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Anteriority Indicator For Managing Fuzzy Dates Based on Maximizing And Minimizing Sets

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Abstract

Many applications of fuzzy set theory require defuzzification and ranking approaches based on alpha level sets because exact membership functions may not always be available. In this article, we have assumed that exact membership functions can be approximated using piecewise linear functions based on alpha level sets and derived two analytical formulas to meet such a requirement. The two formulas are, respectively, extensions of the most widely used centroid defuzzification approach and the maximizing set and minimizing set method. The validity of the two formulas has been examined and verified through a test example and their computational processes have also been illustrated in details.

Keywords: Fuzzy number; Ranking; Centroid; Alpha level sets; Defuzzification.

INTRODUCTION

Recently many authors have studied different methods of maximizing set and minimizing set of fuzzy numbers. Carlsson and Fullér [1] suggested an index of difference based on α -level sets, fuzzy subtraction operation and area measurement. Fullér and Majlender [2] suggested a defuzzification procedure by averaging the α -cuts, which is called averaging level cuts. Yager [3] suggested a valuation method, which ranks fuzzy numbers using valuations and Detyniecki et al. [4]. Filev et al. also suggested a generalized level set defuzzification method. In this paper, the two methods are investigated when explicit membership functions are not known but alpha level sets are available. Two analytical formulas are derived under the assumption that the exact membership functions can be approximated by using piecewise linear functions based on alpha level sets. This formula are of significant importance and provide very useful decision supports for a wide variety of applications of the two methods in engineering and other areas. The main purpose of this article is not to suggest a new method for defuzzification, but to derive analytically formulas for the most widely used centroid defuzzification method and the maximizing set and minimizing set ranking approach when only α - level sets are available.

The paper is organized as follows: In Section 2, this article recalls some fundamental results on centroid defuzzification. In section 3, we assume that exact membership functions can be approximated by using piecewise linear functions. Discussion and comparison of this work and other methods are carried out in this Section. The paper ends with conclusions in Section 4.

CENTROID AND UTILITIES BY α - LEVEL SETS

In this situations that only α - level sets are available, exact membership functions are usually not known. In this paper, we assume that exact membership functions can be approximated by using piecewise linear functions based on α - level sets.

Definition 1. Let A be a fuzzy number represented by α - level sets. Its membership function $f_A(x)$ is approximately defined as

$$\mu_A(x)=\begin{cases}0, & x<x^L_{\alpha_0} \ \ or\ \ x>x^R_{\alpha_n},\\ \alpha_i+\dfrac{\Delta\alpha_i(x-x^L_{\alpha_i})}{x^L_{\alpha_{i+1}}-x^L_{\alpha_i}}, & x^L_{\alpha_i}\le x\le x^L_{\alpha_{i+1}}, i=0,1,\ldots,n-1,\\ 1, & x^L_{\alpha_n}\le x\le x^R_{\alpha_i},\\ \alpha_i+\dfrac{\Delta\alpha_i(x^R_{\alpha_i}-x)}{x^R_{\alpha_i}-x^U_{\alpha_{i+1}}}, & x^R_{\alpha_{i+1}}\le x\le x^R_{\alpha_{i+1}}, i=0,1,\ldots,n-1,\end{cases} \tag{2}$$

where $\Delta\alpha_i=\alpha_{i+1}-\alpha_i,\ i=0,\ldots,n-1$ and $0=\alpha_0<\alpha_1<\ldots<\alpha_n=1$. Under the assumption of piecewise linearity, we have the following theorem.

Theorem 1. Let be a fuzzy number represented by α - level sets, whose membership function is defined by Eq. (2). Then the defuzzified centroid of A can be determined by

$$x_0(A)=\frac{\int_a^d x\,\mu_A(x)dx}{\int_a^d \mu_A(x)dx} \tag{3}$$

where

$$\int_a^d \mu_A(x)dx=\frac{1}{2}\left[x_{\alpha_n}^R-x_{\alpha_n}^L-\sum_{i=1}^{n-1}\alpha_i(x_{\alpha_{i+1}}^R-x_{\alpha_{i+1}}^L)+\sum_{i=1}^{n-1}\alpha_{i+1}(x_{\alpha_i}^R-x_{\alpha_i}^L)\right] \tag{4}$$

$$\int_a^d x\,\mu_A(x)dx=\frac{1}{6}\left[x_{\alpha_n}^{2R}-x_{\alpha_n}^{2L}-\sum_{i=1}^{n-1}\alpha_i(x_{\alpha_{i+1}}^{2R}-x_{\alpha_{i+1}}^{2L})+\sum_{i=1}^{n-1}\alpha_{i+1}(x_{\alpha_i}^{2R}-x_{\alpha_i}^{2L})\right]$$
$$+\frac{1}{6}\sum_{i=0}^{n-1}\Delta\alpha_i(x_{\alpha_i}^R.x_{\alpha_{i+1}}^R-x_{\alpha_i}^L.x_{\alpha_{i+1}}^L) \tag{5}$$

Especially when $\Delta\alpha_i\equiv\frac{1}{n}$ and $\alpha_i=\frac{i}{n}, i=0,\ldots,n$, the equations are simplified as

$$\int_a^d \mu_A(x)dx=\frac{1}{2n}\left[(x_{\alpha_0}^R-x_{\alpha_0}^L)+(x_{\alpha_n}^R-x_{\alpha_n}^L)+2\sum_{i=1}^{n-1}(x_{\alpha_i}^R-x_{\alpha_i}^L)\right] \tag{6}$$

$$\int_a^d x\,\mu_A(x)dx=\frac{1}{6n}\left[(x_{\alpha_0}^{2R}-x_{\alpha_0}^{2L})+(x_{\alpha_n}^{2R}-x_{\alpha_n}^{2L})+2\sum_{i=1}^{n-1}(x_{\alpha_i}^{2R}-x_{\alpha_i}^{2L})\right]+\frac{1}{6n}\sum_{i=0}^{n-1}\left(x_{\alpha_i}^R.x_{\alpha_{i+1}}^R-x_{\alpha_i}^L.x_{\alpha_{i+1}}^L\right) \tag{7}$$

Remark 1. Let $n=1$. Then Eqs. (5), (6) and (3) become

$$\int_a^d \mu_A(x)dx=\frac{1}{2}\left[(x_{\alpha_0}^R-x_{\alpha_0}^L)+(x_{\alpha_n}^R-x_{\alpha_n}^L)\right] \tag{8}$$

$$\int_a^d x\,\mu_A(x)dx=\frac{1}{6}\left[(x_{\alpha_0}^{2R}-x_{\alpha_0}^{2L})+(x_{\alpha_n}^{2R}-x_{\alpha_n}^{2L})+\left(x_{\alpha_0}^R.x_{\alpha_n}^R-x_{\alpha_0}^L.x_{\alpha_n}^L\right)\right] \tag{9}$$

$$x_0(A)=\frac{1}{3}\left[x_{\alpha_0}^L+x_{\alpha_n}^L+x_{\alpha_n}^R+x_{\alpha_0}^R-\frac{x_{\alpha_0}^R.x_{\alpha_n}^R-x_{\alpha_0}^L.x_{\alpha_n}^L}{(x_{\alpha_0}^R-x_{\alpha_0}^L)+(x_{\alpha_n}^R-x_{\alpha_n}^L)}\right] \tag{10}$$

which is exactly the centroid defuzzification formula of trapezoidal fuzzy numbers.

Remark 2. If $x_{\alpha_n}^L=x_{\alpha_n}^R$, then Eqs. (5), (6), and (3) can be simplified as

$$\int_a^d \mu_A(x)dx=\frac{1}{2n}\left[(x_{\alpha_0}^R-x_{\alpha_0}^L)+2\sum_{i=1}^{n-1}(x_{\alpha_i}^R-x_{\alpha_i}^L)\right] \tag{11}$$

$$\int_a^d x\,\mu_A(x)dx=\frac{1}{6n}\left[(x_{\alpha_0}^{2R}-x_{\alpha_0}^{2L})+2\sum_{i=1}^{n-1}(x_{\alpha_i}^{2R}-x_{\alpha_i}^{2L})+\sum_{i=0}^{n-1}\left(x_{\alpha_i}^R.x_{\alpha_{i+1}}^R-x_{\alpha_i}^L.x_{\alpha_{i+1}}^L\right)\right] \tag{12}$$

$$x_0(A)=\frac{1}{3}.\frac{(x_{\alpha_0}^{2R}-x_{\alpha_0}^{2L})+2\sum_{i=1}^{n-1}(x_{\alpha_i}^{2R}-x_{\alpha_i}^{2L})+\sum_{i=1}^{n-1}(x_{\alpha_i}^R.x_{\alpha_{i+1}}^R-x_{\alpha_i}^L.x_{\alpha_{i+1}}^L)}{(x_{\alpha_0}^R-x_{\alpha_0}^L)+2\sum_{i=1}^{n-1}(x_{\alpha_i}^R-x_{\alpha i}^L)} \tag{13}$$

Remark 3. Let $n=1$. Then Eqs. (11)-(13) become

$$\int_a^d \mu_A(x)dx=\frac{1}{2}(x_{\alpha_0}^R-x_{\alpha_0}^L) \tag{14}$$

$$\int_a^d x\,\mu_A(x)dx=\frac{1}{6}\left[(x_{\alpha_0}^{2R}-x_{\alpha_0}^{2L})+(x_{\alpha_0}^R.x_{\alpha_n}^R-x_{\alpha_0}^L.x_{\alpha_n}^L)\right] \tag{15}$$

$$x_0(A)=\frac{1}{3}.\frac{(x_{\alpha_0}^{2R}-x_{\alpha_0}^{2L})+(x_{\alpha_0}^R.x_{\alpha_n}^R-x_{\alpha_0}^L.x_{\alpha_n}^L)}{(x_{\alpha_0}^R-x_{\alpha_0}^L)}=\frac{1}{3}\left(x_{\alpha_0}^R+x_{\alpha_n}^R+x_{\alpha_0}^L\right) \tag{16}$$

Theorem 2. Let , $0<\alpha\leq 1$, be a fuzzy number represented by α - level sets. Its membership function is defined by (2). Then the total utility value of A can be determined by

$$u_T=\frac{(u_M+1-u_G)}{2} \tag{17}$$

where

$$u_M=\frac{\alpha_{j_0+1}.(x_{\alpha_{j0}}^R-x_{\min})-\alpha_{j0}.(x_{\alpha_{j0}+1}^R-x_{\min})}{x_{\alpha_{j0}}^R-x_{\alpha_{j0}+1}^R+(\alpha_{j0+1}-\alpha_{j0})(x_{\max}-x_{\min})} \tag{18}$$

$$u_G=\frac{\alpha_{i_0+1}.(x_{\max}-x_{\alpha_{i0}}^L)-\alpha_{i0}.(x_{\max}-x_{\alpha_{i0}+1}^L)}{x_{\alpha_{i0+1}}^L-x_{\alpha_{i0}}^L+(\alpha_{i0+1}-\alpha_{i0})(x_{\max}-x_{\min})} \tag{19}$$

and $x_{\min}=\inf X$, $x_{\max}=\sup X$, $X=\bigcup_{i=1}^n X_i$, $X_i=\{x|\mu_A(x)>0\}$.

Remark 4. Let n=1. Then Eqs. (18), (19) become

$$u_M=\frac{x_{\alpha_0}^R-x_{\min}}{x_{\alpha_0}^R-x_{\alpha_n}^R+(x_{\max}-x_{\min})} \tag{20}$$

$$u_G=\frac{x_{\max}-x_{\alpha_0}^L}{x_{\alpha_n}^L-x_{\alpha_0}^L+(x_{\max}-x_{\min})} \tag{21}$$

In order to determine which intervals the intersection points G and M lie in, we introduce the following sign functions:

$$S_1(\alpha)=\mu_G(x_\alpha^L)-\alpha \tag{22}$$

$$S_2(\alpha)=\mu_M(x_\alpha^R)-\alpha \tag{23}$$

It can be seen very clearly that

$$\begin{cases}S_1(\alpha)>0, & \alpha\leq\alpha_{i0},\\ S_1(\alpha)<0, & \alpha\geq\alpha_{i0+1}.\end{cases}\quad\text{and}\quad\begin{cases}S_2(\alpha)>0, & \alpha\leq\alpha_{j0},\\ S_2(\alpha)<0, & \alpha\geq\alpha_{j0+1}.\end{cases}$$

So, by checking the signs of the above two functions and observing their changes, $[x^L_{\alpha_{i0}}, x^L_{\alpha_{i0+1}}]$ and $[x^R_{\alpha_{j0+1}}, x^R_{\alpha_{j0}}]$ can be readily determined.

THE PREFERENCE ORDERING OF FUZZY NUMBERS

In this section, we present a new approach for ranking fuzzy numbers based on the distance method. The method not only considers the total utility value of a fuzzy number, but also considers the minimum crisp value of fuzzy numbers. For ranking fuzzy numbers, this study defines a minimum crisp value $x_{\min}$ to be the benchmark. The advantages of the definition of minimum crisp value are two-fold: firstly, the minimum crisp values will be obtained by themselves, and another is it is easy to compute. This new approach for ranking fuzzy numbers not only can compute more quickly and correctly but also ranking the normal, non normal, positive and negative fuzzy numbers. Assume that there are **n** n fuzzy numbers $A_1, A_2, \ldots, A_n$. The proposed method for ranking fuzzy numbers $A_1, A_2, \ldots, A_n$ is now presented as follows

Step 1. Use formulas (17) - (19) to calculate the total utility of each fuzzy numbers A_j, where $1 \le j \le n.$

Step 2. Calculate the maximum crisp value $x_{\min}$ of all fuzzy numbers A_j , where $1 \le j \le n.$

Step 3. Use the point u_T to calculate the ranking value $d_{u_T}(A_j, x_{\min})$ of the fuzzy numbers A_j, where $1 \le j \le n$, as follows:

$$d_{u_T}(A_j, x_{\min}) = |u_T(A_j) - x_{\min}|. \tag{24}$$

From formula (24), we can see that $d_{u_T}(A_j, x_{\min})$ can be considered as the Euclidean distance between the point $(0, u_T(A_j))$ and the point $(x_{\min}, o)$. We can see that the bigger the value of $d_{u_T}(A_j, x_{\min})$, the better the ranking of A_j , where $1 \le j \le n.$

Let A_j is a fuzzy number characterized by (2) and $d_{u_T}(A_j, x_{\min})$ is the Euclidean distance between the point $(0, u_T(A_j))$ and the point $(x_{\min}, 0)$ of its.

Since this article wants to approximate a fuzzy number by a scalar value, thus the researchers have to use an operator $d_{u_T} : F \to R$ (A space of all fuzzy numbers will be denoted by F) which transforms fuzzy numbers into a family of real line. Operator d_{u_T} is a crisp approximation operator. Since ever above defuzzification can be used as a crisp approximation of a fuzzy number, therefore the resultant value is used to rank the fuzzy numbers. Thus, d_{u_T} is used to rank fuzzy numbers. Let $A_1, A_2 \in F$ be two arbitrary fuzzy numbers. Define the ranking of A_1 and A_2 by d_{u_T} on F as follows:

1. $d_{u_T}(A_1, x_{\min}) < d_{u_T}(A_2, x_{\min})$ if only if $A_1 \prec A_2$,
2. $d_{u_T}(A_1, x_{\min}) > d_{u_T}(A_2, x_{\min})$ if only if $A_1 \succ A_2$,
3. $d_{u_T}(A_1, x_{\min}) = d_{u_T}(A_2, x_{\min})$ if only if $A_1 \sim A_2$.

Then, this article formulates the order $\succeq$ and $\preceq$ as $A_1 \succeq A_2$ if and only if $A_1 \succ A_2$ or, $A_1 \sim A_2$, $A_1 \preceq A_2$ if and only if $A_1 \prec A_2$ or $A_1 \sim A_2$. The new ranking index can sort many different fuzzy numbers simultaneously. In addition, the calculation is simple, and the index also satisfies the common properties of ranking fuzzy numbers:

(a) Transitivity of the order relation, i.e. if $A_1 \preceq A_2$ and $A_2 \preceq A_3$, then we should have $A_1 \preceq A_3$.

(b) Compatibility of addition, that is if there is $A_1 \le A_2$ on $\{A_1, A_2\}$, then there is $A_1 + A_3 \le A_2 + A_3$ on $\{A_1 + A_3, A_2 + A_3\}$.

Remark 5. If $A \preceq B$ then $-A \succeq -B$.

Hence, this article can infer ranking order of the images of the fuzzy numbers.

Example 1. Three triangular fuzzy numbers are , and . From Eqs. (10), (17), (20) and (21), we have:

$$x_0(A_1) = \frac{1}{3}\left[x^L_{\alpha_0} + x^L_{\alpha_n} + x^R_{\alpha_n} + x^R_{\alpha_0} - \frac{x^R_{\alpha_0}.x^R_{\alpha_n} - x^L_{\alpha_0}.x^L_{\alpha_n}}{(x^R_{\alpha_0} - x^L_{\alpha_0}) + (x^R_{\alpha_n} - x^L_{\alpha_n})}\right] = 0.3$$

$$u_M(A_1) = \frac{x^R_{\alpha_0} - x_{\min}}{(x^R_{\alpha_0} - x^R_{\alpha_n}) + (x_{\max} - x_{\min})} = 0.45$$

$$u_G(A_1) = \frac{x_{\max} - x^L_{\alpha_0}}{(x^L_{\alpha_n} - x^L_{\alpha_0}) + (x_{\max} - x_{\min})} = 0.79$$

$$u_T(A_1) = \frac{(u_M(A_1) + 1 - u_G(A_1))}{2} = \frac{(0.45 + 1 - 0.79)}{2} = 0.33$$

The defuzzified centroid and utilities based on α - level sets are computed in Table 1, form which it can be seen quite clearly that the results based on α - level sets are identical with those obtained from known membership function. This verifies the validity of the two analytical formulas developed in this study.

Example 2. Consider the three fuzzy numbers $A = (1,2,5)$, $B = (0,3,4)$ and $C = (2,2.5,3)$. By using this new approach $d_{u_T}(A, x_{\min}) = 6.2$, $d_{u_T}(B, x_{\min}) = 6.5$ and $d_{u_T}(C, x_{\min}) = 4.8$. Hence, the ranking order is $C \prec A \prec B$ too. To compare with some of the other methods in [5,6], the reader can refer to Table 2.

All the above examples show the results of this effort to be more efficient and consistent than the previous ranking methods, and overcome the shortcomings of other methods.

CONCLUSION

In this endeavor, the author discusses the problem of defuzzification based on maximizing set and minimizing set of fuzzy numbers and then suggests anteriority about these points of fuzzy numbers. In this paper, the two methods are investigated when explicit membership functions are not known but alpha level sets are available. Numerical examples are offered to test the new method and illustrate their computational processes.

Table.1.

α	A_1		A_2		A_3	
	$A_{1\alpha}^{L}$	$A_{1\alpha}^{R`}$	$A_{2\alpha}^{L}$	$A_{2\alpha}^{R}$	$A_{3\alpha}^{L}$	$A_{3\alpha}^{R}$
0.0	0.20	0.50	0.170	0.58	0.25	0.70
0.1	0.21	0.48	0.185	0.55	0.26	0.67
0.2	0.22	0.46	0.200	0.53	0.28	0.64
0.3	0.23	0.44	0.215	0.50	0.29	0.61
0.4	0.24	0.42	0.230	0.47	0.31	0.58
0.5	0.25	0.40	0.245	0.45	0.32	0.55
0.6	0.26	0.38	0.260	0.42	0.34	0.52
0.7	0.27	0.36	0.275	0.39	0.35	0.49
0.8	0.28	0.34	0.290	0.37	0.37	0.46
0.9	0.29	0.32	0.305	0.34	0.38	0.43
1.0	0.30	0.30	0.320	0.32	0.4	0.40
Centroid	0.30		0.53		0.45	
Total utility	0.33		0.37		0.48	
$d_{u_T}(A_i, x_{\min})$	0.37		0.40		0.50	
Rank	3		2		1	

Fig.1.

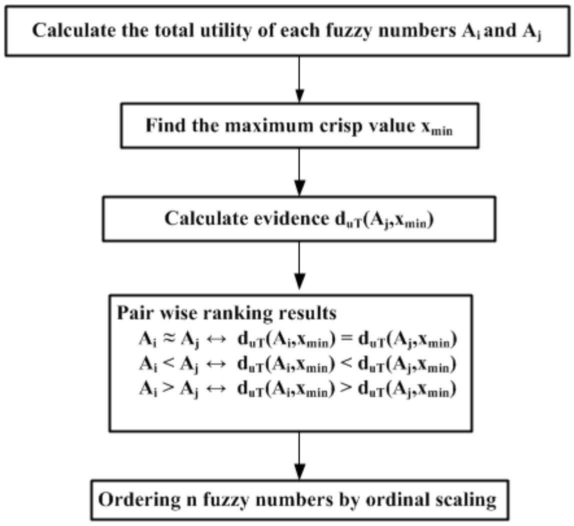

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Table.2. Comparative results of Example 2.

Fuzzy number	New approach	Sign Distance with p=1	Sign Distance with p=2	Distance Minimization	Chu and Tsao
A	6.2	3	2.16	2.50	0.74
B	6.5	3	2.70	2.50	0.74
C	4.8	3	2.70	2.50	0.75
Results	$C \prec A \prec B$	$C \sim A \sim B$	$C \prec A \sim B$	$C \sim A \sim B$	$A \sim B \prec C$