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# The Interactions Between the Excited States in Stabilized Lasers

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### Abstract

Stable lasers are necessary as local oscillators in many applications including coherent communications, high resolution spectroscopy and gravity wave detection. Recent developments in the active stabilization of lasers have led to light sources of extremely narrow linewidth and correspondingly long coherence times. Such stability enables several new laser measurement techniques. Stabilized lasers can provide a probe of the interactions between the excited states of laser transitions and the laser cavity itself. Two derivations of the fundamental laser linewidth are presented that have been used successfully in introductory courses. The cause of the finite linewidth is identified with phase fluctuations in the electric field due to spontaneous emission. This paper will review frequency modulation techniques for stabilized lasers, with emphasis on the relations between the laser linewidth and random processes within the lasing medium. An experiment to test stabilized lasers in the vibration free and microgravity environment of space and applications of ultra-stable lasers in space will be described.

Key words: stabilized lasers, linewidth, laser cavity, frequency modulation, gravity wave detection

## INTRODUCTION

The coherence, phase stability and narrow spectral width of lasers are of paramount importance in several of the most interesting potential scientific applications of these lasers, such as gavity wave detectors using resonant multiple pass interferometry on the Earth [1, 2]. A laser is often described as a monochromatic light source because its bandwidth is much smaller than that of other sources of light. This spectral width results from several different processes. Like any other oscillator, a laser is influenced by fluctuations or noise in its surroundings or occurring during its operation and these fluctuations broaden the laser lineshape. There are two major sources of such noise. Technical noise is a result of the interactions of the laser with its environment; for example, changes of laser cavity length induced by thermal effects or mechanical vibrations or by irregularities in operating conditions, such as changes in the pumping intensity. Stable lasers are required for many experiments attempting to produce and to analyze squeezed states of the radiation field. Quantum noise is the result of randomness in the spontaneous emission processes responsible for laser action. Frequency stable lasers are required as master oscillators in a variety of applications, including coherent communication, high resolution spectroscopy and gravity wave detection [3].

Fluctuations which occur much faster than the time it takes to measure the lineshape broaden the line, slower fluctuations cause the line center to change with time or to drift. In this paper we will focus on the more rapid fluctuations contributing to line broadening. Heliumneon, dye and argon-ion lasers have been successfully stabilized for potential use in these fields. Diode-laser pumped Nd:YAG lasers have also been stabilized to reference interferometers [3]. So that, without a reference oscillator the frequency stability of these lasers is commonly analyzed using the closed loop error signal, which can only provide a lower bound on performance. These lasers also require external elements to force single axial-mode operation and to provide isolation against optical feedback. In laser stabilization it is of interest to develop a scheme where the error signal for the feedback loop is derived from a simple configuration. Often the experiments require in addition to a certain frequency stability and reproducibility also certain intensity stability. Some experiments can not allow frequency modulation of the stabilized laser. Reported on a very simple method to stabilize a single mode dye laser to an absorption line in neon. The method is based on a magneto-optical effect termed forward scattering which is more traditionally utilized in spectroscopic investigations of the properties of atoms [4].

Laser linewidth has been decreasing steadily with improvement in the design of laser systems and in the quality of laser materials. Recently, dramatic reductions in laser linewidth have been achieved by active external stabilization techniques that use feedback to reduce laser phase noise and consequently linewidth. As a result of these developments it is now possible to conceive of laser linewidths that are a fraction of a Hertz [1]. A general property of feedback systems is that the less than ideal elements in the system do not necessarily prevent system performance near the level fixed by the noise of the measurement process. For example, the linewidth of a semiconductor laser-diode pumped solid-state laser Nd:YAG, which has an oscillating frequency of 3x10<sup>14</sup> Hz, has been reduced to 3 Hz; this implies an oscillator stable to one part in 1014. A laser diode-pumped Nd: YAG rod laser that has a frequency jitter in 0.3 sec of less than 10 kHz [5, 6]. Diode-laser pumping avoids the frequency noise associated with flashlamp pumping, and the unidirectional oscillation made possible by the nonplanar ring geometry avoids spatial hole-burning and improves resistance to optical feedback.

Narrow linewidth lasers are of interest in the study of excited state interactions for several reasons. First of all, the laser has become an important tool in the study of the excited state. A principal advantage of a narrow line laser is that it increases the coherence time available for measurements. When the laser linewidth is boad and its correlation time correspondingly short, there can be at most one interaction taking place during a coherence interval. As the linewidth narrows, the correlation time lengthens and more and more interactions occur during the correlation interval; the probe beam begins to resonate with its target. Another reason for the interest in narrow line lasers is that, as the broadening effects of environmental influences on laser linewidth are reduced, intrinsic quantum effects become more important and it becomes possible to study these effects in the laser lineshape. It is possible also to probe the basic interaction responsible for laser emission and interactions between the atoms and the laser cavity. Applications of this system of laser stabilization include precision laser spectroscopy and interferometric gravity wave detectors.

The design of this paper is as follows; we will give a heuristic discussion of the relationship between noise sources and the linewidth of a free running laser following the arguments developed in it [7]. And then, we will describe frequency modulation techniques for actively reducing the linewidth. Finally we will discuss an experiment designed to study the operation of ultra stable lasers in the vibration free and microgravity environment of space.

# MATERIALS

### Linewidth of Free Running Lasers

The linewidth of a free running laser depends upon the characteristics of the laser cavity supporting its oscillation

and upon the power in the coherent radiation emerging from that cavity. The laser cavity can be characterized by looking at the net gain in intensity experienced by a light beam in making one complete circuit of the cavity. Characteristics of the cold laser cavity that a simple Fabry-Perot cavity, shown in Fig. 1; consists of two mirrors, separated by a distance L and enclosing an active medium of length *l*. We assume that one mirror is perfectly reflecting and the other has a reflection coefficient R and transmission (1-R) to allow for some of this circulating energy to escape as a coherent beam. The active medium is characterized by a gain per unit length g and a loss per unit length  $\alpha$ . As the beam makes a round trip in the cavity it experiences a net gain given by

$$G = R \exp \left[2l(g - \alpha)\right] \tag{1}$$



Figure 1. Schematic diagram of a Fabry-Perot laser cavity.

We can define the coherence length for a cold cavity; that is, a cavity which is not being pumped so g=0 as the distance a beam must travel before its intensity is reduced by a factor of 1/e. The number of round trips p, reguired for this condition to be met is

$$G^{P} = e^{-1} \tag{2}$$

so that the coherence length  $L_c$ , is given by

$$L_c = 2Lp = 2L / [2\alpha l - \ln(R)]$$
(3)

And the cavity coherence time is given by  $\tau_c = L_c/c$ . Thus, we would expect that the intensity of a beam in a cold cavity decays exponentially with a lifetime  $\tau_c$ . The loss processes which cause this decay also operate during lasing to produce a loss of coherence in the beam. When the laser is operating continuously, two points along the emergent laser beam, separated by distances much larger than  $L_c$  will not be coherent. The coherence time can be used to define a linewidth for the cavity itself,

$$\Delta \omega_{\rm c} = 2\pi \Delta v_{\rm c} = 1/\tau_{\rm c} \tag{4}$$

The effect of noise on laser linewidth; the electric field at a point in the laser cavity located by r is given by

$$E(\mathbf{r}, t) = \varepsilon [E_0 + \Delta E(t)] e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t + i\varphi(t)}$$
(5)

where  $\varepsilon$  is the polarization vector, k the wave vector defining the wave propagation and  $\Delta E(t)$  and  $\phi(t)$  are random processes representing amplitude and phase noise on the laser field respectively. In general, the various noise sources may induce fluctuations on both the amplitude and the phase of this field but the response of the laser system to each of these types of noise is quite different. A fluctuation in the amplitude of the field will be rapidly damped by the strongly nonlinear laser dynamics acting to restore the amplitude to its mean value. The field fluctuation may oscillate as it is damped out with characteristic frequencies of relaxation oscillations [7]. In view of the very rapid time scale of fluctuations associated with the dye laser's free flowing jet and with plasma movement in the ion laser. It is understandable that efforts to improve their frequency stabilization performance have centered on developing faster tranducers and electronic systems.

No such damping takes place for fluctuations in the phase. Consequently the phase shift over a time interval  $\tau$ ,  $\Delta \phi = \phi(t+\tau) - \phi(t)$ , undergoes a random walk, characterized by the time averages,  $\langle \Delta \phi \rangle = 0$  and by  $\langle \Delta \phi^2 \rangle$  increasing linearly with  $\tau$ , that is by  $\langle \Delta \phi^2 \rangle = \gamma \tau$ . The correlation function for the field can be shown to depend upon  $\langle \Delta \phi^2 \rangle$  exponentially [7] through the expression

$$\langle E(t)E(t+\tau) \rangle \propto \exp\left(-\langle \Delta \varphi^2 \rangle / 2\right)$$
 (6)

so that, since  $\langle \Delta \phi^2 \rangle = \gamma \tau$ ,  $1/\gamma$  becomes a measure of the laser linewidth. Assuming that the phase shifts ocur in discrete jumps, we can represent the variance in the phase shift over time interval  $\tau$  as the product of the number of phase shifts occuring in that interval N( $\tau$ ) and the variance of the single phase shift  $\langle \Delta \phi^2 \rangle$ ,

$$\gamma \tau = \langle \Delta \phi^2 \rangle = N(\tau) \langle \Delta \phi^2 \rangle \tag{7}$$

This implies that the width of the line induced by phase variations is given by

$$2\pi\Delta v_{\text{laser}} = \gamma = [N(\tau) / \tau] < \Delta \phi^2 >$$
(8)

thus, the phase noise induced linewidth is the product of the rate at which phase shifts ocur and the variance in a single phase shift. If we assume, further, that the only noise sources operating are those associated with quantum fluctuations we can estimate these two factors separately. A system which has only quantum noise should, on average, experience one phase shift in a coherence time so that phase shifts will ocur at a rate given by

$$N(\tau) / \tau = \Delta \omega_c \tag{9}$$

The variance in the phase shifts  $\langle \delta \varphi^2 \rangle$ , can be estimated by referring to Fig. 2; where the phase shift,  $\delta \varphi$  of the field is related to the phase angle of the field fluctuation,  $\Delta E Sin(\theta) = E_0 \delta \varphi(\theta)$ . We can calculate  $\langle \delta \varphi^2 \rangle$ from an ensemble average of  $\delta \varphi(\theta)$  assuming that each angle  $\theta$  is equally likely,

$$\langle \delta \varphi^2 \rangle = \int 1/2\pi . \ \delta \varphi^2(\theta) d\theta = 1/2 \ (\Delta E / E_0)^2$$
 (10)

It remains now to calculate the magnitude of the two electric field terms,  $\Delta E$  and  $E_0$ ; they can be estimated from the energy density associated with an electric field E, ( $\epsilon_0 E^2/2$ , where  $\epsilon_0$  is the permittivity of free space). The field  $\Delta E$  is that of a single laser photon of energy hv, so

$$\Delta E^2 = 2hv / \varepsilon_0 V \tag{11}$$

V is the volume over which the energy density is calculated. The field  $E_0$  is that of the emergent laser beam and the energy associated with it can be approximated by the product of the laser output power,  $P_{out}$  and the cavity coherence time; thus

$$E_0^2 = 2P_{out} \tau_C / \varepsilon_0 V \tag{12}$$





Combining the equations (10), (11) and (12) we see that the variance of the phase shift due to quantum noise depends upon the laser parameters through

$$\langle \delta \varphi^2 \rangle = 1/2 \cdot (hv / P_{out} \tau_c) \tag{13}$$

We can now calculate the laser linewidth from equations (8), (9) and (13)

$$\Delta v_{\text{laser}} = h v (\Delta v_{\text{C}})^2 / P_{\text{out}}$$
(14)

This formula was first derived by Schawlow and Townes (apart from a factor two which has since been corrected in their original work) giving the lower limit to the linewidth of a free running laser [8, 9]. If the only processes responsible for broadening the laser line were those due to intrinsic random phase fluctuations in the laser itself, the laser would have a linewidth given by equation (14).

#### **Reduction of Laser Linewidth**

In the free running laser, the cavity serves two distinct roles. One is to define the frequencies of axial modes of the laser beam and the other is to maintain some portion of energy circulating within the cavity to convert energy stored in the pumped medium into coherent radiation. A fraction of the circulating radiation is allowed to escape in the emergent beam. These two roles compete with each other in that higher mirror reflectivity "less loss" is needed to definite a narrow cavity linewidth but high reflectivity allows less energy for the output beam. In active frequency stabilization these roles are separated. An external reference cavity is used to provide the definition of frequency and this reference cavity can be made with the highest quality mirrors available without influencing the laser output power. The laser cavity can be as lossy as needed to provide for efficiency lasing.

A more powerful approach to isolating the pump and test laser stabilities can be obtained in laser systems in which the upper state is fed through collisional or in solid-state systems, through radiationless processes. Spectacular free running laser stability was demonstrated by Zhou et al. [5]. They used a low power laser diode to pump optically a small Nd:YAG oscillator crystal. The primary environmental coupling into such lasers is through the temperature dependent refractive index and gain center. These shifts can be large (several gigahertz) but are relatively slow. The laser linewidth will be approximately equal to the Fourier frequency at which the unity phase modulation index first occurs.

In order to provide for this separation of roles it is necessary that the reference cavity track excursions of the laser frequency from a central reference frequency and convert these excursions into an error signal that can be used to shift that laser frequency back to the reference value. An electro-optic device to generate this error signal was developed by Drever et al. [10]. In designing laser frequency control systems, to reduce cavity line center accuracy problems and minimize frequency drift it is appropriate to consider modulation techniques. A preliminary optical phase modulation sideband scheme was suggested earlier. However, in the work, the modulation sidebands were transmitted with the laser carrier through the control cavity and some difficulties of profile asymmetry were noticed. The idea of this device is simple and elegant. Light from the laser is modulated electro-optically (at tens of MHz) with a small modulation index so that the modulated output consists of a carrier frequency and two sidebands 180 degrees out of phase with each other. Refer to Fig. 3. The modulated light is passed through a polarizing beam splitter and quarter wave plate to be incident on a high finesse reference cavity. Light from the carrier passes into the cavity but the sidebands are sufficiently shifted to be reflected from the cavity. The carrier light admitted to the cavity resides there; on average, for a time equal to the cavity round trip time times the finesse. The purpose of the experiment was to investigate the precision with which lasers can be servo locked to an interferometer cavity, using the heterodyne between the two independent lasers as the diagnostic tool. To assure that the lasers were truly independently locked to the cavity and that they had no unexpected mutual interactions or possibility for coupling (Fig. 3) [10]. The light emerging from the front of the cavity consists of the reflected sidebands plus the time averaged carrier

signal; this light is mixed on a detector where the slowly varying signal at the modulation frequency is extracted, demodulated, and used as an error signal to be fed back to the laser where it controls the frequency by means of a piezoelectric crystal mounted on top of the laser crystal. This servo system reduces the laser noise over the servo bandwidth producing a laser with extremely narrow linewidth.



**Figure 3.** Schematic diagram of the frequency modulation laser stabilization technique. The single laser line (carrier) is modulated by an electro-optic modulator to add two out of phase sidebands. The sidebands are reflected from the reference cavity while the carrier is averaged within the cavity. The signals are mixed on a detector to generate an electronic error signal which is fed back to the laser [10].

Research directed by Byer et al. At the Ginzton Laboratory of Stanford University over the last decade has led to the development of an extremely stable, diode pumped solid-state laser [2, 11, 12]. This laser, now in commercial production has a free running linewidth of 10 kHz. During the same period, work under whom has refined methods for the active stabilization of lasers primarily through the development of the frequency locking technique. Using the feedback stabilization technique developed the Stanford group has recently been able to reduce the linewidth of their Nd:YAG laser to 3 Hz. The point to be made from the above rather detailed presentation is the following. When the servo gain for a given laser source can be extended to a sufficiently high frequency that the integrated residual phase variations are well below 1 rad, the intrinsic frequency noise variations of the laser have been totally suppressed by the servo control system. The frequency noise properties of the total laser servo frequency discriminator system are then essentially controlled by the measurement of noise and frequency stability attributes of optical frequency discriminator employed [1].

# APPLICATIONS

The limits to stability of lasers in terrestrial laboratories is set by environmental and microseismic noise and by gravity induced distortions in the optical devices used as frequency references. Stabilized laser oscillators will operate even more stably in space where vibrational and gravitational effects are significantly reduced. Furthermore, space is the appropriate environment for a variety of important applications of ultra stable lasers.

#### Gravity wave detection

One of the most revolutionary applications of technology emerging from this program is a family of gravitational wave detectors able to probe interactions among the most massive entities in our universe. Using laser gravitational wave interferometers in space to detect gravitational pulses with periods up to a few hours, events involving the motion of large masses, such as the collisions of black holes, can be studied. Ground based systems on a smaller scale are now being designed but such devices will have a low frequency cutoff of about 100 Hz. In a proposed experiment titled Laser Gravitational Wave Observation in Space (LAGOS), a laser operating at the fundamental linewidth limit will allow the measurement of gravity waves in a one million kilometer interferometer in orbit around the sun. This experiment is enable specifically by the recent advances in solid-state laser technology including the very recent studies on laser stabilization.





Figure 4. Temporal analysis of the beat frequency heterodyned down to low frequency: (a) the beat frequency is  $\sim$ 1 Hz and 512 points, 40 msec/point, 20.48 sec recording time, (b) the beat frequency is 0.6 Hz and still appears nicely sinusoidal, (c) the beat frequency is  $\sim$ 0.4 Hz and some phase reversal may be seen, (d) the beat frequency has been adjused to be  $\sim$ 0 Hz. There is an 8 sec period during which the phase is stable within 1 rad. Each laser linewidth needs to be  $\sim$ 50 mHz to give such a beat [1].

The initial conception and much early development of this stabilization method arose during work on gravitational radiation detectors at Glasgow and in practice, it has proved very satisfactory in this application. The argon-ion laser is frequency locked to one of the resonant modes of a triangular ring cavity with two sides 10 m long and the third 0.05 m, formed between mirrors mounted on freely suspended masses in a large vacuum system [13]. Subsidiary servo systems control the orientation and low frequency seismic motions of the masses. A second smilar 10 m cavity with its long axis perpendicular to that of the first could be used as a frequency analyzer to assess the stabilization achived (Fig. 4). The phase modulation technique was used to give a servo control signal for a piezoelectric transducer supporting one of the cavity mirrors, which adjused the cavity length to maintain it precisely in resonance with the laser light. Analysis of the feedback voltage applied to piezo transducer gives an upper limit to the residual frequncy fluctuations of the laser with respect to the resonance frequency fluctuations occuring around 1 kHz, the main region of interest for the planned gravity wave experiments, stabilization factors of greater than 104 were achieved [10].

### **Optical clock technology**

From the measurement science point of view, the considerations of the previous section lead issues offer the prospect of a few milliherz laser linewidth and stability. This is to be achieved by serious line splitting of a cavity resonance. Suppose that we take 10<sup>5</sup> Hz for the cavity resonance width. Then to reach the shot noise level we have to divide the resonance width into 10<sup>8</sup> part. A natural idea for avoiding the direct feedback is the use of ring cavity resonator modes in which the specular reflection is not autocollimated with the input beam. Three and four mirror ring resonator systems are being used successfully for this application [1, 14]. The laser oscillator used in the Stanford University NASA laser inspace technology experiment has a potential for an accuracy of one part in 10<sup>17</sup>. This extraordinary stability coupled with recent demonstration of sub-Kelvin temperatures and subdoppler spectroscopic techniques make it possible to conceive of an optical clock having a short term oscillator or flywheel based on a solid-state laser stabilized, in the long term by interrogating a laser cooled atom or a trapped ion. The flywheel oscillator in the atomic clocks in current use are based on quartz oscillators. Replacing these with an optical frequency oscillator represents a significant advance in clock technology.

One key point in attaining the interesting long term stability not to mention accurate locking to the cavity resonances, depends on the accuracy and stability of the phase modulation process. For example, even order sidebands clearly will lead to a systematic frequency shift of the lock point. One distinctive feature of these optical systems is that the wavelength is small compared with the apparatus size. Because many of the parasitic effects are of interferometric origin, one tends to have serious thermally driven changes in the systematic offsets in the optical locking systems.

## Laser cooled atoms

Another, equally fundamental study involves the use of stable laser sources to probe the behavior of laser cooled atoms in the reduced gravity environment of space. Attempts to study the behavior of small ensembles of atoms at temperatures in the tens of micro-Kelvin are hindered by the short observation times available in the 1 g environment. Observation times in space can increase the observation time by a factor from 10 to 100.

## **RESULTS AND DISCUSSIONS**

We have described a reflection mode laser frequency stabilizing system and described a new high speed operating regime for such a system where the detected signal is proportional to phase rather than frequency changes. Experiments with beter technical solutions to several of the optical and electronic challenges are in preparation. The most interesting regime for applications is the deep periodic spiking which can be used for increasing the intensity of short pulses. In this case, the spontaneous emission plays an important part in the stabilization of the spiking. However, describe that the prospect of subhertz laser linewidth in the optical Region of the spectrum is now a likely possibility since, as shown in this paper, frequency locking to a single reference cavity can be accomplished at the few tens of millihertz level.

Particularly the theoretical results of the work described in predicted that the forward scattering experimental configuration should be applicable to laser frequency stabilization. The theoretical results showed that the forward scattering light intensity has a laser frequency dependence. We have used these dependencies for frequency stabilization. A motivation to the present work was to study the forward scattering stabilization, because of its extreme simplicity.

Active stabilization of the monolithic Nd:YAG oscillator may be achieved by the control of temperature, pressure, or electric field through the Kerr effect. Applications for the oscillator include metrology, fiber-optic sensing, and very high resolution spectroscopy.

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# REFERENCES

- Salomon C, Hils D, Hall JL. 1988. Laser stabilization at the millihertz level. Journal of Optical Society of America, 5 (8): 1576-1587.
- [2] Nilsson AC, Gustafson EK, Byer RL. 1989. Eigenpolarization theory of monolithic nonplanar ring oscillators. IEEE Journal of Quantum Electronics, 25 (4): 767-789.
- [3] Day T, Gustafson EK, Byer RL. 1990. Active frequency stabilization of a 1.062-μm, Nd:GGG, diode-laser-pumped nonplanar ring oscillator to less than 3 Hz of relative linewidth. Optics Letters, 15 (4): 221-223.
- [4] Fellman T, Lindberg A, Stahlberg B. 1994. Laserfrequency stabilization using forward scattering. Applied Physics, B59 (6): 631-633.

- [5] Zhou B, Kane TJ, Dixon GJ, Byer RL. 1985. Efficient, frequency-stable laser-diode-pumped Nd:YAG laser. Optics Letters, 10 (2): 62-64.
- [6] Stute U, Kling R, Hermsdarf J. 2007. Interaction between electrical arc and Nd:YAG laser radiation. CIRP Annals-Manufacturing Technology, 56 (1): 197-200.
- [7] Jacobs SF. 1979. How monochromatic is laser light? American Journal of Physics, 47 (7): 597-601.
- [8] Schawlow AL, Townes CH. 1958. Infrared and optical masers. Physical Review, 112 (6): 1940-1949.
- [9] Davarcioğlu B. 1991. Study of Radiation by Solid-State Lasers. Proc of ISRP-5, Ruder Boskovic Institute, Dubrovnik-Yugoslavia, Ed. Bradley DA. 1, 43-51.
- [10] Drever RWP, Hall JL, Kowalski FV, Hough J, Ford GM, Munley AJ, Word H. 1983. Laser phase and frequency stabilization using an optical resonator. Applied Physics, B31 (1): 97-105.
- [11] Kane TJ, Byer RL. 1985. Monolithic, unidirectional single-mode Nd:YAG ring laser. Optics Letters, 10 (2): 65-67.
- [12] Kane TJ, Nilsson AC, Byer RL. 1987. Frequency stability and offset locking of a laser-diode-pumped Nd:YAG monolithic nonplanar ring oscillator. Optics Letters, 12 (3): 175-177.
- [13] Wilson JIB. 1990. Argon laser deposition of microelectronic materials. Turkish Journal of Physics, 14 (1): 24-39.
- [14] Eichler HJ, Koltchanov IG, Liu B. 1995. Numerical study of the spiking instability caused by modulation frequency detuning in an actively mode-locked solid-state laser. Applied Physics, B61 (1): 81-88.