International Journal of Natural and Engineering Sciences 9 (1): 13-17, 2015
ISSN: 1307-1149, E-ISSN: 2146-0086, www.nobel.gen.tr

Investigating the Performance of Fuzzy, PID and LQR Controllers for Control of Airplane Pitch Angle

Reza TAMASKANI[1*] Amin BAZZAZI[2] Abbas MOHAMMADI[3] Mojtaba AJOUDANI[4]
[1] Young Researchers and Elite Club, Bandargaz Branch, Islamic Azad University, Bandargaz, Iran
[2] Gorgan Branch, Islamic Azad University, Gorgan, Iran
[3] Department of Electrical Engineering, University Of Guilan, Rasht, Iran
[4] Department of Electrical Engineering, Bandargaz Branch, Islamic Azad University, Bandargaz, Iran

*Corresponding author:
E-mail: reza_tamaskani@yahoo.com

Received: January 02, 2015
Accepted: February 11, 2015

Abstract

Airplane automated control is one of the indispensable components of the airplane. One of the controllable deviations in airplane is controlling along the transverse axis. The main objective of this paper is to select the appropriate controller for this purpose. Three controllers i.e. PID, fuzzy and LQR controllers are designed to control the pitch angle. After modeling and obtaining mathematical relationships required for the desired control system, these controllers are applied separately to the mathematical model of airplane. To evaluate the performance of the controllers, a square wave with amplitude of 11.5 degree and period of 7 seconds is considered to be able to compare the time response of the controllers. Finally it is concluded that Fuzzy controller has lower speed due to lack of overshoot. LQR and PID controllers' performance are almost identical but in practice the LQR controller is better to follow the desired pitch angle.

Keywords: Airplane automatic controller, Fuzzy controller, LQR controller

INTRODUCTION

Airplanes today rely heavily on automated control systems. Development of automated control systems plays a crucial role in civil and military aviation. Accordingly, an automatic control system controls the pitch angle in harsh weather conditions so that the plane pitch angle is maintained in desired value without pilot intervention [1]. Auto pilot is a part of the flight control system. Role of this system is a mode that airplane can maintain its tendency in each flight condition without interference of the aircraft pilot [2]. Design of an automatic pilot system needs control theory to understand the concept of stabilized derivatives that are specific to an aircraft in flight [3].

Many works has been done to control the aircraft pitch angle and still continue to work in the future [4-8]. This angle is controlled with the horizontal rudder in the tail of the plane. Changing the elevator angle up or down will cause the nose of plane go up or down. Looking more precisely, changing elevator angle changes the aerodynamic forces that it creates non-linear equations. These equations must be linearized at the operating point that is the stability derivatives in certain flight is conditions [9].

Several works have been conducted to improve the fuzzy controller in [10-14]. Vick and Cohen in [15] conducted a height control system with a PID controller combined with fuzzy controller. In this paper, in order to follow the optimal pitch angle for the airplane, three controllers i.e. PID, Fuzzy and LQR controllers are designed separately and at the end these three controllers are compared with each other.

Dynamics of System

In this section, pitch controlled model obtained from longitudinal equations of airplane is briefly presented. To reduce the complexity of the equations, they are divided into two longitudinal and transverse equations [16]. Pitch control system is shown in Figure 1.

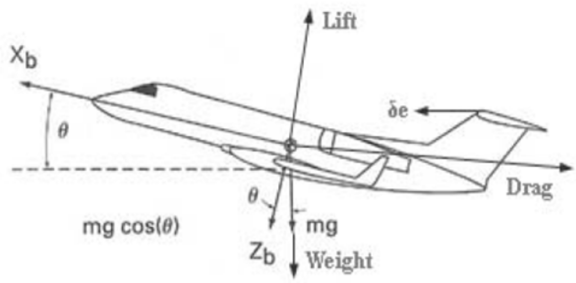

Figure 1. Pitch Control System of airplane.

In the Figure 1, shapes X_b, Y_b and Z_b represent the resultant aerodynamic forces. θ and φ represent the rotation of airplane around the longitudinal and transverse axis, and δ_e represents the deviation of elevator angle. Forces and velocity components are shown as the body fixed coordinates in Figure 2.

Aerodynamic torques for roll (φ), pitch (θ) and yaw (ψ) are L, M and N respectively. Also p, q and r show the rates of these angles, respectively. α and β represent the attack and Sideslip angles, respectively. To model the plane data of [9] is used. In the following, longitudinal stability derivatives are given in Table 1.

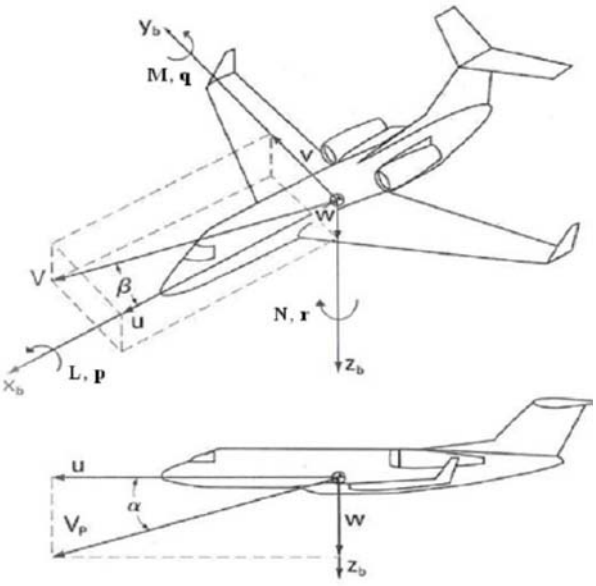

Figure2. The concept of torques, forces and speeds in a body fixed coordinates.

Table1. Indices of longitudinal stability derivatives.

longitudinal derivatives	Indices		
	X axis force (S^{-1})	Z Axis force (F^{-1})	Pitch torque (FT^{-1})
Roll speeds	$X_u = -0.045$	$Z_u = -0.369$	$M_u = 0$
Yaw speeds	$X_w = 0.036$ $X_{\dot{w}} = 0$	$Z_w = -2.02$ $Z_{\dot{w}} = 0$	$M_w = -0.05$ $M_{\dot{w}} = -0.051$
Attack angle	$X_\alpha = 0$ $X_{\dot{\alpha}} = 0$	$Z_\alpha = -355.4$ $Z_{\dot{\alpha}} = 0$	$M_\alpha = -8.8$ $M_{\dot{\alpha}} = -0.8976$
Pitch rate	$X_q = 0$	$Z_q = 0$	$M_q = -2.05$
Elevator deviation	$X_{\delta_e} = 0$	$Z_{\delta_e} = -28.1$	$M_{\delta_e} = -11.84$

Before writing equations, it is assumed that the aircraft is flying at a constant altitude and speed. Therefore, the propulsion and drag forces neutralize the weight and Lift. Another assumption is that the change in pitch angle will not change the speed of the aircraft. According to Figures 1 and 2, dynamic equations including the forces and torques are presented in three equations 1, 2 and 3:

$$X - mg.\sin\theta = m(\dot{u} + qv - rv) \quad (1)$$
$$Z + mg.\cos\theta.\cos\varphi = m(w + p\dot{v} - qu) \quad (2)$$
$$M = I_y\dot{q} + rq(I_x - I_z) + I_{xz}(p^2 - r^2) \quad (3)$$

These equations are complex. Assumptions 4 to 9 can be considered with a good approximation:

$$p = \dot{\varphi} - \dot{\psi}.\sin\theta \quad (4)$$
$$q = \dot{\theta}.\cos\varphi + \dot{\psi}.\cos\theta.\sin\varphi \quad (5)$$
$$r = \dot{\psi}.\cos\theta.\cos\varphi - \dot{\theta}.\sin\varphi \quad (6)$$
$$\dot{\theta} = q.\cos\varphi - r.\sin\varphi \quad (7)$$
$$\dot{\varphi} = p + q.\sin\varphi.\tan\theta + r.\cos\theta.\tan\varphi \quad (8)$$
$$\dot{\psi} = (q.\sin\varphi + r.\cos\varphi)\sec\theta \quad (9)$$

For the controller design, the above equations needed to bee linearized. A commonly used technique to investigate the stability and linearization of the equations is the theory of small disturbances. This theory is used for disturbance with no big amplitude like rotational lagging and divergence that are used for two reasons:

1. Most of the aerodynamic positions are close to linear mode.

2. To analyze the disturbed flight, it is compulsory to consider the disturbance equivalent to a small linear or rotational velocity.

In the linearization with small disturbance theory, it is assumed that the plane is in stable or steady-state flight mode and some disturbances cause the plane to be out of this state. As a result, each of the flight variables has two parts, one is related to the variable in steady-state flight mode which is shown with index 0 and the other one is related to the disturbance and shown by Δ. Equations 10 and 15 represent the linear velocity, angular velocity, Euler angles, position, torques and operators, respectively.

$$u = u_0 + \Delta u \ , \ v = v_0 + \Delta v \ , \ w = w_0 + \Delta w \quad (10)$$
$$p = p_0 + \Delta p \ , \ q = q_0 + \Delta q \ , \ r = r_0 + \Delta r \quad (11)$$
$$\psi = \psi_0 + \Delta\psi \ , \ \theta = \theta_0 + \Delta\theta \ , \ \varphi = \varphi_0 + \Delta\varphi \quad (12)$$
$$X = X_0 + \Delta X \ , \ Y = Y_0 + \Delta Y \ , \ Z = Z_0 + \Delta Z \quad (13)$$
$$M = M_0 + \Delta M \ , \ N = N_0 + \Delta N \ , \ L = L_0 + \Delta L \quad (14)$$
$$\delta = \delta_0 + \Delta\delta \quad (15)$$

By entering new values of the variables in the original equations and taking into account the following assumptions:

The product of small disturbances is ignored.

The second power of small disturbances is ignored.

Reference flight conditions is symmetric and driving forces are unchanged:

$$w_0 = v_0 = P_0 = q_0 = r_0 = \phi_0 = \psi_0$$

Because the disturbance angle is small, sine of the angle is equal to its argument and cosine of the angle is equal to 1. Therefore, Euler angles will be orthogonal.

Nonlinear equations will change into linear equations according to 16 to 17:

$$\left(\frac{d}{dt} - X_u\right)\Delta u - X_w.\Delta w + (g.\cos\theta_0)\Delta\theta = X.\delta_e.\Delta\delta_e \quad (16)$$

$$-Z_u.\Delta u + \left[(1 - Z_w)\frac{d}{dt} - Z_w\right]\Delta w - \left[(u_0 + Z_q)\frac{d}{dt} - g.\sin\theta_0\right]\Delta\theta = Z_{\delta_e}.\Delta\delta_e \quad (17)$$

$$-M_u.\Delta u - \left(M_w\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\theta = M_{\delta_e}.\Delta\delta_e \quad (18)$$

Multiplying the above equation and placing stability derivatives, the transfer function 19 is obtained for pitch rate relative to the elevator deviation:

$$\frac{\Delta q(s)}{\Delta\delta_e(s)} = \frac{-(\Delta\delta_e + M_{\dot{\alpha}}Z_{\delta_e}/u_0)s - (M_\alpha Z_{\delta_e}/u_0 - M_{\delta_e}Z_\alpha/u_0)}{s^2 - (M_q + M_{\dot{\alpha}} + Z_\alpha/u_0)s + (Z_\alpha M_q/u_0 - M_\alpha)} \quad (19)$$

Regarding equations 20 to 22, transfer function of the aircraft pitch angle control can be obtained in 23 and 24.

$$\Delta q = \Delta\dot{\theta} \quad (20)$$

$$\Delta q(s) = s.\,\Delta\theta(s) \quad (21)$$

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{1}{s}.\frac{\Delta q(s)}{\Delta\theta(s)} \quad (22)$$

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{1}{s}.\frac{-(\Delta\delta_e + M_{\dot{\alpha}} Z_{\delta_e}/u_0)s - (M_\alpha Z_{\delta_e}/u_0 - M_{\delta_e} Z_\alpha/u_0)}{s^2 - (M_q + M_{\dot{\alpha}} + Z_\alpha/u_0)s + (Z_\alpha M_q/u_0 - M_\alpha)} \quad (23)$$

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{11.7304s + 22.578}{s^3 + 4.9676s^2 + 12.941s} \quad (24)$$

On the other hand, the state space matrices are required for the LQR controller that is obtained such as equations 25 and 26.

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix}-2.02 & 1 & 0\\ -6.9868 & -2.9476 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\theta\end{bmatrix} + \begin{bmatrix}0.16\\ 11.7304\\ 0\end{bmatrix}[\Delta\delta_e]$$

(25)

$$y = [0 \quad 0 \quad 1]\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\theta\end{bmatrix} + [0] \quad (26)$$

Controllers Design

In this section, design of three feedback PID, Fuzzy and LQR controllers is described.

Pid Controller Design

If we expect improvement of transient and steady-state response, we need to use two lead and lag controllers at the same time [17]. PID controller is a controller with three terms where K_P is proportional gain, K_I integral gain and K_D derivative gain. The transfer function of this controller is like 27:

$$G(s) = K_p + \left(1 + \frac{1}{T_i s} + T_d s\right) \quad (27)$$

Parameters of the controller are selected as $K_P = 4.15$, $K_I = 0.04$ and $K_D = 0.9$.

Fuzzy Controller Design

In controller design for a flying vehicle we are faced with issues such as the uncertainty of aerodynamic parameters, coupling of equations for motion of the body, presence of very non-linear and time-varying equations [18]. So if we need to consider all abovementioned parameters in controller design based on classic logic, that would be a difficult and to some extent impossible task. Therefore, a method is needed that these parameters would not influence on difficulty and calculation cost of the problem. Another problem is that is effect of dynamic factors on parameters like attack angle, side slip angle and etc is not clear. As a result, a method has to be used for controller design so that it is not sensitive to the variation of these parameters. Fuzzy controller is one of the most useful controllers which described in the following.

Fuzzy systems are based on human knowledge and experience. Heart of a fuzzy system is a database that is consisted of if-then fuzzy rules and it is derived from human experience in different conditions of controlling a process which is somewhat unknown in terms of dynamic and structure that cannot be modeled.

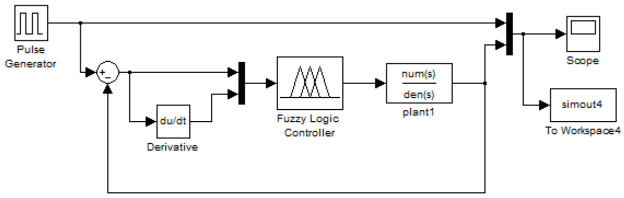

Figure3. Fuzzy controller system of pitch angle.

Figure 3 shows the layout of a Fuzzy controller. In the fuzzy controller, pitch angle is sampled and is compared with the desired value to produce the error value. The designed Fuzzy controller has two inputs and one output. The inputs are error value and the rate of changes of error or error derivative that the control commands or elevator deviation angle is produced according to the table (2).

Table2. Fuzzy controller if-then rules.

	Error (e)	Error rate e_d	Pitch angle
1	N	N	N
2	N	Z	N
3	N	P	N
4	Z	N	N
5	Z	Z	Z
6	Z	P	P
7	P	N	P
8	P	Z	P
9	P	P	P

The controller has two inputs and one output, each of them are consisted of three triangular membership function of p, z and n, and each of them indicates positive, zero and negative. Therefore, the membership functions of the fuzzy controller inputs and output are defined as in Figure 4, 5 and 6. To avoid saturation of elevator angle deviation, the maximum size of deviation is considered ±57 degree (±1 radians).

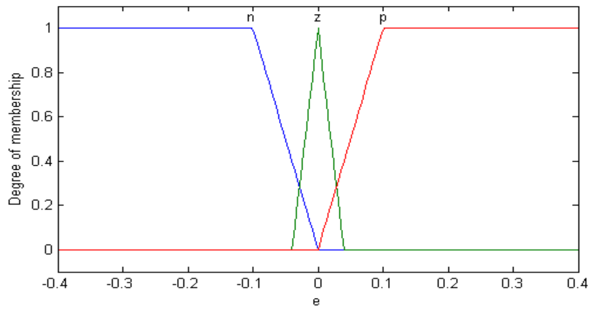

Figure4. Membership functions of error input.

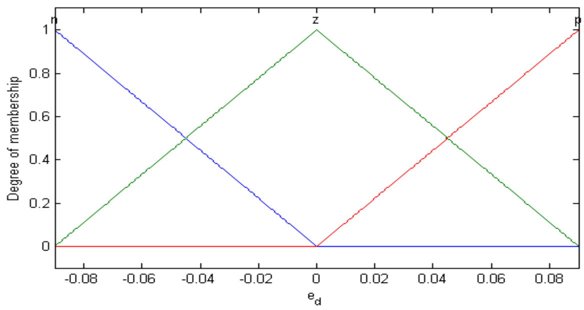

Figure5. Membership function of error rate input.

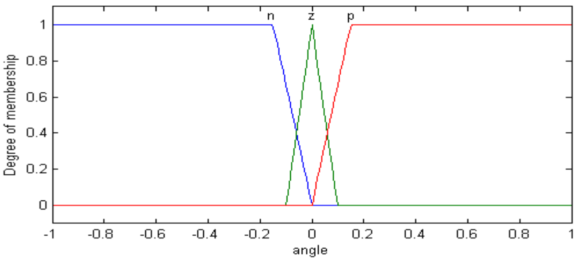

Figure6. Membership functions of output (elevator angle).

LQR Controller Design

In this section, the optimal linear quadratic regulator (LQR) method is used for controller design. This method aims to minimize the cost function J with an appropriate choice of matrices Q, R [19].

$$J(U) = \int_0^\infty (X^T Q X + U^T R U + 2X^T N U)dt \tag{28}$$

Matrices Q, R are real and positive matrices. It is noted that the optimal feedback control law is defined as $U = -K*X$ and all state variables can reach the equilibrium point. After solving the above equations, the desired value of gain K will be calculated as follow:

K=[-0.57044 1.6929 22.361]

In order to reduce the steady-state error of the system after reference command, the value of N should be added that this amount is equal to 22.36. Figure 7 shows the composition of K matrix feedback among this controller.

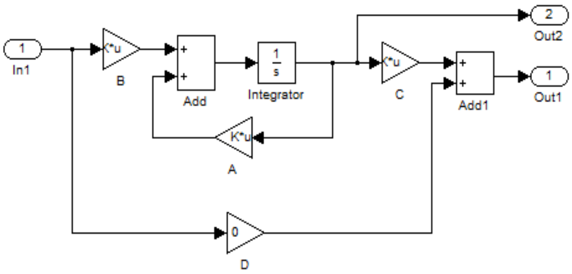

Figure7. State feedback control system with LQR gain.

Simulation And Analysis Of Findings

In this section, a square command as reference is applied to the inputs of the controllers to evaluate the performance of these three controllers. Reference wave amplitude is 11.5 ° (0.2 radians) with a period of 7 seconds. At first, response to each controller is shown separately in Figures 8, 9 and 10, and for comparison, all of the responses are shown in Figure 1.

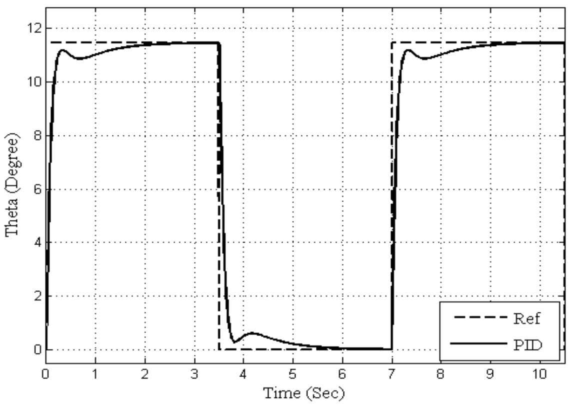

Figure8. Time response of PID controller for pitch angle.

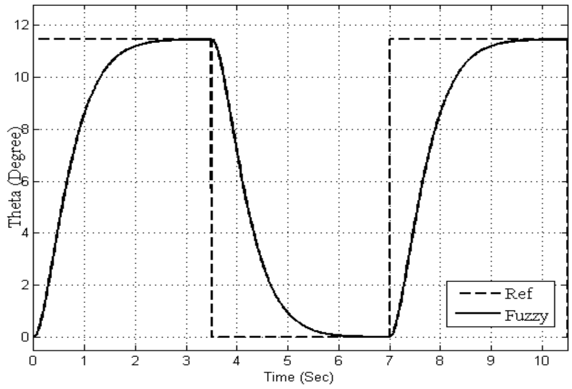

Figure9. Time response of Fuzzy controller for pitch angle.

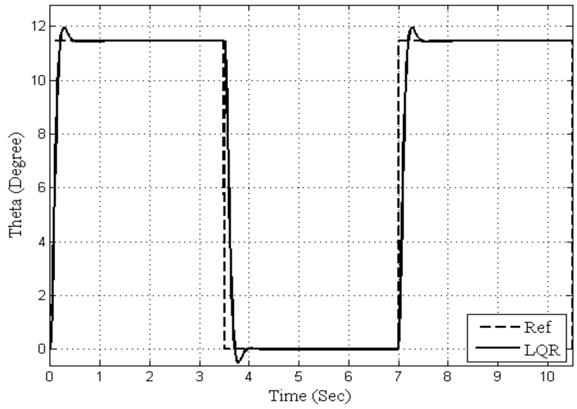

Figure10. Time response of LQR controller for pitch angle.

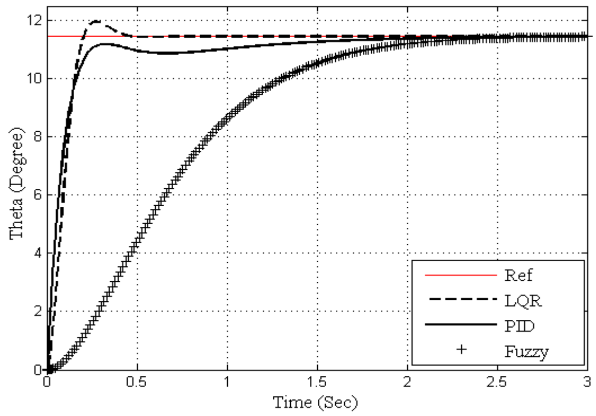

Figure11. Time response of three controllers for pitch angle.

The data obtained from these controllers can be found in Table (3).

Table3. Time response indices of three controllers.

Time response indices	Pitch angle controllers		
	PID	FUZZY	LQR
Rise time (s)	0.211	1.54	0.133
Settling time (s)	0.638	2.23	0.524
Overshoot (%)	0	0	4.63
Steady-state error (%)	0.008	0.7	0.01

CONCLUSION

In this paper, after modeling the transfer function of airplane the pitch angle, three controllers i.e. PID, Fuzzy and LQR controllers are designed that all three controllers are able to seek the desired value of the reference signal. But using the obtained results in the simulation of each controller, it is concluded that the Fuzzy controller has very lower speed to track the desired value. On the other side, LQR and PID controllers have similar time response but according to the obtained diagrams it can be said that LQR controller has better performance with respect to these three controllers. To further continuing the work in the future, it is proposed that LQR and Fuzzy controllers are combined so that the Fuzzy controller generates gain matrix with different K at any time. This action causes the parts of the control process that is near to the system stability, that initial large gain K is not utilized. This is due to the lower production cost of large gains for controllers that work for a long time.

REFERENCES

[1] Nelson R.C. 1998, *Flight Stability and Automatic Control*, McGraw Hill, Second Edition.

[2] Thomas J. Redling 2001, Integrated Flight Control System; A New Paradigm for an Old Art", *IEEE Aerospace and Electronic Systems Society (AESS) Systems Magazine*.

[3] Stojiljkovic, B., Vasov, L., Mitrovic, C., Cvetkovic D., 2009. The Application of the Root Locus Method for the Design of Pitch Controller of an F-104A Aircraft", *Journal of Mechanical Engineering, 55*.

[4] Pavle Boskoski, Biljana Mileva, Stojche Deskoski, 2005, Auto Landing Using Fuzzy Logic", *6th International PhD Workshop on Systems and Control*, Slovenia.

[5] Choe, D. Lee, Y. Cho, S. 2005. Nonlinear Pitch Autopilot Design with Local Lines Linear System Analysis", *International Conference on Automatic Control and Systems Engineering*, Cairo, Egypt.

[6] Wahid, N. Rahmat, M.F. Jusoff, K. 2010. Comparative Assessment using LQR and Fuzzy Logic Controller for a Pitch Control System", European Journal of Scientific Research, 42(2), 184 – 194.

[7] Bates, D. G., Kureemun, R., & Postlethwaite, I. 2001. Quantifying the Robustness of Flight Control Systems Using Nichols Exclusion Regions and the Structured Singular Value", *Proc InstnMech Engrs*, 215.

[8] Ekprasit Promtun, 2009, Sridhar Seshagiri, "Sliding Mode Control of Pitch Rate of an F-16 Aircraft", International Journal on Applied Science, Engineering and Technology, 5(5).

[9] ROSKAM, 2001 .Airplane Flight Dynamic and Automatic Flight Control.

[10] Badaei, A.R., Mortazavi M., 2009, Moradi M.H. "Classical and fuzzy – genetic autopilot design for unmanned aerial vehicle.

[11] Arrofiq, M. Saad, N. 2010. Control of Induction Motor Using Modified Fuzzy Logic Method", 2010 IEEE International Conference on System Man and Cybernetics, Istanbul.

[12] Kuzelkaya, M. I., Eksin, & E. Yesil, 2003. Self-tuning of PID-type Fuzzy Logic Controller Coefficient via Relative Rate Observer", Engineering Applications of Artificial Intelligence, 16, 227-236.

[13] Jianwen, C., Zhanjun, H., Jingcun, S., Changjian, P., 2011, Simulation of Fuzzy Self-tuning PID Control Based on Simulink", Applied Mechanics and Materials, 52, 54, 1644-1649, 2011.

[14] Yu B., Zhu H., Xue C. 2011. Research on Adaptive Fuzzy PID Synchronous Control Strategy of Double-Motor", International Journal of Intelligent Systems and Applications, 5, 28- 33,

[15] Vick A, Cohen K. 2009, "Longitudinal Stability Augmentation Using a Fuzzy logic Based PID Controller", The 28th North American Fuzzy Information Processing Society Annual Conference, USA,.

[16] Khaleel Qutbodin, 2010, Merging Autopilot/Flight Control and Navigation-Flight Management Systems", American Journal of Engineering and Applied Sciences, 629-630.

[17] Myint M., Oo H.K., Naing Z.M., & Myint Y.M., 2008, PID Controller for Stability of Piper Cherokee's Pitch Displacement using MATLAB", International Conference on Sustainable Development: Issues and prospects for the GMS, China.

[18] Schumacher C., Singh S. N. (2000). "Nonlinear Control of Multiple UAVs in Close-coupled Formation Flight", *In Proceedings of the AIAA Guidance, Navigation, and Control Conference*, pp. 14-17, Denver, CO.

[19] Bryson A. E. 1969. Applied Optimal Control: "Optimization, Estimation, & Control.