented an analytical solution for the natural frequencies, mode shapes and orthogonality condition, of a free–free beam with large off-set masses connected to the beam by torsion springs. The study lays the foundation for investigations into the dynamics and vibration control of multi-link articulated systems such as the Space Shuttle Remote Manipulator.

In this paper the equations of motion of two coupled masses supported by two springs and coupled with another spring by parallel massless bars system were derived. The set of equations including the effect of coupling on the time response of the motion for the two masses is examined theoretically and numerically with the presence of damping and without damping for different cases including free and forced vibration. The dynamic response equations of the system was studied and solved using computer simulation program by considering two independent coordinates to describe the motion as shown in Figure (1) assuming that the system vibrates in a vertical plane.

MODEL ANALYSIS

Introduction

By referring to figure 1, the system consist of two masses; m1 and m2, two springs with coefficients k1 and k2 and two bars coupled with a spring of coefficient kc, and required two independent coordinates to describe its motion by assuming that it vibrates in a vertical plane.

Assume that θ_1 (t) and θ_2 (t) are independent coordinates used to specify the motion of the system, thus, the system has two degrees of freedom . The motion of the system is completely described by the coordinates θ_1 (t) and θ_2 (t) which define the position of the masses m1 and m2 at any time from the respective equilibrium positions . The external forces F_1(t) and F_2(t) act on the masses m1 and m2 respectively.

There are two equations of motion for the two degree of freedom system, one for each mass. If a harmonic solution is assumed for each coordinate, the equations of motion lead to a frequency equation that gives two natural frequencies for the system. If a suitable initial excitation given, the system vibrates at one of these natural frequencies.

The system has two normal modes of vibration corresponding to the two natural frequencies. If an arbitrary initial excitation given to the system, the resulting free vibration will be a superposition of the two normal modes of vibration.

However, if the system vibrates under the action of an external harmonic force, the resulting forced harmonic vibration takes place at the frequency of the applied force.

The coupling coefficient is introduced to relate the coupling spring kc with the other springs k1 and k2.

System descriptions and Assumptions

Two masses (m1,m2) are supported by two springs (k1,k2) and connected through a mechanical coupling by two parallel bars connected by a spring as shown in Figure (1). For simplification purpose it is assumed that the bars are massless.

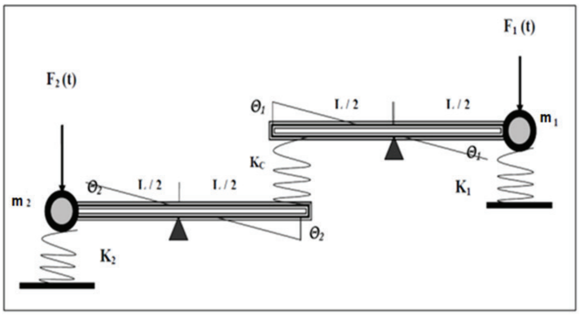

Figure 1. Schematic diagram of two coupled masses

The system equations of motion

The kinetic energy equation (T) of the system can be described as follows:

$$T = \frac{1}{2} m_1 \left(\frac{L}{2}\right)^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \left(\frac{L}{2}\right)^2 \dot{\theta}_2^2 = \frac{1}{8} m_1 L^2 \dot{\theta}_1^2 + \frac{1}{8} m_2 L^2 \dot{\theta}_2^2$$

Where θ is the angular velocity of the coordinates.

The potential energy equation (V) of the system is:

$$V = \frac{1}{2} K_1 \left(\frac{L}{2}\right)^2 \theta_1^2 + \frac{1}{2} K_2 \left(\frac{L}{2}\right)^2 \theta_2^2 + \frac{1}{2} K_c \left(\frac{L}{2}\theta_1 - \frac{L}{2}\theta_2\right)^2$$
$$= \frac{1}{8} K_1 L^2 \theta_1^2 + \frac{1}{8} K_2 L^2 \theta_2^2 + \frac{1}{2} K_c \left(\frac{L}{2}\theta_1 - \frac{L}{2}\theta_2\right)^2$$

The Lagrange function is introduced as follows, (Burton 1994, [12]:

L = T- V

By substitution instead of the two types of energy:

$$L = \frac{1}{8} m_1 L^2 \dot{\theta}_1^2 + \frac{1}{8} m_2 L^2 \dot{\theta}_2^2 - \frac{1}{8} K_1 L^2 \theta_1^2 + \frac{1}{8} K_2 L^2 \theta_2^2$$
$$- \frac{1}{2} K_c \left(\frac{L}{2}\theta_1 - \frac{L}{2}\theta_2\right)^2$$

The Lagrange principle stated that:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = Q_{nc}\text{, and } \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = Q_{nc}$$

Where non-conservative forces (Qnc) equal zero.
The resulting equations are :

$$m_1\ddot{\theta}_1 + (K_1+K_c)\theta_1 - K_c\theta_2 = 0$$

$$m_2\ddot{\theta}_2 - K_c\theta_1 + (K_2+K_c)\theta_2 = 0$$

So equations (4) & (5) are the coupled differential equations of the system and can be arranged as follows:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} K_1+K_c & -K_c \\ -K_c & K_2+K_c \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (6)$$

The system free vibration analysis

By referring to Figure (1), the assumptions considered to solve the system equations are:

1- $F_1(t) = F_2(t) = 0$
2- Massless bars
3- The two masses m1 and m2 can oscillate harmonically with the same frequency ( ) and phase angle ( ) but with different amplitudes, so that it is possible to have harmonic motion as follows:

$\theta_1(t) = \theta_1 \cos(\omega t + \varphi)$ and $\theta_2(t) = \theta_2 \cos(\omega t + \varphi)$ (7)

Where θ_1, θ_2 are constants that denote the maximum amplitudes of θ_1 (t) and θ_2 (t) , and φ is the phase angle.

By differentiating and substituting in equations (4) and (5), the obtained equations are :

$$\left.\begin{aligned} &[\{-m_1w^2+(k_1+k_c)\}\theta_1 - k_c\theta_2]\cos(\omega t+\phi) = 0 \\ &[-k_c\theta_1 + \{-m_2w^2+(k_2+k_c)\}\theta_2]\cos(\omega t+\phi) = 0 \end{aligned}\right\}$$

Since equation (8) must be satisfied for all values of time t , the terms between brackets must be zero .This yields:

$$\left.\begin{aligned} &\{-m_1w^2+(k_1+k_c)\}\theta_1 - k_c\theta_2 = 0 \\ &-k_c\theta_1 + \{-m_2w^2+(k_2+k_c)\}\theta_2 = 0 \end{aligned}\right\} \quad (9)$$

Which represent two simultaneous homogeneous algebraic equations in the unknowns and .

It can be seen that equation (9) is satisfied by the trivial solution = = 0, which implies that there is no vibration . For a nontrivial solution of and the determinant of the coefficients of and must be zero :

$$\text{Det}\begin{bmatrix} \{-m_1\omega^2+(k_1+k_c)\} & -k_c \\ -k_c & \{-m_2\omega^2+(k_1+k_c)\} \end{bmatrix} = 0$$

$$(m_1m_2)w^4 - \{(k_1+k_c)m_2+(k_2+k_c)m_1\}w^2 + \{(k_1+k_c)(k_2+k_c)-k_c^2\} = 0$$

Equation (10) called the frequency or characteristic equation because solution of this equation yields the frequencies or the characteristic values of the system. The roots of equation (10) are given by:

$$w_1^2, w_2^2 = \frac{1}{2}\left\{\frac{(k+k_c)m_2+(k+k_c)m_1}{m_1m_2}\right\} \pm \frac{1}{2}\left[\left\{\frac{(k+k_c)m_2+(k+k_c)m_1}{m_1m_2}\right\}^2 - 4\left\{\frac{(k+k_c)+(k+k_c)-k_c^2}{m_1m_2}\right\}\right]^{0.5}$$

This shows that it is possible for the system to have a nontrivial harmonic solution of the form of equation (7) when is equal to and given by equation (11), where and the natural frequencies of the system .

Denote the values of and , corresponding to as and , and those corresponding to as and . Further, since equation (9) is homogenous, only the ratios can be found .

$r_1 = \{\theta_2^{(1)}/\theta_1^{(1)}\}$ *and* $r_2 = \{\theta_2^{(2)}/\theta_1^{(2)}\}$ can be found .

For $w^2 = w_1^2$ and $w^2 = w_2^2$, equation (9) gives:

$$r_1 = \frac{\theta_2^{(1)}}{\theta_1^{(1)}} = \frac{-m_1w_1^2+(k_1+k_c)}{k_c} = \frac{k_2}{-m_2w_1^2+(k_2+k_c)} \text{ and } r_2 = \frac{\theta_2^{(2)}}{\theta_1^{(2)}} = \frac{-m_1w_2^2+(k_1+k_c)}{k_c} = \frac{k_2}{-m_2w_2^2+(k_2+k_c)}$$

The normal modes of vibration corresponding to w_1^2 and w_2^2 can be expressed respectively as :

$$\theta^{(1)} = \begin{Bmatrix} \theta_1^{(1)} \\ \theta_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} \theta_1^{(1)} \\ r_1\theta_1^{(1)} \end{Bmatrix}, \quad \theta^{(2)} = \begin{Bmatrix} \theta_1^{(2)} \\ \theta_2^{(2)} \end{Bmatrix} = \begin{Bmatrix} \theta_1^{(2)} \\ r_2\theta_1^{(2)} \end{Bmatrix}$$

Where , and are the modal vectors of the system, and the constants can be determined by the following initial conditions:

$$\theta_1^{(1)} = \left[\{\theta_1^{(1)}\cos\phi_1\}^2 + \{\theta_1^{(1)}\sin\phi_1\}^2\right]^{\frac{1}{2}} = \frac{1}{(r_2-r_1)}\left[\{r_2\theta_1(0)-\theta_2(0)\}^2 + \frac{\{-r_2\dot{\theta}_1(0)-\dot{\theta}_2(0)\}^2}{w_1^2}\right]^{\frac{1}{2}} \text{ and}$$

$$\theta_1^{(2)} = \left[\{\theta_1^{(2)}\cos\phi_2\}^2 + \{\theta_1^{(2)}\sin\phi_2\}^2\right]^{\frac{1}{2}} = \frac{1}{(r_2-r_1)}\left[\{-r_1\theta_1(0)-\theta_2(0)\}^2 + \frac{\{r_1\dot{\theta}_1(0)-\dot{\theta}_2(0)\}^2}{w_2^2}\right]^{\frac{1}{2}} \quad (14)$$

$$\theta_1 = \tan^{-1}\left\{\frac{\theta_1^{(1)}\sin\phi_1}{\theta_1^{(1)}\cos\phi_1}\right\} = \tan^{-1}\left\{\frac{-r_2\dot{\theta}_1(0)+\dot{\theta}_2(0)}{w_1[r_2\theta_1(0)-\theta_2(0)]}\right\}$$

$$\theta_2 = \tan^{-1}\left\{\frac{\theta_1^{(2)}\sin\phi_2}{\theta_1^{(2)}\cos\phi_2}\right\} = \tan^{-1}\left\{\frac{r_1\dot{\theta}_1(0)+\dot{\theta}_2(0)}{w_2[-r_2\theta_1(0)-\theta_2(0)]}\right\} \quad (15)$$

Time response and power spectral density estimation

To find the time response of the system, two cases were considered :

Case .1 Free vibration response

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} K_1+K_c & -K_c \\ -K_c & K_2+K_c \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Equation (16) represent a two coupled second order differential equations , which can be expressed as a system of coupled first order differential equations and solved by using the MATLAB program after substituting the values of masses and stiff nesses which assumed for the system.

Then the power spectral density estimation can be found using the Matlab program using what designated as (PSD function).

Case 2: Forced vibration response

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} K_1+K_c & -K_c \\ -K_c & K_2+K_c \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} \vartheta_1\sin\omega t \\ \vartheta_2\sin\omega t \end{Bmatrix} \quad (17)$$

equation (17) represents a two coupled second order differential equations , which can be expressed as a system of coupled first order differential equations and solved by using the MATLAB program after substituting the values of masses, stiff nesses, ( ), and which assumed for the system.

The power spectral density estimation can be found using the Matlab program using what designated as (PSD function).

Coupling coefficient

Assuming the equations of motion to be as follows:

$$m_1\ddot{\theta}_1 + (K+K_c)\theta_1 - K_c\theta_2 = \mathbf{F_0\,exp(i\,\omega\,t)}$$

$$m_2\ddot{\theta}_2 - K_c\theta_1 + (K+K_c)\theta_2 = \mathbf{0}$$

For the steady state motion, assume a solution of the form:

$\theta_1(t) = \theta_1 \exp(i\,\omega\,t)$ and $\theta_2(t) = \theta_2 \exp(i\,\omega\,t)$

and defining a coupling coefficient α as:

$\alpha = \mathbf{K_c / (K+K_c)}$, or $\mathbf{K_c = (\alpha / 1 - \alpha) * K}$

For Kc = 0 (weak coupling) then = 0 and = 1 for K = 0 (the middle connector is a rigid rod).

In order to solve and simplify the solution of equations, assume m1 = m2 = 1, K =1, Fo = 1, and b (The damping constant) = 1, then:

$$\theta_1{}^2 = \frac{(1/1-\alpha)-w^2)^2+w^2}{((1-w^2)^2+w^2)((((1+\alpha)/(1-\alpha))-w^2)^2+w^2)}$$

$$\theta_2{}^2 = \frac{(\alpha/1-\alpha)}{((1-w^2)^2+w^2)((((1+\alpha)/(1-\alpha))-w^2)^2+w^2)}$$

So, we can draw the amplitudes of the motion of the two masses as a function of frequency for different values of for two cases :

Case 1 : with damping , b =1

Case 2 : without damping , b = 0

The amplitudes of motion For $m_1 = m_2 = m$, $k_1 = k_2 = k$, $F = F_0 \exp(i\omega t)$ can be expressed as:

$$\Theta_1 = [(k + kc\ ) - m\omega^2]\ Fo/(k - m\,\omega^2)(k + 2kc - m\,\omega^2) \quad (23)$$

$$\Theta_2 = k_c\ Fo\ /(k - m\omega^2)(k + 2kc - m\,\omega^2). \quad (24)$$

The above equations for **Fo**=1 becomes:

$$\Theta_1 = ((1/1-\alpha) - \omega^2)/(1-\omega^2)((1+\alpha/1-\alpha)-\omega^2) \quad (25)$$

$$\Theta_2 = (\alpha/1-\alpha)^{0.5}/(1-\omega^2)((1+\alpha/1-\alpha)-\omega^2) \quad (26)$$

The amplitudes of motion of the two masses as a function of frequency for different values of α can be drawn.

Steady state response of the system

The steady state response of the system is studied when the mass m1 is excited by the force F1 (t)=Fo exp(iωt) and assuming that m1= m2 ,k1 =k2 =kc =k and no damping. Substitute the above values in equation (17) results in:

$$\Theta_1(\omega) = (-m\omega_2 + 2k)F_{o1}\ /(-m\omega_2 + 3k)(-m\omega_2 + k)$$

$$\Theta_2\ (\omega) = kF_{o1}\ /(-m\omega_2\ + 3k)(-m\omega_2 + k)$$

By defining $\omega_{12} = k/m$ and $\omega_{22} = 3k\ /m$, the above equations can be expressed as:

$$\Theta_1(\omega) = \{2-(\ \omega/\omega_1)_2\}Fo/k[(\omega_2/\omega_1)_2-(\omega/\omega_1)_2][1-(\omega/\omega_1)_2]$$

$$\Theta_2(\omega) = Fo\ /k[(\omega_2/\ \omega_1)_2 - (\omega/\ \omega_1)_2[1-(\omega/\ \omega_1)_2]$$

The variation of the quantities (Θ1*k/F01)and (Θ2*k/F01) with the frequency ratio (ω/ ω1) is plotted in figure(20).

RESULTS and DISCUSSIONS

1. Case study: To find the natural frequencies and mode shapes for any system, a case study can be assumed for the masses and stiffness matrices values, then using the Matlab program to find the Eigen values (natural frequencies) and Eigen vectors (modal vectors).

The case study assuming that m1 = m2 = 50 kg, K1 = K2 = 500 N/m and Kc = 1000 N/m.

So that that mass matrix and stiffness matrix will be appeared as:

$$\text{Mass matrix} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$$

$$\text{Stiffness matrix} = \begin{bmatrix} K_1 + K_c & -K_c \\ -K_c & K_2 + K_c \end{bmatrix} = \begin{bmatrix} 1500 & -1000 \\ -1000 & 1500 \end{bmatrix}$$

By using the Matlab program to solve the set of equations, the following results are obtained:

$$\text{Eigen values } w^2 = \begin{bmatrix} 7.0711 & 0 \\ 0 & 3.1623 \end{bmatrix} \text{ and Modal vectors} = \begin{bmatrix} -0.7071 & -0.7071 \\ 0.7071 & -0.7071 \end{bmatrix}$$

Based on the above results, the modal vectors and mode shapes can be easily obtained and presented graphically for different cases of masses and stiffness.

2. It can be seen from figure 2 where (m1=m2=constant, k1=k2=constant, kc=varies) and figure 3 (m1=m2=constant, k1=k2=varies, kc=constant) that when the system vibrates in its first mode, the displacement of the two masses have the same magnitudes with opposite signs. Thus the motion of m1 and m2 are 180o out of phase. In this case the midpoint of the middle spring remains stationary for all time, such point is called a node.

Also it can be seen that when the system vibrates in its second mode, the amplitudes of the two masses remain the same. This implies that the length of the middle spring remains constant. Thus the motions of m1 and m2 are in phase.

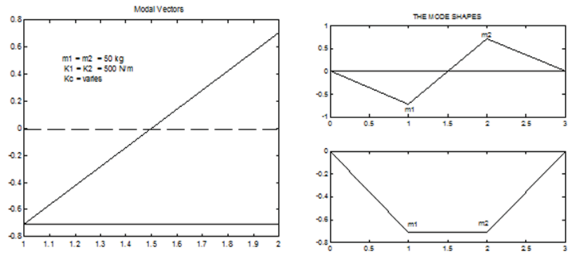

Figure 2. Modal vectors and mode shapes when (m1=m2=constant, k1=k2 =constant, kc = varies).

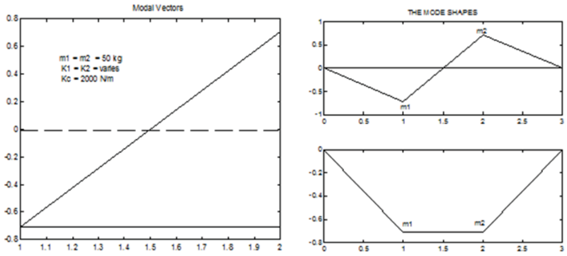

Figure 3. Modal vectors and mode shapes when (m1=m2=constant, k1=k2=varies, kc = constant).

3. Figures (4 and 5) for the case (m1 = m2 = constant, k1 = varies, k2 = varies, kc = constant) show that when the system vibrates in its first mode, the displacement of the two masses have different magnitudes and same signs and move in phase with each other, but when it vibrates in its second mode, the displacement of the two masses have different magnitudes and opposite signs. Thus the motions of m1 and m2 are out of phase. In this mode the midpoint of the middle spring remains stationary for all time. It can be noted that when k1 and k2 are replaced also the mode shapes are replaced according to k1 and k2.

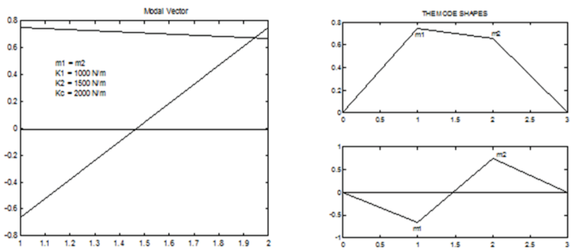

Figure 4. Modal vectors and mode shapes when (m1=m2=constant, k1 less than k2, kc = constant).

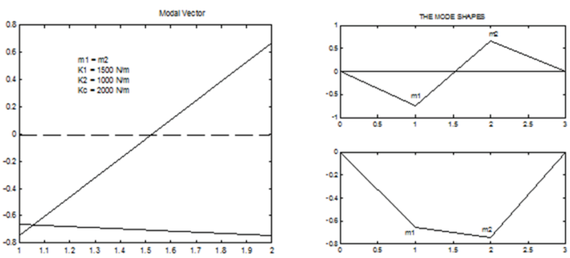

Figure 5. Modal vectors and mode shapes when (m1=m2=constant, k1 greater than k2, kc = constant).

4. In figure (6) for the case (m1=varies, m2 =varies, k1 = varies, k2 = varies, kc = varies). It can be seen from this figure that the two masses vibrate in the first mode with different amplitudes and opposite signs and vibrate in the second mode with different amplitudes and same signs.

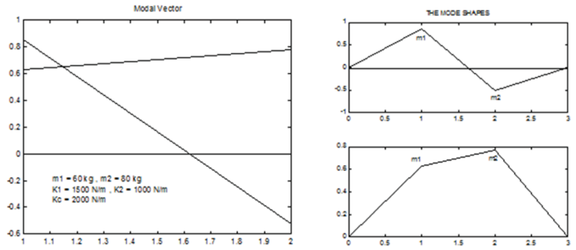

Figure 6. Modal vectors and mode shapes when (m1=varies, m2=varies, k1 =varies, k2=varies, kc=varies).

5. Figures(7,8) represent the free vibration time response of the system for the case when (m1=m2, kc= constant and different values for k1=k2) in each case. From these figures it can be noticed that differs from because of applying the initial conditions. The magnitudes of and decrease as the stiffness of the two springs increase which can be noticed in the PSD figures.

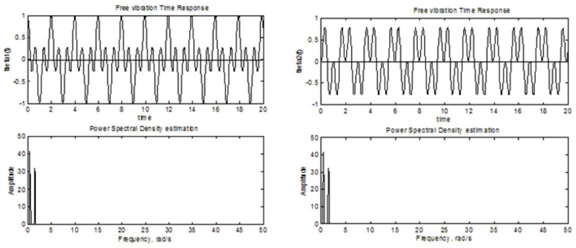

Figure 7. The free vibration time response and Power Spectral Density estimation for and when (m1=m2=constant, k1=k2=500 N/m, kc = constant).

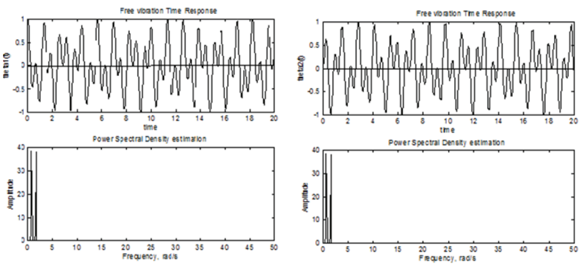

Figure 8. The free vibration time response and Power spectral density estimation for and when (m1=m2=constant, k1=k2=1000 N/m, kc = constant).

6. The amplitude of m2 decreases because of the increase of k2. this result is shown clearly in figure(9) for the case when (m1=m2=constant, k1 less than k2, kc = constant).

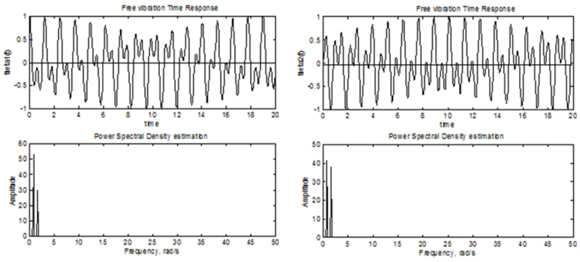

Figure 9. The free vibration time response and Power spectral density estimation for and when (m1=m2=constant, k1 less than k2, kc = constant).

7. Figure (10) shows the free vibration time response and PSD for different masses and different spring's stiffness. The figure shows different responses depending on the values of the masses and spring's stiffness.

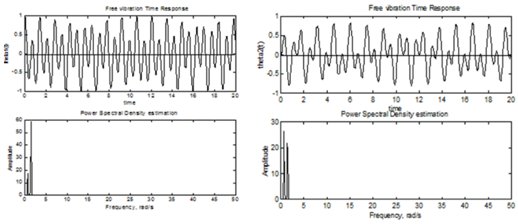

Figure 10. The free vibration time response and Power spectral density estimation for and when (m1=varies, m2=varies, k1 =varies, k2=varies, kc=varies).

8. When the system is subjected to forced harmonic excitation, its vibration response takes place at the same frequency of that of the excitation. This result can be identified in figures (11 and 12). When the masses m1 and m2 are subjected to harmonic excitation separately many amplitudes arise which are differ from that of free vibration. This result is for the case when (m1=m2= 50 kg, k1=k2= 500 N/m, kc = 2000 N/m, F1 and F2 varies).

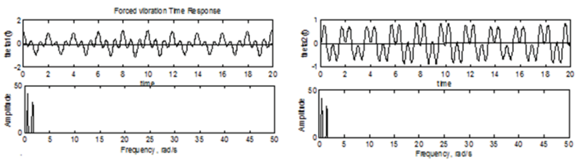

Figure 11. The forced vibration time response and Power Spectral Density estimation for and when (m1=m2=constant, k1=k2=constant, kc = constant, F1 = 100 sin 10t, F2= 0).

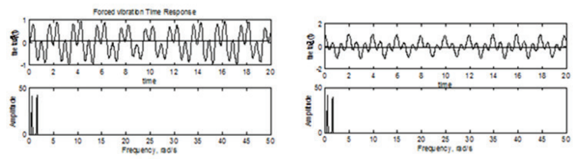

Figure 12. The forced vibration time response and Power Spectral Density estimation for and when (m1=m2=constant, k1=k2=constant, kc = constant, F1 = 0, F2 = 100 sin 10t).

9. In figures (13, 14) the values of k1 and k2 are changed and increased from that of the previous case for m1=m2 and same value of kc = constant. These figures show that when k1 and k2 increase the amplitudes that arise in the time response decrease in comparison with that of the previous case and the case of free vibration.

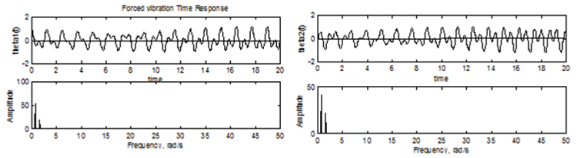

Figure 13. The forced vibration time response and Power Spectral Density estimation for and when (m1=m2=constant, k1 less than k2, kc = constant, F1 = 100 sin 10t, F2= 0).

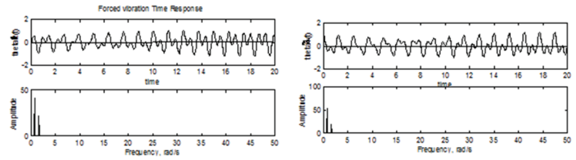

Figure 14. The forced vibration time response and Power Spectral Density estimation for and when (m1=m2=constant, k1 less than k2, kc = constant, F1 =0, F2= 100 sin 10t).

10. When the mass m2 is increased from that of m1 and at the same time is subjected to harmonic excitation , new time response results differs from that for free vibration depending on the value of the subjected force and its location whether on m1 or m2. This result are shown in figure (15).

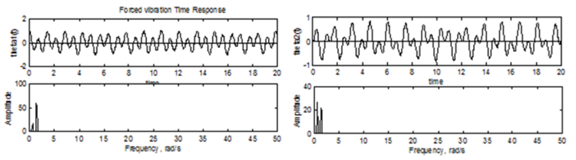

Figure 15. The forced vibration time response and Power Spectral Density estimation for and when (m1=60 kg, m2=80 kg, k1 = 1500 N/m, k2, kc =1000 N/m, F1 = 100 sin 10t, F2= 0).

11. For the case where the two masses are subjected to the same harmonic excitation; figure (16) shows that the response of the system seems to be looks like that of free vibration for the case when (m1=m2, k1=k2, kc = constant, F1=F2= 100 sin 10t). And figure (17) shows that the response of the system is identical to free vibration response when m1 differs from m2 and k1 also differs from k2 and kc = constant.

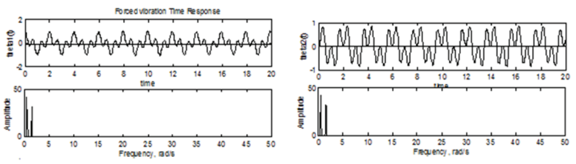

Figure 16. The forced vibration time response and Power Spectral Density estimation for and when (m1=m2=constant, k1=k2=constant, kc = constant, F1 = F2=100 sin 10t).

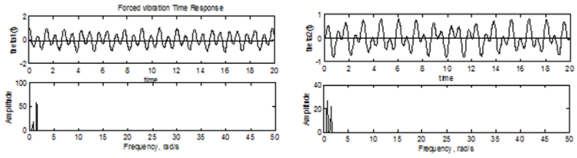

Figure 17. The forced vibration time response and Power Spectral Density estimation for and when (m1=varies,m2=varies,k1 =varies,k2=varies, kc=constant,F1=F2=100 sin 10t).

12. Figure (18) shows the variation of the amplitudes of and with frequency for different coupling coefficient ( ) with the presence of damping. This figure reveals that the amplitudes decrease as increase. Also it can be noticed that is greater than for the same values of .

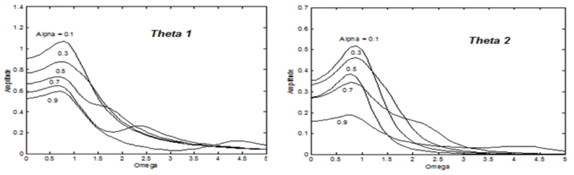

Figure 18. The variation of Theta1( ) and Theta2( ) amplitudes versus Omega for different coupling coefficient ( ).

13. The amplitude of the motion with the frequency for different coupling coefficients(0,0.2,0.4,0.6,1) are plotted in figure (19), for the case of no damping and assumption of simple case where (k1=k2=kc=1 and m1=m2=1, F0=1). This figure shows that = 0 when (= 0 and 1). Also and become infinite when = or = .

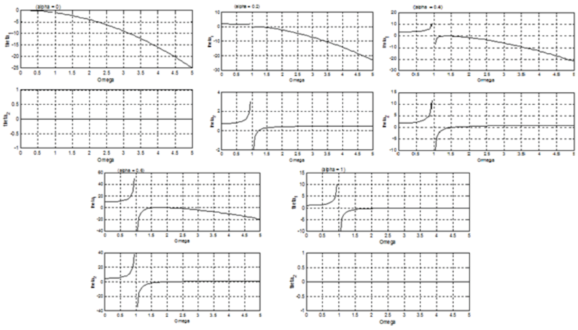

Figure 19. The amplitude of the motion versus Omega for different coupling coefficient.

In figure (20), the response of Θ1 and Θ2 are shown in terms of the dimensionless parameter ω / ω1 , where ω1 was selected arbitrarily.

It can be seen that Θ1 and Θ2 become infinite when ω2 = ω12 or ω2= ω22 , Thus there are two resonance conditions for the system, one at ω1 and other one at ω2 . At all other values of ω the amplitude of vibration are finite.

It can be noticed from this figure that there is a particular value of the frequency ω at which the vibration of the first mass m1 (where the force F1(t) is applied) is reduced to zero. This characteristic forms the basis of the dynamic vibration of the absorbers.

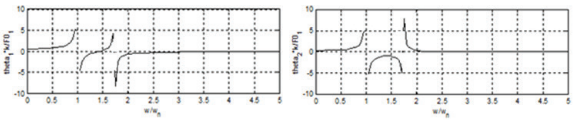

Figure 20. The variation of amplitudes with frequency ratio (Undamped amplitude ratio versus frequency ratio).

CONCLUSIONS

1) For free vibration of the system, the displacement (magnitude and direction of motion) of the two masses depends on the values of the springs stiffnesses.

2) For free vibration the magnitudes of and decrease as the stiffness of the two springs increase.

3) For forced vibration (harmonic excitation), the system vibrates at the same frequency of the excitation force.

4) The amplitudes of vibration of masses in forced vibration decrease with the spring stiffness (k1 and k2) increase in comparison with that of the free vibration.

5) There are two resonance conditions for the system, one at ω1 and the other one at ω2.

6) The basis of the dynamic vibration of the absorber systems implies that there is a particular value of the frequency ω at which the vibration of the mass is reduced to zero.

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