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# Compressing-Bending Deformation of Base-Slab of Dry-Dock Settling Basin Resting on Elastic Foundation

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**Abstract**

This research deals with the bending deformation resulted from the longitudinal-transverse forces on the dock settling basin bottom resting on elastic foundation. Subgrade reaction coefficient of the contact problem is presented as a third order parabola which is computed by Fuss-Winkler model and fourth order ordinary differential equation. The general solution of this equation is possible by considering the values of boundary conditions and Maclaurin's series method. The main characteristic of this method is using of the zero values as one of the boundary conditions values and derivatives of Y(x) that gained by successive integration. Some equations are represented to compute the values of bending moment, shearing force and deformation at any arbitrary point of the settling basin bottom section. A new method of computing the rate and quality of the dock settling basin deformation is presented in this research. The results are compared to the results of the some other researchers.

***Keywords:*** Dry-Dock Settling Basin, Compressing-Bending Deformation, Elastic Foundation, Maclaurin's Series, Boundary Condition

## INTRODUCTION

The computation of interior forces and deformations of base-slabs of dry dock settling basins due to applied forces and subgrade reaction is surveyed in this article. Winkler (1867) proposed the first model of beam on an elastic foundation based on pure bending beam theory, later Pasternak in 1954 proposed the shear model based on the assumption of pure shear of the beam. Both of these two models take an extreme point of view on the deformation behavior of the beam. Biot (1937) considered the problem of bending, under a concentrated load, of infinite flexible beams on a homogeneous elastic-isotopic subgrade. Terzaghi (1955) established a number of equations to calculate the modulus of subgrade reaction for cohesive and cohesionless soils, depending on plate load test results. Klepikov (1967) proposed a computation method for obtaining the deformations of flexible base-slabs of dock settling basins of ship entering sluice. In this method, the base-slab is divided into several parts and a coefficient of subgrade reaction acting along each part is assumed to be linear. In general the problem is bringing out to equation system and is solved by applying ultimate finite element method. The solution is generated by using the initial parameters and interactions follow. Kocitcin (1971) considered the variation of the coefficient of subgrade reaction in a nonlinear curve form. He considered the form of the curve to be dependent on the deflection of structure foundation in a convex or concave parabolic form. Then, he resolved the problem using a fourth order differential equation with special boundary conditions. Simvulidi (1978) offered a computation method for obtaining elastic deformations of the foundation of engineering structures. In his method, the subgrade reaction modulus is computed through fourth order polynomials. According to shear function theorem, all the loads applied to the beam are substituted by a uniformly distributed pressure and deformations are computed by solving a fourth order differential equation. The author mainly used the elastic half-space theory and solved differential soil-structure interaction problems. Qorbunov–Posadov et al. (1984) tried a computation method for flexible beam-elements on an elastic subgrade by using elastic half-space theorem. The half-space is generally characterized by the deformation model and it's Poisson's Ratio. The author solved the problem by considering the subgrade reaction modulus in the form of an eighth order polynomial. Yankelevsky et al. (1988) presented an iterative procedure for the analysis of beams resting on nonlinear elastic foundation based on the exact solution for beams on a linear elastic foundation. Yin (2000) derived the governing ordinary differential equation for a reinforced Timoshenko beam on an elastic foundation. Guo and Wietsman (2002) made an analytical method, accompanied by a numerical scheme, to evaluate the response of beams on the space-varying foundation modulus, which is called the modulus of subgrade reaction (Kz=Kz(x)). Mammadov et al. (2004) performed experiments for determining the deformation by hypothesizing of flexible subgrade beams of different geometric forms within a finite confined layer.

In this research article, an analytical method, applied to static analysis of base-slab of the dock settling basin resting on elastic foundation, is suggested to compute the values of vertical subgrade reactions, deflections, bending moments and shear forces along the substructure. Finally, the values obtained through the application of this method are compared with the values obtained through the use of other methods found in the literature.

## THEORETICAL BACKGROUND

In the article, the deformation computation of flexible base-slab of dry dock settling basin influenced by the loads at its length-width is considered. Flexible base-slab is as a beam on an elastic foundation with constant rigidity. The supporting foundation soil is considered to be an elastic, isotropic, and homogeneous continuum with constant thickness *H* and having $E_s$ and $v_s$, modulus of elasticity and Poisson's ratio, respectively. The subsoil thickness *H* can be view as a depth from the subgrade surface to the assumed rigid layer, or as a depth to which the deformations due to beam loading are not significant. Changeable coefficient of subgrade reaction of Fuss-Winkler is used as a mechanical model. Though the Fuss-Winkler foundation in an extremely simple model of real foundations, such as soil, it is widely used because of its simplicity. The coefficient of the subgrade reaction of ground, basic parameter of this model, along the base-slab according to parabolic non-linear equation is accepted changeable. A typical dry dock settling basin construction is shown in Figure 1. It is assumed that the dock settling basin depth is considered *h* and it is not filled up with water, as shown in Figure 1. The dock settling basin walls and its base slab are affected by the forces generated by a variety of factors. The forces applied to the walls include lateral pressure of backfill and the weight of concrete walls. The base slab of the settling basin is subjected to the internal forces transferred from the walls (including axial and shearing forces and moment at the ends of base slab). A schematic view of the mentioned forces is shown in Figure 2. The computation scheme of the static contact problem is shown in the figure 3. According to this scheme base-slab is affected by the (q), created by the weight along its length, regular load. The forces resulted from influence on the lateral walls the initial and end transverse section of the base-slab is affected by the $Q_o(Q_L)$ Force, $M_o(M_L)$ moment on its width and by the and (N) compressing force on its lengths. In order to compute the differential equation of the flexible base-slab at the compressing-bending in this complex loading scheme, we can write bending moment and shearing force at the arbitrary cross section:

$$\begin{cases} M(x) = -Q_o \cdot x + M_o + N \cdot [Y(x) - Y_o] + M_{qr}(x) - q \cdot \dfrac{x^2}{2} \\ Q(x) = -Q_o + N \cdot Y^{/}(x) + Q_{qr}(x) - q \cdot x \end{cases} \tag{1}$$

In equation (1), $M_{qr}(x)$ and $Q_{qr}(x)$ are bending moment and shearing force which are created at the arbitrary cross section of the base-slab of the dock settling basin from the reactive resistance of the base-slab.

According to last equation if we write the equation of the beam bended arrow then we have:

$$E.J \frac{d^3Y(x)}{dx^3} = -Q(x) = Q_o - N.Y^{/}(x) - Q_{qr}(x) + q.x \tag{2}$$

If we get the differential of the equation according to (x) once more, then:

$$E.J \frac{d^4Y(x)}{dx^4} = q - N.Y^{/}(x) - q_{qr}(x) \tag{3}$$

$q_{qr}(x)$ is the intensity of the reactive resistance of the base-slab. According the Fuss-Winkler model:

$$q_{qr} = -k(x)Y(x) \tag{4}$$

k(x) has been the changeable coefficient of subgrade reaction along the base-slab and Y(x) is the deflection at the arbitrary cross section of the base-slab of the dock settling basin. k(X) is accepted in the following trinomial square parabolic equation.

$$K(x) = K_o - \frac{4(K_o - K_s)}{L}x + \frac{4(K_o - K_s)}{L^2}x^2 \tag{5}$$

$k_o$, $k_l = k_o$ are the coefficient of subgrade reaction at the beginning and end transverse section of the base-slab of the dock settling basin.

$K_s$ is the coefficient of subgrade reaction in the middle of the flexible beam.

If we consider the (4) and (5) equations in equation (3), we can write the differential equation of the base-slab of the dock settling basin at the compressing-bending as follow:

$$Y^{IV}(X) = \overline{q} - \upsilon^2.Y^{I}(X) - (a_o - a_1 x + a_2 x^2)Y(x) \tag{6}$$

The following terms were accepted as conditionally signs:

$$a = \frac{q}{E.J}, [m^{-3}]\ ;\ v^2 = \frac{N}{E.J}\ ;\ [m^{-2}]\ ;\ a_o = \frac{k_o}{E.J}\ [m^{-4}]$$

$$a_1 = \frac{4(k_o - k_s)}{E\ .J};\ [m^{-5}];\ a_2 = \frac{4(k_o - k_s)}{L^2 E.J};\ [m^{-6}] \tag{7}$$

*E.J* is the bending sharpness of the base-slab of the dock settling basin.

According to the equation (6), we can accept the following boundary conditions for the left beginning transverse section of the base-slab of the dock settling basin.

$$Y(0) = Y_o;\ Y^{/}(0) = \theta_o;\ Y^{I}(0) = -\frac{M_o}{E.J} = -\overline{M}_o\ ;$$

$$Y^{///}(0) = \frac{Q_o}{E.J} - \upsilon^2\theta_o = \overline{Q}_o - \upsilon^2\theta_o \tag{8}$$

In the last equation $Y_o$, $\theta_o$, $M_o$ and $Q_o$ are the beginning parameters. They are the deflection, rotating angle, bending moment and shearing force at the initial cross section of the base-slab of the dock settling basin. Equation (6) is an ordinary fourth order differential equation. It can't be solved in the quadrature. Different approximate methods are used to solve this equation. The principal methods as the variation methods of constructions mechanics, A.N.Krilov numerical computation method, Series method, Picard consequence limit method etc are used in the computation of this equation [1,2,3,4,5,6,7].

## ANALYSIS CONSIDERING OF THE METHOD

Series method is used in the computation of (6)-(8) static contact problems. If we compute the Y(x) function in the form of the Mackloran series, so we write:

$$Y(x) = Y(0) + Y'(0)\frac{x}{1!} + Y'(0)\frac{x^2}{2!} + Y'''(0)\frac{x^3}{3!} + Y^{IV}(0)\frac{x^4}{4!} + ....... + Y^{(n)}(0)\frac{x^n}{n!} + ....... \quad (9)$$

The zero values of Y(0(, Y$'$(0), Y$''$(0), Y$'''$(0( are accepted as initial conditions of equation (8). Four order derivative value of Y(x) function is found by using the (6) differential equation. The zero values of derivative of Y(x) function more than Four-order, by considering the initial boundary conditions equation (8), can be computed by consequence differentiating of the differential equation (6). If we substitute the zero values of all of the derivatives in (9) Mackloran series and we group the gained functions according to the four beginning parameters and intensity of the uniformly distributed then we can solve the equation problem as follow:

$$Y(x) = Y_o F_1(x) + \theta_o F_2(x) - \frac{M_o}{E.J} F_3(x) + \frac{Q_o}{E.J} F_4(x) + \frac{q}{E.J} F_5(x) \quad (10)$$

These functions $F_1(x)$, $F_2(x)$, $F_3(x)$, $F_4(x)$ are the Four special solutions, homogeneous independent linear, of (6) differential equation. $F_5(x)$ function is a heterogeneous special solution of the (6) differential equation. So the unknown functions are specified by the following continuous, rapid converging series. In the last equation $Y_o$ deflection, $\theta_o$ rotate angle (slop), $M_o$ bending moment and $Q_o$ shearing force are the initial parameters at the beginning of the base-slab.

According to the (10) general solution, we should find the following formulas to compute the rotating angle, bending moment and shearing force at the arbitrary transverse section of the base-slab of dock settling basin:

$$\begin{cases} \theta(x) = Y_o F_1'(x) + \theta_o F_2'(x) - \overline{M}_o F_3'(x) + \overline{Q}_o F_4'(x) + \overline{q} F_5'(x) \\ \frac{M(x)}{E.J} = Y_o F_1''(x) + \theta_o F_2''(x) - \overline{M}_o F_3''(x) + \overline{Q}_o F_4''(x) + \overline{q} F_5''(x) \\ \frac{Q^{Leng.}(x)}{E.J} = Y_o\left[F_1'''(x) + v^2 F_1'(x)\right] + \theta_o\left[F_2''(x) - v^2 F_2'(x)\right] - \\ -\overline{M}_o\left[F_3''(x) + v^2 F_3'(x)\right] + \overline{Q}_o\left[F_4''(x) + v^2 F_4'(x)\right] + \overline{q}\left[F_5'''(x) + v^2 F_5'(x)\right] \end{cases} \quad (11)$$

The first three derivatives of $F_j(x)$ function in the last equation are specified by the successive differentiate of (11) equations.

$$F_1'(x) = \frac{dF_1(x)}{dx}, F_1'(x) = \frac{d^2F_1(x)}{dx^2}, F_1'''(x) = \frac{d^3F_1(x)}{dx^3}, \text{etc}$$

$Y_o$ and $\theta_o$ in the (10)-(40) formulas are the unknown initial parameters. The boundary conditions at the right side of the base-slab are used to find these parameters:

$$\frac{M(L)}{E.J} = -\frac{M_L}{E.J} \quad ; \quad \frac{Q^{Leng.}(L)}{E.J} = \frac{Q_L}{E.J} \quad (12)$$

According to the last two lines of the (11) equation, (12) Conditions is written as follow:

$$\begin{cases} \frac{M(L)}{E.J} = Y_o F_1''(L) + \theta_o F_2''(L) - \overline{M}_o F_3''(L) + \overline{Q}_o F_4''(L) + \overline{q} F_5''(L) = -\frac{M_L}{E.J} = -\frac{M_o}{E.J} = -\overline{M}_o \\ \frac{Q^{Leng.}(L)}{E.J} = Y_o\left[F_1'''(L) + v^2 F_1'(L)\right] + \theta_o\left[F_2'''(L) + v^2 F_2'(L)\right] - \\ -\overline{M}_o\left[F_3''''(L) + v^2 F_3'(L)\right] + \overline{Q}_o\left[F_4'''(L) + v^2 F_4'(L)\right] + \overline{q}\left[F_5'''(L) + v^2 F_5'(L)\right] = \frac{Q_L}{E.J} = \frac{Q_o}{E.J} = \overline{Q}_o \end{cases} \quad (13)$$

If we accept the following substitutes:

$$\begin{cases} \overline{M}_o\left[F_3''(L) - 1)\right] - \overline{Q}_o F_4''(L) - \overline{q} F_5''(L) = \alpha_1(L) \\ F_1''''(L) + v^2 F_1'(L) = \alpha_2(L) \\ F_2'''(L) + v^2 F_2'(L) = \alpha_3(L) \\ \overline{M}_o\left[F_3''''(L) + v_2 F_3'(L)\right] - \overline{Q}_o\left[1 - F_4''(L) - v^2 F_4'(L)\right] - \overline{q}\left[F_5'''(L) + \upsilon^2 F_5'(L)\right] = \alpha_4(L) \end{cases} \quad (14)$$

In this case, we get the following equation system to find the unknown parameters:

$$\begin{cases} Y_o F_1''(L) + \theta_o F_2'' L) = \alpha_1(L) \\ Y_o \alpha_2(L) + \theta_o \alpha_3(L) = \alpha_4(L) \end{cases} \quad (15)$$

If we solve this equation system according to the unknown parameters, we shall receive:

$$\begin{cases} Y_o = \frac{\alpha_4(L) F_2'' L) - \alpha_1(\mathrm{L})\,\alpha_3(\mathrm{L})}{\alpha_2(L) F_2'' L) - \alpha_3(\mathrm{L})\,F_1''(\mathrm{L})} \\ \theta_o = \frac{\alpha_4(L) F_1''(L) - \alpha_1(\mathrm{L})\,\alpha_2(\mathrm{L})}{\alpha_2(L) F_2''(L) - \alpha_3(\mathrm{L})\,F_1''(\mathrm{L})} \end{cases} \quad (16)$$

According to the general solution of (8) equation, we consider special situation of the problem. If we accept that: length force N=0 or $v^2 = \frac{N}{E.J} = 0,$ and we substitute the changeable coefficient of subgrade reaction with integral mean value as follow:

$$k_{ave.} = \frac{1}{L}\int_o^l k(x)dx = \frac{1}{L}\int_0^l\left[k_o - \frac{4(k_o - k_s)}{L}x + \frac{4(k_o - k_s)}{L^2}x^2\right]dx = \frac{k_o + 2k_s}{3}$$

In this case

$k_o = k_s = k_l = k_{ave.} \Longrightarrow a_o = a_{ave.}$ və $a_1 = a_2 = 0$

So the basic functions of the (8) general solution of the problem are written as follow:

$$\begin{cases} F_1(x) = 1 + \sum_{n=1}^{\infty}(-1)^n \frac{a_{ave.}^n \cdot x^{4n}}{(4n)!}; \\ F_2(x) = x + \sum_{n=1}^{\infty}(-1)^n \frac{a_{ave.}^n \cdot x^{4n+1}}{(4n+1)!}; \\ F_3(x) = \frac{x^2}{2!} + \sum_{n=1}^{\infty}(-1)^n \frac{a_{ave.}^n \cdot x^{4n+2}}{(4n+2)!}; \\ F_4(x) = \frac{x^3}{3!} + \sum_{n=1}^{\infty}(-1)^n \frac{a_{ave.}^n \cdot x^{4n+3}}{(4n+3)!}; \\ F_5(x) = \frac{x^4}{4!} + \sum_{n=1}^{\infty}(-1)^n \frac{a_{ave.}^n \cdot x^{4n+4}}{(4n+4)!}. \end{cases}$$

Thus the offered method makes it possible to solve the contact problem of compressing-bending deformation of base-slab of the dry dock settling basin and internal forces at any arbitrary transverse section of the base-slab.

## GENERAL COMPARISONS OF THE RESULTS AND METHOD SUPERIORITY

To evaluate the suitability of the suggested method, the values of bending moments obtained from this method are compared with values predicted by other methods proposed by different researchers. This comparison has been undertaken for a base element with a length of 10 m, as shown in Figure 3. The coefficient of subgrade reaction (K) is considered as parabolic form along the base-slab (this causes the complexity of the solution of (4) equation in the article) but, in other mentioned methods by other authors in this article, the coefficient of subgrade reaction (K) is considered as a constant value, whereas we consider (k) non-linear along the beam (base-slab), so parameters are more accurate than the parameters of other authors' methods.

Picard limit of sequence method is used for the solution of (4) equation whereas the other authors have used simple methods for the solution of their equations (as mentioned in the literature such as Civil Engineering Variation Mechanics Method, Krilov (1931) Simple Computation Method, Series Method), i.e. the solution method of the problem in this article is completely different from those methods.

## CONCLUSION

The Fuss-Winkler foundation model has been applied to the static structural analysis and, also Analytical calculations are carried out in order to obtain the compressing-bending behavior of the base-slab of the dry dock settling basin resting on elastic foundation and to clarify the effect of Fuss-Winklers' foundation. The results obtained from this method and results obtained from other researches are plotted together to check the accuracy of the used Maclaurin's Series method.

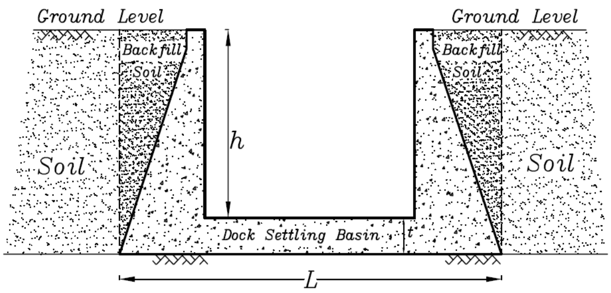


*Fig.1.* Forces applied on the dry dock settling basin.

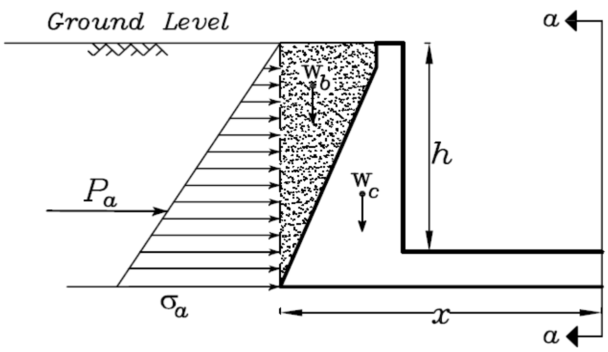


*Fig.2.* Diagram of forces effect on the dry dock settling basin.

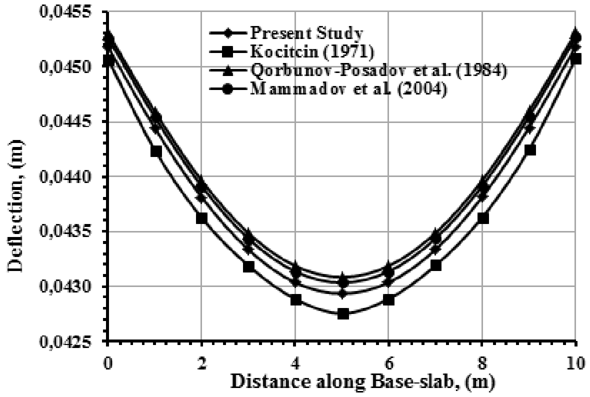


*Fig.4.* Deflection curves along base-slab under a uniformly distributed load.

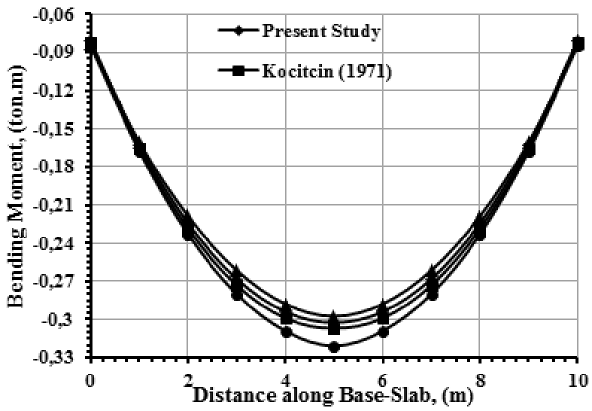


*Fig.5.* Bending moment curves along base-slab under a uniformly distributed load.

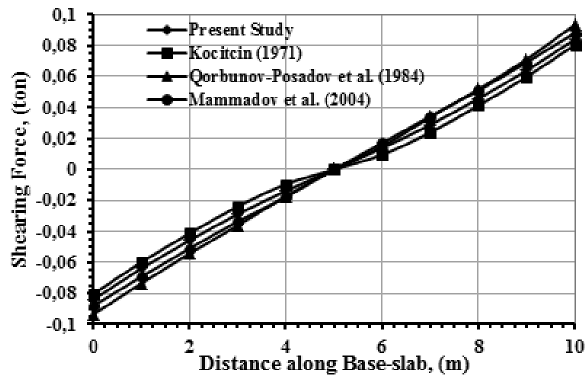


*Fig.6.* Shearing force curves along base-slab under a uniformly distributed load.

As shown in Figure 4, maximum deflection values gained by the Kocitcin (1971) method. The maximum deflection obtained from the present study for a simply supported base-slab of the dry-dock settling basin, under a uniformly distributed load, is 3.02% less than the associated value of Kocitcin (1971) method. The minimum values of bending moment as shown in Figure 5 compare well to the values determined by Qorbunov-

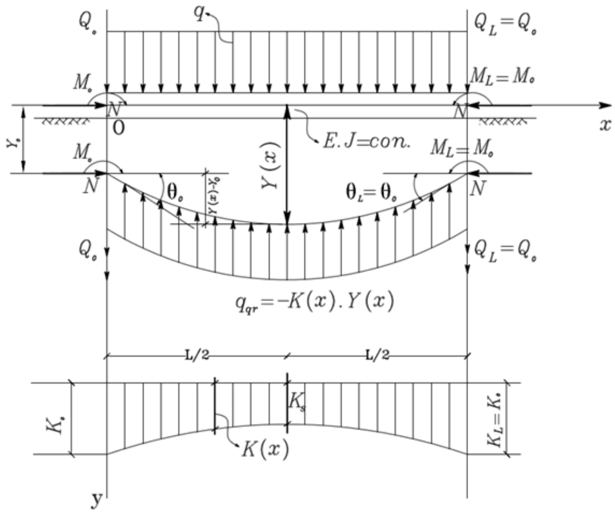


**Fig.3.** Deformation scheme of base-slab of the dry dock settling basin.

Posadov et al. (1984) method. The maximum bending moment value obtained from the present study is 3.192% more than the associated value predicted by Qorbunov-Posadov et al. (1984) method. The variations of shear force estimated by the different method, along simply supported base-slab are shown in Figure 6. It is seen that the minimum value is estimated by Kocitcin (1971) method. The maximum shear force for a simply supported of the dry-dock settling basin, under a uniformly distributed load, computed from present study is about 4.24% more than those of Kocitcin (1971) method. By carrying out these comparisons, it can be seen that the suggested method calculates interior forces and deformations of base-slab of the dry-dock settling basin that are in close agreement with published methods. This method helps us to compute the resistance of reinforcement construction and also determine the demanding reinforcements. Also the represented method can be used to compute the deformation of deep foundation.

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