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Comparison of Fuzzy Numbers by Using a Statistical Index

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Abstract

Many fuzzy numberscomparison approaches are developed in the literature for multi attribute decision-making problem. Almost all of theexisting approaches focus on quantity measurement of fuzzy numbers for ranking purpose. In this paper, the researchers have proposed a new method for ranking fuzzy numbers. Here we compare fuzzy numbers using a distance measure. This interval can be used as a crisp set approximation. Ranking fuzzy numbers using parametric interval has been done previously by other researchers. In the present paper, a novel approach of parametric interval is proposed. We obtain this parametric interval by using the statistical principles of fuzzy numbers. This method can effectively rank various fuzzy numbers and addresses the deficiencies of previous techniques. Some examples to illustrate the advantages of this method are outlined in this article.

Key words: Fuzzy numbers, Fuzzy arithmetic, Fuzzy variance, Ranking, Parametric interval.

INTRODUCTION

In many applications of fuzzy set theory, to solve a problem, we must select one of the possible solutions [1]. Weusually choose the best and the most appropriate solution. In orderto do this, we need to rank fuzzy numbers. Various methods have been proposed for ranking fuzzy numbers. One of these methods is the use of valuation techniques [2].

A general method for comparing fuzzy numbers is its relationship with a scalar quantity which is used to compare fuzzy numbers. Another method for ranking fuzzy numbersis to use the classification intervals. In this method, the fuzzy numbers are ranked in a rough interval [3]. The approximation of an interval representation of a fuzzy number may have many useful applications. By utilizing this method, it is possible to apply some results derived from interval number analysis.Various authors (Saneifard, 2009) have studied the crisp approximation of fuzzy sets and proposed a rough theoretic definition of the crisp approximation, called the nearest interval approximation of a fuzzy set [4].

The different approachesto the crisp approximation of fuzzy sets,is presented inpreviousstudies [5] However, the fuzzy numbers approximation methods are not unique.

Reviewing previous studies, we have proposed a new parametric interval for ranking fuzzy numbers in this paper. One of the methods comparing two fuzzy numbers is the use of a suitable index for these numbers. In other words, we compare the index of two fuzzy numbers instead of comparing themselves. In the present article, we introduce a fuzzy index to provide a new method for ranking fuzzy numbers. We need to review an improved algorithmfor this fuzzy index. This algorithm usesfuzzy arithmeticand fuzzy principles and provides parametric interval. Therefore applying this fuzzy index, a new method for ranking fuzzy numbers is proposed in this paper. Since this fuzzy index is a statistical index, the algorithm can be used for positive fuzzy numbers. The reason is obvious, because all variables and data are always a positive value in statistical problems.

This paper is organized as follows: in Section 2, this article recalls some fundamental results on fuzzy numbers. In Section 3 a statistical fuzzy index (fuzzy variance) is described. Proposed method for ranking fuzzy numbers is in Section 4. In Section 5, numerical examples of using the method above have been given.The paper ends with conclusions in Section 6.

Preliminaries and Basic Efinitions

Fuzzy numbers

Let X be a nonempty and universe set, A fuzzy set $\mathcal{A} = [a, b, c, d]$, $a, b, c, d \in \mathbb{R}$ and $a \le b \le c \le d$ in X is characterized by its membership function, $\mu_A: X \to [0,1]$ and $\mu_A(x)$is interpreted as the degree of membership of element x in fuzzy set $\mathcal{A}$ for each $x \in X$ [6]. It is clear that $\mathcal{A}$ is completely determined by the set of tuples: $\mathcal{A} = \{(x, \mu_A(x): x \in X\}$the membership function $\mu_A(x)$ is a continuous mapping from R to the closed interval [0,1]; $\mu_A(x) = 0$ for all $x \in (-\infty, a]$ and $x \in [d, +\infty)$, $\mu_A(x)$ is strictly increasing on $x \in [a,b]$; $\mu_A(x) = 1$ for $x \in [b, c]$; $\mu_A(x)$ is strictly decreasing on $x \in [c, d]$ [7]. $\tilde{B} = (e, f, g)$ is defined as a triangular fuzzy number if$\mu_B(x)$is given by:

$$\mu_B(x) = \begin{cases} (x-e)/(f-e), & e \le x \le f, \\ (x-g)/(f-g), & f \le x \le g, \\ 0, & otherwise. \end{cases} \quad (1)$$

Or

$$\mu_B = \begin{cases} \mu_B^L(x), & e \le x \le f, \\ \mu_B^R(x), & f \le x \le g, \\ 0, & otherwise. \end{cases} \quad (2)$$

Where $\mu_B^L : [e, f] \longrightarrow [0,1]$ and $\mu_B^R : [f, g] \longrightarrow [0, 1]$ based on basic theories of fuzzy number. B is a normal fuzzy number if $\mu_B(x) = 1$. Where B is a non-normal fuzzy number if $0 \le \mu_B \le 1$ therefore, the extended fuzzy number B can be denoted as (e, f, g, μ_B). The image –B of B can be expressed by $(-g, -f, -e, \mu_B)$ [8].

α-cut:(the interval of confidence for the level of α)

The α-cut of a fuzzy number $\tilde{A}$ can be defined as [9]:

$$A_\alpha = \{x \mid \mu_A(x) \ge \alpha\} \quad x \in R,\ \alpha \in [0,1] \tag{3}$$

Where A_α is a non-empty bounded closed interval contained in R, it can be further denoted by $A_\alpha = [\underline{A}_\alpha, \overline{A}_\alpha]$ where $\underline{A}_\alpha$ and $\overline{A}_\alpha$ are respectively the lower and upper bounds of the closed interval and they are functions of α defined for any fuzzy intervals. If B= (e, f,g) be a triangular fuzzy number, the α- cut of B can be expressed as:

$$B_\alpha = [\underline{B}_\alpha, \overline{B}_\alpha] = [(f-e)\alpha + e, (f-g)\alpha + g], \alpha \in [0,1] \tag{4}$$

Fuzzy arithmetic

A non-fuzzy number r can be expressed as (r,r, r) [10] by the extension principle, the fuzzy sum ⊕ and fuzzy subtraction ⊖ of any two triangular fuzzy numbers are also triangular fuzzy number, given any two positive triangular fuzzy numbers, $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ and a positive real number r, some main operations of fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be expressed as follows:

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3), \tag{5}$$

$$\tilde{A} \ominus \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3), \tag{6}$$

$$\tilde{A} \otimes \tilde{B} = (a_1 b_2,\ a_2 b_2, a_3 b_3), \tag{7}$$

$$\tilde{A} \otimes r \cong (a_1 r, a_2 r, a_3 r), \tag{8}$$

And Equations 9-12 are the standard fuzzy-arithmetic operation rules [11].

$$(\tilde{A} \oplus \tilde{B})_\alpha = [\tilde{A}_\alpha \oplus \tilde{B}_\alpha] = [\underline{A}_\alpha + \underline{B}_\alpha, \overline{A}_\alpha + \overline{B}_\alpha], \tag{9}$$

$$(\tilde{A} \ominus \tilde{B})_\alpha = [\tilde{A}_\alpha \ominus \tilde{B}_\alpha] = [\underline{A}_\alpha - \underline{B}_\alpha, \overline{A}_\alpha - \overline{B}_\alpha], \tag{10}$$

$$(\tilde{A} \otimes \tilde{B})_\alpha = [\tilde{A}_\alpha \otimes \tilde{B}_\alpha] = [\underline{A_\alpha}.\underline{B_\alpha}, \overline{A_\alpha}.\overline{B_\alpha}], \tag{11}$$

$$(\tilde{A} \otimes \tilde{B})_{E_\alpha} = \begin{cases} [\overline{A}_\alpha^{\,2}, \underline{A}_\alpha^{\,2}], & \overline{A} < 0, \\ [\underline{A}_\alpha^{\,2}, \overline{A}_\alpha^{\,2}], & \underline{A} > 0, \\ [0, \max(\underline{A}_\alpha^{\,2}, \overline{A}_\alpha^{\,2})], & 0 \in [\underline{A}, \overline{A}], \end{cases} \tag{12}$$

Provide A Method of Fuzzyvariance

Let $\tilde{X}$ be a fuzzy variable that has triangular fuzzy variants $\tilde{x}_j = (a_j, b_j, c_j)$ where, $j = 1, 2, 3, \dots, N$. Applying Eq. (4),

$$(\tilde{X}_j)_\alpha = [\underline{x_j}_\alpha, \overline{x_j}_\alpha] = [(b_j - a_j)\alpha + a_j, (b_j - c_j)\alpha + c_j], \alpha[0,1]. \tag{13}$$

Define the α-cut of the fuzzy mean of $\tilde{\mathbf{X}}$

The α − cut offuzzymeanis definedas follows [12] .

$$(\tilde{\mu}_{\tilde{x}})_\alpha = \frac{1}{N}\sum_{j=1}^{N} \tilde{x}_{j_\alpha} = [\underline{\mu_{\tilde{x}}}_\alpha, \overline{\mu}_{\tilde{x}\alpha}] = [A_L\alpha + B_L, A_U\alpha + B_U], \alpha \in [0,1]. \tag{14}$$

Where,

$$\underline{\mu_{\tilde{x}}}_\alpha = \frac{1}{N}\sum_{j=1}^{N} \underline{x_j}_\alpha,\ \overline{\mu}_{\tilde{x}\alpha} = \frac{1}{N}\sum_{j=1}^{N} \overline{x_j}_\alpha,$$

and

$$A_L = \frac{1}{N}\sum_{j=1}^{N}(b_j - a_j) = \mu_b - \mu_a, \quad B_L = \frac{1}{N}\sum_{j=1}^{N} a_j = \mu_a,$$

and

$$A_U = \frac{1}{N}\sum_{j=1}^{N}(b_j - c_j) = \mu_b - \mu_c, \quad B_U = \frac{1}{N}\sum_{j=1}^{N} c_j = \mu_c.$$

This allows one to obtain the membership functions of $\tilde{\mu}_{\tilde{X}}$ as

$$f_{\tilde{\mu}_{\tilde{X}}} = \begin{cases} f^L_{\tilde{\mu}_{\tilde{X}}}(x) = (x - B_L)/A_L, & B_L \le x \le C, \\ f^R_{\tilde{\mu}_{\tilde{X}}}(x) = (x - B_U)/A_U, & C \le x \le B_U, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

Where $f^L_{\tilde{\mu}_{\tilde{X}}}(x)$ and $f^R_{\tilde{\mu}_{\tilde{X}}}(x)$ are the left and right membership functions, respectively.

And $C = \frac{1}{N}\sum_{j=1}^{N} b_j = \mu_b$.

Define theα-cut of the fuzzy variance of $\tilde{\mathbf{X}}$:

Let $\tilde{M}_{\tilde{x}_j} = \tilde{x}_j - \mu_{\tilde{X}}$ and apply Eq. (12),

$$(\tilde{M}_{\tilde{x}_j})^2_\alpha = \begin{cases} [(\overline{M}_{\tilde{x}_j})^2_\alpha, (\underline{M}_{\tilde{x}_j})^2_\alpha], & (\overline{M}_{\tilde{x}_j})_\alpha < 0, \\ [(\underline{M}_{\tilde{x}_j})^2_\alpha, (\overline{M}_{\tilde{x}_j})^2_\alpha], & (\underline{M}_{\tilde{x}_j})_\alpha > 0, \\ [0, \max((\underline{M}_{\tilde{x}_j})^2_\alpha, (\overline{M}_{\tilde{x}_j})^2_\alpha)], & 0 \in [\underline{M}_{\tilde{x}_{j_\alpha}}, \overline{M}_{\tilde{x}_{j_\alpha}}]. \end{cases} \tag{16}$$

Where

$$(\underline{M}_{\tilde{x}_j})_\alpha = \underline{x_j}_\alpha - \overline{\mu_{\tilde{x}}}_\alpha = (b_j - a_j - A_U)\alpha + (a_j - B_U),$$

and

$$(\overline{M}_{\tilde{x}_j})_\alpha = \overline{x_j}_\alpha - \underline{\mu_{\tilde{x}}}_\alpha = (b_j - c_j - A_L)\alpha + (c_j - B_L).$$

Now define the α-cut of the fuzzy variance of $\tilde{X}$ by [13].

$$(\tilde{\sigma}_{\tilde{x}}^{\,2})_\alpha = \frac{1}{N}\sum_{j=1}^{N}(\tilde{x}_j - \tilde{\mu}_{\tilde{x}})^2_\alpha = [(\underline{\sigma_{\tilde{x}}})^2_\alpha, (\overline{\sigma_{\tilde{x}}})^2_\alpha]. \tag{17}$$

That this equation is abbreviated to be [14].

$$V_\alpha(A) = (\tilde{\sigma}_{\tilde{x}}^{\,2})_\alpha [G_L\alpha^2 + H_L\alpha + I_L, G_U\alpha^2 + H_U\alpha + I_U], \alpha \in [0,1]. \tag{18}$$

Where

$$G_L = \frac{1}{N}\sum_{j=1}^{N}\begin{cases}(b_j - c_j - A_L)^2, & \left(\overline{M}_{\tilde{x}_j}\right)_\alpha < 0,\\ (b_j - a_j - A_U)^2, & (\underline{M}_{\tilde{x}_j})_\alpha > 0,\\ 0, & 0 \in [(\underline{M}_{\tilde{x}_j})_\alpha, \left(\overline{M}_{\tilde{x}_j}\right)_\alpha]\end{cases}$$

$$= \frac{1}{N}\sum_{j=1}^{N}\begin{cases}2(b_j - c_j - A_L)(c_j - B_L), & (\overline{M}_{\tilde{x}_j})_\alpha < 0,\\ 2(b_j - a_j - A_U)(a_j - B_U), & (\underline{M}_{\tilde{x}_j})_\alpha > 0,\\ 0, & 0 \in [\left(\underline{M}_{\tilde{x}_j}\right)_\alpha, \left(\overline{M}_{\tilde{x}_j}\right)_\alpha].\end{cases}$$

$$I_L = \frac{1}{N}\sum_{j=1}^{N}\begin{cases}(c_j - B_L)^2, & (\overline{M}_{\tilde{x}_j})_\alpha < 0,\\ (a_j - B_U)^2, & (\underline{M}_{\tilde{x}_j})_\alpha > 0,\\ 0, & 0 \in [\left(\underline{M}_{\tilde{x}_j}\right)_\alpha, \left(\overline{M}_{\tilde{x}_j}\right)_\alpha].\end{cases}$$

G_U

$$= \frac{1}{N}\sum_{j=1}^{N}\begin{cases}(b_j - a_j - A_U)^2 & \left(\overline{M}_{\tilde{x}_j}\right)_\alpha < 0,\\ (b_j - c_j - A_L)^2 & (\underline{M}_{\tilde{x}_j})_\alpha > 0,\\ (b_j - c_j - A_L)^2 & 0 \in \left[(\underline{M}_{\tilde{x}_j})_\alpha, \left(\overline{M}_{\tilde{x}_j}\right)_\alpha\right], \left|(\underline{M}_{\tilde{x}_j})_\alpha\right| \le \left|\left(\overline{M}_{\tilde{x}_j}\right)_\alpha\right|,\\ (b_j - a_j - A_U)^2 & 0 \in \left[(\underline{M}_{\tilde{x}_j})_\alpha, \left(\overline{M}_{\tilde{x}_j}\right)_\alpha\right], \left|(\underline{M}_{\tilde{x}_j})_\alpha\right| \ge \left|\left(\overline{M}_{\tilde{x}_j}\right)_\alpha\right|.\end{cases}$$

$$= \frac{1}{N}\sum_{j=1}^{N}\begin{cases}2(b_j - a_j - A_U)(a_j - B_U), & (\overline{M}_{\tilde{x}_j})_\alpha < 0,\\ 2(b_j - c_j - A_L)(c_j - B_L), & (\underline{M}_{\tilde{x}_j})_\alpha > 0,\\ 2(b_j - c_j - A_L)(c_j - B_L), & 0 \in [(\underline{M}_{\tilde{x}_j})_\alpha, (\overline{M}_{\tilde{x}_j})_\alpha], \left|(\underline{M}_{\tilde{x}_j})_\alpha\right| \le \left|(\overline{M}_{\tilde{x}_j})_\alpha\right|,\\ 2(b_j - a_j - A_U)(a_j - B_U), & 0 \in [(\underline{M}_{\tilde{x}_j})_\alpha, (\overline{M}_{\tilde{x}_j})_\alpha], \left|(\underline{M}_{\tilde{x}_j})_\alpha\right| \ge \left|(\overline{M}_{\tilde{x}_j})_\alpha\right|.\end{cases}$$

I_U

$$= \frac{1}{N}\sum_{j=1}^{N}\begin{cases}(a_j - B_u)^2, & (\overline{M}_{\tilde{x}_j})_\alpha < 0,\\ (c_j - B_L)^2, & (\underline{M}_{\tilde{x}_j})_\alpha > 0,\\ (c_j - B_L)^2, & 0 \in [(\underline{M}_{\tilde{x}_j})_\alpha, (\overline{M}_{\tilde{x}_j})_\alpha], \quad \left|(\underline{M}_{\tilde{x}_j})_\alpha\right| \le \left|(\overline{M}_{\tilde{x}_j})_\alpha\right|,\\ (a_j - B_u)^2, & 0 \in [(\underline{M}_{\tilde{x}_j})_\alpha, \left(\overline{M}_{\tilde{x}_j}\right)_\alpha], \quad \left|(\underline{M}_{\tilde{x}_j})_\alpha\right| \ge \left|(\overline{M}_{\tilde{x}_j})_\alpha\right|.\end{cases}$$

The membership function of ${\tilde{\sigma}^2}_{\tilde{x}}$ are the presented as [15]

$$f_{{\tilde{\sigma}^2}_{\tilde{x}}}(x) = \begin{cases}\{-H_L + [{H_L}^2 - 4G_L(I_L - x)]^{1/2}\}/2G_L, & I_L \le x \le J,\\ \{-H_U + [{H_U}^2 - 4G_U(I_U - x)]^{1/2}\}/2G_U, & J \le x \le I_u,\\ 0, & \text{otherwise.}\end{cases}$$

Where, $J = \frac{1}{N}\sum_{j=1}^{N}(b_j - c)^2$.

The Preference Comparison of Fuzzy Numbers

In this section we propose a novel technique for comparison of fuzzy number associated with the parametric distance. If $\widetilde{A}$ is a fuzzy variable that has triangular fuzzy variants $\tilde{x}_j = (a_j, b_j, c_j)$, where $j = 1,2,3,\dots,N$ and $[\underline{A}_\alpha, \overline{A}_\alpha]$ be itsα-cutthen $V_\alpha(A)$ be it's the parametric interval. Since every parametric interval can be used as a crisp set approximation of a fuzzy number therefore, the resulting interval is used to rank the fuzzy numbers.Thus, $V_0(A)$ is used to rank fuzzy numbers.

Definition 1. Let $\mathcal{F}$[L] represent the set of closed interval in R; the interval order is as follows:

$\forall a_1, b_1, a_2, b_2 \in R$.

Verifying $a_1 \le b_1$ and $a_2 \le b_2$, $[a_1, b_1] \le [a_2, b_2]$, if and only if $a_1 \le a_2$ and $b_1 \le b_2$.

Let $\mathcal{A}$ and $\mathcal{B} \in \mathcal{F}[L]$, this paper also defines $\mathcal{UTM}(\mathcal{A},\mathcal{B})$ and $\mathcal{LEA}(\mathcal{A},\mathcal{B})$ the utmost and the least of two intervals as follows:

$\mathcal{UTM}(\mathcal{A},\mathcal{B}) = \mathcal{A} \vee \mathcal{B}$,
$\mathcal{LEA}(\mathcal{A},\mathcal{B}) = \mathcal{A} \wedge \mathcal{B}$,

Where the utmost and the least of the intervals are defined in the following way:

$\forall a_1, b_1, a_2, b_2 \in R$ Verifying$a_1 \le b_1$ and $a_2 \le b_2$,

$[a_1, b_1] \vee [a_2, b_2] = [a_1 \vee a_2, b_1 \vee b_2]$,

$[a_1, b_1] \wedge [a_2, b_2] = [a_1 \wedge a_2, b_1 \wedge b_2]$.

Let A and $B \in \mathcal{F}[L]$denotetwo arbitrary fuzzy numbers and $V_\alpha(A) = [a_1, a_2]$ and $V_\alpha(B) = [b_1, b_2]$ express theα-cut fuzzy variance of A and B; respectively. Define the comparison of A and B by $V_0(.)$ on F, are defined as [16].

$\mathcal{LEA}(\mathcal{A},\mathcal{B}) = \mathcal{UTM}(\mathcal{A},\mathcal{B})$ if and only if A~B, $\mathcal{LEA}(\mathcal{A},\mathcal{B}) < UTM(\mathcal{A},\mathcal{B})$ if and only if $A \prec B$, $V_\alpha(B) \subset V_\alpha(A)$ if and only if $A \succ B$.

Then, this attempt formulates the order $\succcurlyeq$ and $\preccurlyeq$ as $A \succcurlyeq B$if and only if $A \succ B$ or $A \sim B$, and $A \preccurlyeq B$ if and only if$A \prec B$ or$A \sim B$.

Remark 1. For two arbitrary fuzzy numbers A and Bthis states that, $V_\alpha(A + B) = V_\alpha(A) + V_\alpha(B)$.

Proof. For $\alpha \in [0, 1]$, let

$V_\alpha(A) = \left({\tilde{\sigma}_{\tilde{x}}}^2\right)_\alpha = [G_L\alpha^2 + H_L\alpha + I_L, G_U\alpha^2 + H_U\alpha + I_U], \alpha \in [0,1]$

and

$V_\alpha(B) = \left({\tilde{\sigma}_{\tilde{y}}}^2\right)_\alpha = [\acute{G}_L\alpha^2 + \acute{H}_L\alpha + \acute{I}_L, \acute{G}_U\alpha^2 + \acute{H}_U\alpha + \acute{I}_U], \alpha \in [0,1]$.

Therefore,

$$V_\alpha(A) + V_\alpha(B) = [G_L + H_L\alpha + I_L, G_U\alpha^2 + H_U\alpha + I_U] + [\acute{G}_L\alpha^2 + \acute{H}_L + \acute{I}_L, \acute{G}_U\alpha^2 + \acute{H}_U\alpha + \acute{I}_U]$$

$= [(G_L + \acute{G}_L)\alpha^2 + (H_U + \acute{H}_U)\alpha + (I_U + \acute{I}_U)]$

$= V_\alpha(A + B)$.

Numerical Examples

In this section this study, several numerical examples are given to check the proposed method.

Example 1. Let A, B, C be three fuzzy variables that have triangular fuzzy variants

$\tilde{x}_1 = (0.1,0.2,0.3)$, $\tilde{x}_2 = (0.3,0.45,0.6)$, $\tilde{x}_3 = (0.5,0.6,0.7)$,

and

$\tilde{y}_1 = (0.5,0.25,0.35)$, $\tilde{y}_2 = (0.2,0.3,0.4)$, $\tilde{y}_3 = (0.35,0.45,0.55)$

and

$\tilde{z}_1 = (0.18,0.25,0.5)$, $\tilde{z}_2 = (0.32,0.5,0.7)$, $\tilde{z}_3(0.4,0.5,0.8)$, therefore, applying Eq. (5), A=(0.9,1.25,1.6), B=(0.7,1,1.3) and C=(0.9,1.25,2) as shown in Table 1. According Eq. 18, the ranking index values are obtained, $V_0(A) = [0,0.15], V_0(\mathrm{B}) = [0,0.08]$ and $V_0(c) = [0,0.2]$. Ranking order of fuzzy numbers is $\mathrm{C} \succ \mathrm{A} \succ \mathrm{B}$. (See Table1)

Table 1. Fuzzy numbers A, B, C in Example 1.

Fuzzy Numbers	$\boldsymbol{V_0}(.)$
A=(0.9,1.25,1.6)	[0,0.15]
B=(0.7,1,1.3)	[0,0.08]
C=(0.9,1.25,2)	[0,0.2]
Results	C≻A ≻B

Example 2. Consider the data

$\tilde{x}_1 = (3{,}3.5{,}4), \tilde{x}_2 = (2{,}3.5{,}5)$, and

$\tilde{y}_1 = (2{,}3{,}3.5), \ \tilde{y}_2 = (4{,}4{,}4.5)$,

therefore applying Eq. (5) observe the two symmetric triangular fuzzy numbers A= (5,7,9) and B=(6,7,8) as shown in Table 2. Through the proposed approach in this paper, the ranking index values can be obtained as $V_0(\mathrm{A}) = [0{,}4.25]$ and $V_0(\mathrm{B}) = [0{,}3.13]$, then the ranking order of fuzzy numbers result is $A \succ B$.(See Table2).

Table 2. Fuzzy numbers A, B in Example 2.

Fuzzy Numbers	$\boldsymbol{V_0}(.)$
A=(5,7,9)	[0,4.25]
B=(6,7,8)	[0,3.1]
Results	A ≻B

Example 3. Let the data

$\tilde{x}_1 = (1{,}1{,}1), \tilde{x}_2 = (2{,}2{,}2)$ and

$\tilde{y}_1 = (2, 2, 2), \tilde{y}_2 = (4{,}4{,}4)$.

By using Eq. (5) and this new approach, for A=(3,3,3) and B=(6,6,6) (Table3), $V_0(\mathrm{A}) = [0.25{,}0.25]$ and $V_0(\mathrm{B}) = [1{,}1]$, hence the ranking order is $\mathrm{B} \succ \mathrm{A}$.

Table 3. Fuzzy numbers A, B in Example 3.

Fuzzy Numbers	$\boldsymbol{V_0}(.)$
A=(3,3,3)	[0.25,0.25]
B=(6,6,6)	[1, 1]
Results	B ≻A

CONCLUSION

There are many methods for ranking fuzzy numbers. however statistical science indexes are used less in ranking fuzzy numbers. There are shortcoming and limitations in pervious ranking methods.

In this study, to eliminate these shortcomings, we have proposed another parametric interval index that can effectively rank various fuzzy numbers. The calculations of the proposed method are far simpler than other approaches. Finally, examples were presented to illustrate the advantages of this method.

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