

Performance Analysis of Single and Multiple Estimation in MIMO Rician Frequency-Flat Fading Channels

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Abstract

In this paper, the performance of the single-estimation (SE) and multiple-estimation (ME) is investigated in multiple-input multiple-output (MIMO) Rician frequency-flat fading channels using the maximum likelihood (ML) technique, the new shifted scaled least squares (SSLS) estimator, and the minimum mean square error (MMSE) estimator. The closed form equations are obtained for mean square error (MSE) of the estimators in SE and ME cases under optimal training. Analytical and numerical results show that the ML estimator has lower error in the case of ME than SE. Moreover, it is seen that the performance of SSLS and MMSE channel estimators in the ME case is better than SE particularly at high signal to noise ratios (SNRs) and for weak line of sight (LOS) propagation paths. At low SNRs and for strong LOS propagation paths, and also for small numbers of sub-blocks used for channel estimation, the SSLS estimator is better than ML as well as the MMSE estimator is better than both estimators but in other cases the ML estimator is better than SSLS (MMSE). The un-equal power allocation is also investigated analytically and numerically. Simulation results show that exponential power allocation is an appropriate method in ME case.

Keywords: Rician fading, multiple-estimation, un-equal power allocation, shifted scaled least squares, minimum mean square error

INTRODUCTION

Multiple-input multiple-output (MIMO) system provides substantial benefits in both increasing system capacity and improving its immunity to deep fading in the channel [1, 2]. To take advantage of these benefits, the accurate channel state information (CSI) is required at the receiver and/or transmitter. In the coherent receivers [1], channel equalizers [3], and transmit beamformers [4], the perfect knowledge of the channel is usually needed.

Due to low complexity and better performance, training-based channel estimation (TBCE) is widely used in practice for quasi-static or slow fading channels, e.g., indoor MIMO channels [5-10]. However, in outdoor MIMO channels where channels are under fast fading, the channel tracking and estimating algorithms as the wiener least mean squares (W-LMS) [11], Kalman filter [12, 13], recursive least squares (RLS) [14], generalized RLS (GRLS) [15], and generalized LMS (GLMS) [16] are used.

In [5], the performance of the least squares (LS), scaled LS (SLS), minimum mean square error (MMSE), and relaxed MMSE (RMMSE) estimators is studied in the Rayleigh fading MIMO channel using TBCE scheme. The MMSE channel estimator has the best performance among the estimators, because it employs more a-priori knowledge about the channel. In [6-8], it is assumed that the MIMO channel has Rician distribution. Rician fading is characterized by the factor K which is the power ratio of the line of sight (LOS) and the diffused components.

When $K=0$, it represents Rayleigh fading, and no fading when $K \rightarrow \infty$. Rician fading, thus, can be considered a general fading model for land mobile channels. An interesting result in [6] is that the optimal training sequence length can be considerably smaller than the number of transmitter antennas in systems with strong spatial correlation. For MIMO Rician flat fading channels, the new shifted scaled least squares (SSLS) channel estimator is presented in [8]. It is seen that this estimator has the best performance among the LS-based estimators in Rician channel model. Nevertheless, the MMSE channel estimator has lower error than that of SSLS in Rician fading channel model especially at high signal to noise ratios (SNRs) and spatial correlations [7].

In [9], the performances of the time-multiplexed (TM) and superimposed (SI) schemes have been compared in MIMO channel estimation. It is shown that in fast fading channels and/or for many receiver antennas, the SI scheme is better than TM but in other cases this scheme suffers from a higher estimation error. In part II of this paper [10], to improve the performance of the SI scheme a decision directed approach is applied.

In order to perform the individual channel estimation at the destination, in [17], the SI training strategy is applied into the MIMO amplify-and-forward (AF) one-way relay network (OWRN). The discussion is restricted to the case of a slow, frequency-flat block fading model. A specific suboptimal channel estimation algorithm is applied in [17] using the optimal training sequences and to verify the Bayesian Cramér-

Rao lower bound (CRLB) results the normalized mean square error (MSE) performance for the estimation is provided.

In this paper, TBCE method is studied in the frequency-flat Rician fading MIMO channels. First, the single-estimation (SE) is considered and the minimum MSE is obtained for maximum likelihood (ML), SSSL, and MMSE estimators under optimal training. Then, multiple-estimation (ME) is investigated in these estimators. In ME case, the multiple estimates of the channel during received N sub-blocks are combined optimally. The optimal weight coefficients are achieved for all estimators. Furthermore, the minimum MSE under optimal training is obtained for aforementioned estimators.

Simulation results show that all estimators have better performance in the ME case than SE case especially at high SNRs and low Rice factors. At low SNRs and high Rice factors as well as for small numbers of sub-blocks used for channel estimation, the MMSE is better than SSSL (and ML). However, at high SNRs and low Rice factors and for large numbers of sub-blocks, the ML estimator is better than SSSL (and MMSE). Therefore, the SSSL and MMSE estimators are appropriate for Rician fading channels with a short coherence time (fast fading). For Rician fading channels with a long coherence time (slow fading), however, the ML estimator is better than SSSL (and MMSE).

The un-equal power allocation is also considered in this paper. Using the SSSL and MMSE estimators, it is shown that in linear power allocation the results are analogous to the uniform power allocation. Nevertheless, in exponential power allocation the channel estimation errors are lower than the uniform power allocation.

Notation: $(\cdot)^H$ is reserved for Hermitian, $(\cdot)^*$ for the complex conjugate, $(\cdot)^{-1}$ for the matrix inverse, $(\cdot)^T$ for the matrix transpose, \otimes for the Kronecker product, $tr\{\cdot\}$ for the trace of a matrix. $E\{\cdot\}$ is the mathematical expectation, \mathbf{I}_m denotes the $m \times m$ identity matrix, $\|\cdot\|_F$ denotes the Frobenius norm. $\text{vec}(\cdot)$ stacks all the columns of its matrix argument into one tall column vector.

SYSTEM MODEL

It is considered a MIMO system with t transmitter and r receiver antennas. The frequency-flat block fading model is assumed for MIMO channel. It means that the channel response is fixed within one block and can change from one block to another one randomly. Each transmitted block has N sub-blocks which contain training and data symbols as shown in Fig. 1. The frame structure is the same for all Tx antennas. Training and data symbols are located in the first and end part of the sub-blocks, respectively. In practice, the channel is estimated using training symbols in the training phase. Then, the results are used for data detection. To estimate the MIMO channel in each sub-block, it is required that $n_p \geq t$ training signals are transmitted by each transmitter antenna. The $r \times n_p$ complex received signal matrix can be expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{V} \quad (1)$$

where \mathbf{X} and \mathbf{V} are the complex t -vector of transmitted sequences on the t transmit antennas and r -vector of additive receiver noise, respectively. The elements of noise matrix are independently and identically distributed (i.i.d.) complex Gaussian random variables as $\mathcal{CN}(0, 1)$.

In MIMO Rician fading channels with K as Rice factor, the $r \times t$ matrix of channel, \mathbf{H} , is defined in the following form [18, 19]:

$$\mathbf{H} = \sqrt{\frac{1}{K+1}} \mathbf{H}_{\text{Ray}} + \sqrt{\frac{K}{K+1}} \mathbf{H}_{\text{LOS}} \quad (2)$$

The matrix \mathbf{H}_{Ray} explains the Rayleigh component of the channel and the matrix \mathbf{H}_{LOS} describes the channel mean value or LOS component of the channel.

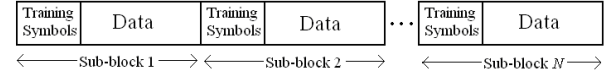


Figure 1. Frame structure for each Tx antenna in a MIMO channel

The MIMO channel model of (1) can be expressed in the following vector form:

$$\mathbf{y} = \tilde{\mathbf{X}}\mathbf{h} + \mathbf{v} \quad (3)$$

where $\mathbf{y} = \text{vec}(\mathbf{Y})$, $\mathbf{v} = \text{vec}(\mathbf{V})$, $\tilde{\mathbf{X}} = \mathbf{X}^T \otimes \mathbf{I}_r$ and

$$\mathbf{h} = \text{vec}(\mathbf{H}) = \sqrt{\frac{1}{1+K}} \mathbf{h}_{\text{Ray}} + \sqrt{\frac{K}{1+K}} \mathbf{h}_{\text{LOS}} \quad (4)$$

where $\mathbf{h}_{\text{Ray}} = \text{vec}(\mathbf{H}_{\text{Ray}})$, $\mathbf{h}_{\text{LOS}} = \text{vec}(\mathbf{H}_{\text{LOS}})$ and equation $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A}) \text{vec}(\mathbf{B})$ is applied. The elements of \mathbf{h}_{Ray} are identically distributed complex Gaussian random variables with the zero mean and the unit variance and \mathbf{h}_{LOS} is a deterministic vector. So, \mathbf{h} will be a complex normally distributed vector, denoted as $\mathbf{h} \sim \mathcal{CN}(\mathbf{m}, \mathbf{C}_h)$ where \mathbf{C}_h and \mathbf{m} are the co-variance matrix of the channel vector \mathbf{h} and the mathematical expectation vector of \mathbf{h} , respectively. Using (4), it is simple to show that

$$\mathbf{m} = E\{\mathbf{h}\} = \sqrt{\frac{K}{K+1}} \mathbf{h}_{\text{LOS}} \quad (5)$$

Also, the correlation matrix of the channel vector \mathbf{h} can be computed as follows:

$$\mathbf{R}_h = E\{\mathbf{h}\mathbf{h}^H\} = \frac{1}{1+K} E\{\mathbf{h}_{\text{Ray}} \mathbf{h}_{\text{Ray}}^H\} + \frac{K}{1+K} \mathbf{h}_{\text{LOS}} \mathbf{h}_{\text{LOS}}^H \quad (6)$$

$$= \frac{1}{1+K} \mathbf{R}_{h_{\text{Ray}}} + \frac{K}{1+K} \mathbf{h}_{\text{LOS}} \mathbf{h}_{\text{LOS}}^H$$

Then, the co-variance matrix of the channel vector \mathbf{h} will be as:

$$\begin{aligned} \mathbf{C}_h &= \mathbf{R}_h - E\{\mathbf{h}\} E\{\mathbf{h}\}^H = \mathbf{R}_h - \mathbf{m}\mathbf{m}^H \\ &= \frac{1}{1+K} \mathbf{R}_{h_{\text{Ray}}} \end{aligned} \quad (7)$$

SINGLE CHANNEL ESTIMATION

In this section, it is supposed that the number of sub-blocks used for channel estimation is $N = 1$. First, the ML channel estimator is probed. Then, the performance of the SSSL and MMSE channel estimators is investigated.

ML Channel Estimator

In classical estimation, the channel is assumed to be unknown deterministic. For linear model of (3), the ML estimator which maximizes the joint probability distribution function (pdf) of (8) is optimal [20].

$$P(\mathbf{y}; \mathbf{h}) = \frac{1}{\pi^{(rn_p)} \det(\mathbf{C}_v)} \times \exp[-(\mathbf{y} - \tilde{\mathbf{X}}\mathbf{h})^H \mathbf{C}_v^{-1} (\mathbf{y} - \tilde{\mathbf{X}}\mathbf{h})] \quad (8)$$

Clearly, for noise vector in (3), the co-variance matrix is

$$\mathbf{C}_v = \mathbf{R}_v = E\{\mathbf{v} \mathbf{v}^H\} = \mathbf{I}_{rn_p} \quad (9)$$

Therefore, the ML estimator which is equal with the LS estimator in model (3) can be defined in the following form:

$$\hat{\mathbf{h}}_{ML} = \arg \min_{\mathbf{h}} (\mathbf{y} - \tilde{\mathbf{X}}\mathbf{h})^H (\mathbf{y} - \tilde{\mathbf{X}}\mathbf{h}) \quad (10)$$

By differentiating $(\mathbf{y} - \tilde{\mathbf{X}}\mathbf{h})^H (\mathbf{y} - \tilde{\mathbf{X}}\mathbf{h})$ with respect to \mathbf{h} and setting the result equal to zero, the result is

$$\hat{\mathbf{h}}_{ML} = (\tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^H \mathbf{y} \quad (11)$$

Using equation $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A}) \text{vec}(\mathbf{B})$, (11) can be expressed in the following matrix form:

$$\hat{\mathbf{H}}_{ML} = \mathbf{Y} \mathbf{X}^H (\mathbf{X} \mathbf{X}^H)^{-1} \quad (12)$$

The error of the estimator is

$$J_{ML} = E\{\|\mathbf{h} - \hat{\mathbf{h}}_{ML}\|_F^2\} = r \text{tr}\{(\mathbf{X} \mathbf{X}^H)^{-1}\} \quad (13)$$

It is shown that the optimal training matrix should satisfy the following equation [5, 8]:

$$\mathbf{X} \mathbf{X}^H = \frac{p}{t} \mathbf{I}_t \quad (14)$$

Using (13) and (14), the error of ML estimator is minimized as follows:

$$J_{ML(\min)} = \frac{t^2 r}{p} \quad (15)$$

where p is a given constant value as the total power of training matrix \mathbf{X} . This estimator achieves the classical CRLB, hence, it is efficient. However, the ML estimator utilizes only received signals and transmitted symbols that are given at the receiver. It has no knowledge about the channel.

SSLS Channel Estimator

Consider (3), the SSLS channel estimator can be expressed in the following form

$$\hat{\mathbf{h}}_{SSLS} = \gamma \hat{\mathbf{h}}_{LS} + \mathbf{b} \quad (16)$$

where $\hat{\mathbf{h}}_{LS}$ is the LS estimation of the channel. The SSLS estimator is the shifted type of SLS [5] which has been proposed in [8]. The scaling factor, γ , and the shifting vector, \mathbf{b} , have to be obtained so that the MSE, $J_{SSLS} = E\{\|\mathbf{h} - \hat{\mathbf{h}}_{SSLS}\|_F^2\}$, is minimized.

Using (16), the MSE of the SSLS estimator can be computed as follows

$$\begin{aligned} J_{SSLS} &= E\left\{\left\|\mathbf{h} - \gamma \hat{\mathbf{h}}_{LS} - \mathbf{b}\right\|_F^2\right\} \\ &= \text{tr}\left\{E\{(\mathbf{h} - \gamma \hat{\mathbf{h}}_{LS} - \mathbf{b})(\mathbf{h} - \gamma \hat{\mathbf{h}}_{LS} - \mathbf{b})^H\}\right\} \\ &= (1-\gamma)^2 \text{tr}\{\mathbf{R}_h\} + (\gamma-1) \text{tr}\{\mathbf{m} \mathbf{b}^H \\ &\quad + \mathbf{b} \mathbf{m}^H\} + \text{tr}\{\mathbf{b} \mathbf{b}^H\} + \gamma^2 J_{LS} \end{aligned} \quad (17)$$

By differentiating (17) with respect to γ and \mathbf{b} and setting the results equal to zero, the results are

$$-2(1-\gamma) \text{tr}\{\mathbf{R}_h\} + \text{tr}\{\mathbf{m} \mathbf{b}^H + \mathbf{b} \mathbf{m}^H\} + 2\gamma J_{LS} = 0 \quad (18)$$

$$(\gamma-1) \mathbf{m}^* + \mathbf{b}^* = 0 \quad (19)$$

Using (19), the SSLS estimator of (16) can be rewritten as (20) and using (18) and (19), the scaling factor can be written as (21).

$$\hat{\mathbf{h}}_{SSLS} = \gamma \hat{\mathbf{h}}_{LS} + (1-\gamma) \mathbf{m} \quad (20)$$

$$\gamma = \frac{\text{tr}\{\mathbf{C}_h\}}{\text{tr}\{\mathbf{C}_h\} + J_{LS}} \quad (21)$$

According to [8], optimal training for LS (that is equal with ML in this paper) and SSLS estimators is identical. Using (19) and (21), the MSE (17) under optimal training minimizes as follows:

$$J_{SSLS(\min)} = \frac{r t^2 \text{tr}\{\mathbf{C}_h\}}{r t^2 + p \text{tr}\{\mathbf{C}_h\}} \quad (22)$$

MMSE Channel Estimator

For linear model of (3), the MMSE channel estimator of \mathbf{h} is given by [20]

$$\hat{\mathbf{h}}_{MMSE} = \mathbf{m} + \mathbf{A}(\mathbf{y} - \tilde{\mathbf{X}} \mathbf{m}) \quad (23)$$

where

$$\mathbf{A} = \mathbf{C}_h \tilde{\mathbf{X}}^H (\tilde{\mathbf{X}} \mathbf{C}_h \tilde{\mathbf{X}}^H + \mathbf{I}_{rt})^{-1} = (\mathbf{C}_h^{-1} + \tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^H \quad (24)$$

The performance of the MMSE channel estimator is measured by the error matrix $\boldsymbol{\varepsilon} = \mathbf{h} - \hat{\mathbf{h}}_{MMSE}$, whose pdf is Gaussian with zero mean and the following covariance matrix:

$$\mathbf{C}_\varepsilon = \mathbf{R}_\varepsilon = E\{\boldsymbol{\varepsilon} \boldsymbol{\varepsilon}^H\} = (\mathbf{C}_h^{-1} + \tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \quad (25)$$

The MMSE estimation error is given by

$$\begin{aligned} J_{MMSE} &= E\left\{\left\|\mathbf{h} - \hat{\mathbf{h}}_{MMSE}\right\|_F^2\right\} = E\{\text{tr}\{\boldsymbol{\varepsilon} \boldsymbol{\varepsilon}^H\}\} \\ &= \text{tr}\{E\{\boldsymbol{\varepsilon} \boldsymbol{\varepsilon}^H\}\} = \text{tr}\{\mathbf{R}_\varepsilon\} = \text{tr}\{(\mathbf{C}_h^{-1} + \tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1}\} \end{aligned} \quad (26)$$

To minimize (26) subject to the transmitted power constraint $\text{tr}\{\mathbf{X} \mathbf{X}^H\} = p$ or $\text{tr}\{\tilde{\mathbf{X}}^H \tilde{\mathbf{X}}\} = r p$, the Lagrange multiplier method is used. The problem can be written as follows:

$$\begin{aligned} L(\tilde{\mathbf{X}}^H \tilde{\mathbf{X}}, \eta) &= \text{tr}\{(\mathbf{C}_h^{-1} + \tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1}\} \\ &\quad + \eta [\text{tr}\{\tilde{\mathbf{X}}^H \tilde{\mathbf{X}}\} - r p] \end{aligned} \quad (27)$$

where η is the Lagrange multiplier. By differentiating (27) with respect to $\tilde{\mathbf{X}}$ and setting the result equal to zero, it is obtained that the optimal training matrix should satisfy the following equation:

$$\mathbf{C}_h^{-1} + \tilde{\mathbf{X}}^H \tilde{\mathbf{X}} = \frac{1}{\sqrt{\eta}} \mathbf{I}_{rt} \quad (28)$$

Using the constraint $\text{tr}\{\tilde{\mathbf{X}}^H \tilde{\mathbf{X}}\} = r p$, it can be readily shown that the optimal probing must satisfy the following equation:

$$\tilde{\mathbf{X}}^H \tilde{\mathbf{X}} = \frac{tr\{\mathbf{C}_h^{-1}\} + r p}{r t} \mathbf{I}_{r t} - \mathbf{C}_h^{-1} \quad (29)$$

Substituting (29) back into (26), the MSE will be minimized as

$$J_{MMSE(\min)} = \frac{r^2 t^2}{tr\{\mathbf{C}_h^{-1}\} + r p} \quad (30)$$

MULTIPLE CHANNEL ESTIMATION

In order to improve the performance of the estimators, the multiple estimates of the channel during received N sub-blocks are combined. In this section, it is assumed that the channel response is fixed within N sub-blocks. In other words, the coherent time of the channel is enough to use N sub-blocks for channel estimation. Suppose that N estimates $\hat{\mathbf{h}}_1, \dots, \hat{\mathbf{h}}_N$ of the MIMO channel are obtained based on the training matrices $\mathbf{X}_1, \dots, \mathbf{X}_N$, respectively. The results are combined in the following linear method:

$$\hat{\mathbf{h}}_{ME} = \sum_{n=1}^N a_n \hat{\mathbf{h}}_n \quad (31)$$

where the optimal weight coefficients a_1, \dots, a_N are obtained so that the MSE (32) is minimized subject to $\sum_{n=1}^N a_n = 1$.

$$J_{ME} = E \left\{ \left\| \mathbf{h} - \sum_{n=1}^N a_n \hat{\mathbf{h}}_n \right\|_F^2 \right\} \quad (32)$$

Then, the optimization problem is

$$\min_{a_1, \dots, a_N} E \left\{ \left\| \mathbf{h} - \sum_{n=1}^N a_n \hat{\mathbf{h}}_n \right\|_F^2 \right\} \quad ST \quad \sum_{n=1}^N a_n = 1 \quad (33)$$

In this section, the problem (33) will be solved considering the ML, the SLS, and the MMSE channel estimators.

Multiple ML Estimation

Using (3), the ML estimator (11) can be rewritten as

$$\hat{\mathbf{h}}_{ML} = \mathbf{h} + (\tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^H \mathbf{v} \quad (34)$$

Using (34) and the constraint $\sum_{n=1}^N a_n = 1$, the error of the multiple ML estimation will be written as

$$\begin{aligned} J_{Multiple ML} &= E \left\{ \left\| \mathbf{h} - \sum_{n=1}^N a_n \hat{\mathbf{h}}_n \right\|_F^2 \right\} \\ &= E \left\{ \left\| \mathbf{h} - \sum_{n=1}^N a_n (\mathbf{h} + (\tilde{\mathbf{X}}_n^H \tilde{\mathbf{X}}_n)^{-1} \tilde{\mathbf{X}}_n^H \mathbf{v}_n) \right\|_F^2 \right\} \\ &= E \left\{ \left\| \sum_{n=1}^N a_n (\tilde{\mathbf{X}}_n^H \tilde{\mathbf{X}}_n)^{-1} \tilde{\mathbf{X}}_n^H \mathbf{v}_n \right\|_F^2 \right\} \\ &= E \left\{ tr \left\{ \left(\sum_{n=1}^N a_n \mathbf{E}_n \tilde{\mathbf{X}}_n^H \mathbf{v}_n \right) \left(\sum_{m=1}^N a_m \mathbf{E}_m \tilde{\mathbf{X}}_m^H \mathbf{v}_m \right)^H \right\} \right\} \\ &= tr \left\{ \sum_{n=1}^N \sum_{m=1}^N a_n a_m^* \mathbf{E}_n \tilde{\mathbf{X}}_n^H \mathbf{E} \{ \mathbf{v}_n \mathbf{v}_m^H \} \tilde{\mathbf{X}}_m \mathbf{E}_m \right\} \\ &= tr \left\{ \sum_{n=1}^N |a_n|^2 \mathbf{E}_n \right\} \end{aligned} \quad (35)$$

where $\tilde{\mathbf{X}}_n = \mathbf{X}_n^T \otimes \mathbf{I}_r$, $\mathbf{E}_n = (\tilde{\mathbf{X}}_n^H \tilde{\mathbf{X}}_n)^{-1}$, and the latter one is obtained using the following equation:

$$E \{ \mathbf{v}_n \mathbf{v}_m^H \} = \begin{cases} \mathbf{I}_{r n p} & ; n = m \\ \mathbf{0} & ; n \neq m \end{cases} \quad (36)$$

Then, for multiple ML estimation, the problem (33) can be written as

$$\min_{a_1, \dots, a_N} tr \left\{ \sum_{n=1}^N |a_n|^2 \mathbf{E}_n \right\} \quad ST \quad \sum_{n=1}^N a_n = 1 \quad (37)$$

The ML estimator is unbiased. The constraint in (37) guarantees that the multiple ML estimation is also unbiased.

To solve (37), the Lagrange multiplier method is used. The problem can be written as

$$L(a_1, \dots, a_N, \eta) = tr \left\{ \sum_{n=1}^N |a_n|^2 \mathbf{E}_n \right\} + \eta \left\{ \sum_{n=1}^N a_n - 1 \right\} \quad (38)$$

To find a_1, \dots, a_N , the partial derivatives of (38) with respect to a_i ($i = 1, 2, \dots, N$) are computed. Then, the results are set equal to zero. Finally, the optimal weight coefficients in the multiple ML estimation are obtained from:

$$a_n = \frac{1}{tr\{\mathbf{E}_n\} \sum_{l=1}^N 1/tr\{\mathbf{E}_l\}} \quad ; \quad n = 1, \dots, N \quad (39)$$

It is straightforward to show that under optimal training for ML estimator

$$tr\{\mathbf{E}_n\} = tr\{(\tilde{\mathbf{X}}_n^H \tilde{\mathbf{X}}_n)^{-1}\} = r t^2 / p_n \quad (40)$$

where p_n is the total power of training matrix \mathbf{X}_n which is used during the training phase in the sub-block n . Suppose that $p_n = k_n p$ is the transmitted power during the n -th

($n = 1, \dots, N$) training period and $p_{tot} = \sum_{n=1}^N p_n = N \times p$ is the total transmitted power during the N training periods. Then $\sum_{n=1}^N k_n = N$ and using (40), the optimal weight coefficients (39) can be rewritten as

$$a_n = \frac{1}{(r t^2 / k_n p) \sum_{l=1}^N (p_l / r t^2)} = \frac{k_n p}{\sum_{l=1}^N p_l} = \frac{k_n}{N} \quad (41)$$

Using (40) and (41), under optimal training, the MSE (35) is minimized as follows

$$J_{Multiple ML(\min)} = \frac{r t^2}{p N^2} \sum_{n=1}^N k_n = \frac{r t^2}{N p} \quad (42)$$

Comparing (42) and (15), it is seen that in the multiple ML estimation, the error reduces by the number of sub-blocks N which is used for channel estimation. It is notable that the error (42) is independent of p_n , the transmitted power during the n -th training period. It means that for uniform training powers and non-uniform training powers during N training periods, the error is the same.

Multiple SSSL Estimation

The SSSL channel estimator (20) can be rewritten as

$$\hat{\mathbf{h}}_{SSLS} = \gamma \mathbf{h} + \gamma (\tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^H \mathbf{v} + (1-\gamma) \mathbf{m} \quad (43)$$

Using (43), the MSE of multiple SSSL estimator is expressed as

$$\begin{aligned} J_{Multiple\ SSSL} &= E \left\{ \left\| \mathbf{h} - \sum_{n=1}^N a_n \hat{\mathbf{h}}_n \right\|_F^2 \right\} \\ &= E \left\{ \left\| \mathbf{h} - \sum_{n=1}^N a_n (\gamma_n \mathbf{h} + \gamma_n \mathbf{E}_n \tilde{\mathbf{X}}_n^H \mathbf{v}_n + (1-\gamma_n) \mathbf{m}) \right\|_F^2 \right\} \\ &= E \left\{ \left\| (1 - \sum_{n=1}^N a_n \gamma_n) \mathbf{h} - \sum_{n=1}^N a_n \gamma_n \mathbf{E}_n \tilde{\mathbf{X}}_n^H \mathbf{v}_n - \sum_{n=1}^N a_n (1-\gamma_n) \mathbf{m} \right\|_F^2 \right\} \\ &= E \{ tr \{ ((1 - \sum_{n=1}^N a_n \gamma_n) \mathbf{h} - \sum_{n=1}^N a_n \gamma_n \mathbf{E}_n \tilde{\mathbf{X}}_n^H \mathbf{v}_n - \sum_{n=1}^N a_n (1-\gamma_n) \mathbf{m}) \\ &\quad \times ((1 - \sum_{n=1}^N a_n \gamma_n) \mathbf{h} - \sum_{n=1}^N a_n \gamma_n \mathbf{E}_n \tilde{\mathbf{X}}_n^H \mathbf{v}_n - \sum_{n=1}^N a_n (1-\gamma_n) \mathbf{m})^H \} \} \end{aligned} \quad (44)$$

Using (9), (36), $\sum_{n=1}^N a_n = 1$, and with some calculations the result is

$$\begin{aligned} J_{Multiple\ SSSL} &= (1 - \sum_{n=1}^N a_n \gamma_n) (1 - \sum_{n=1}^N a_n^* \gamma_n^*) tr \{ \mathbf{C}_h \} \\ &\quad + \sum_{n=1}^N |a_n|^2 |\gamma_n|^2 tr \{ \mathbf{E}_n \} \end{aligned} \quad (45)$$

It is noteworthy that the elements of \mathbf{h} and \mathbf{v} are independent of each other and $E\{\mathbf{v}\} = 0$. Moreover, \mathbf{m} and \mathbf{C}_h are independent of N while $\tilde{\mathbf{X}}$ and \mathbf{v} are dependent to N . The optimization problem is

$$\min_{a_1, \dots, a_N} J_{Multiple\ SSSL} \quad ST \quad \sum_{n=1}^N a_n = 1 \quad (46)$$

The SSSL estimator is biased. The constraint in (46) results in that the multiple SSSL estimation is also biased. Using the Lagrange multiplier method, the result is

$$\begin{aligned} L(a_1, \dots, a_N, \eta) &= (1 - \sum_{n=1}^N a_n \gamma_n) (1 - \sum_{n=1}^N a_n^* \gamma_n^*) tr \{ \mathbf{C}_h \} \\ &\quad + \sum_{n=1}^N |a_n|^2 |\gamma_n|^2 tr \{ \mathbf{E}_n \} + \eta \left\{ \sum_{n=1}^N a_n - 1 \right\} \end{aligned} \quad (47)$$

By differentiating (47) with respect to a_i ($i = 1, 2, \dots, N$) and setting the results equal to zero the result is

$$-\gamma_i (1 - \sum_{n=1}^N a_n^* \gamma_n^*) tr \{ \mathbf{C}_h \} + a_i^* |\gamma_i|^2 tr \{ \mathbf{E}_i \} + \eta = 0 \quad (48)$$

In general, equation (48) cannot be solved analytically. Nevertheless, in the uniform power allocation $p_1 = \dots = p_N = p_{tot} / N = p$ where $\gamma_1 = \dots = \gamma_N = \gamma$ and $\mathbf{E}_1 = \dots = \mathbf{E}_N = \mathbf{E}$, (48) can be rewritten as:

$$-\gamma (1 - \gamma^*) tr \{ \mathbf{C}_h \} + a_i^* |\gamma|^2 tr \{ \mathbf{E} \} + \eta = 0 \quad (49)$$

Using $\sum_{n=1}^N a_n = 1$, the result for Lagrange multiplier will be

$$\eta = \gamma (1 - \gamma^*) tr \{ \mathbf{C}_h \} - \frac{|\gamma|^2}{N} tr \{ \mathbf{E} \} \quad (50)$$

Substituting (50) back into (49), the result is

$$a_n = \frac{1}{N} ; \quad n = 1, \dots, N \quad (51)$$

Using (21), (40), and (51), it is shown that under optimal training the MSE (45) is minimized as

$$\begin{aligned} J_{Multiple\ SSSL\ (min)} &= \left(\frac{rt^2}{p tr \{ \mathbf{C}_h \} + rt^2} \right)^2 tr \{ \mathbf{C}_h \} \\ &\quad + \frac{rt^2 p}{N} \left(\frac{tr \{ \mathbf{C}_h \}}{p tr \{ \mathbf{C}_h \} + rt^2} \right)^2 \end{aligned} \quad (52)$$

When $N=1$, (52) reduces to the special case of (22) for single channel estimation with the SSSL estimator. According to (52), it is seen that the error decreases when the number of sub-blocks N increases.

In the non-uniform power allocation, $p_n = k_n p$,

$$p_{tot} = \sum_{n=1}^N p_n = N \times p, \quad \sum_{n=1}^N k_n = N, \quad \text{suppose that } a_n = k_n / N$$

for $n = 1, 2, \dots, N$. It is shown numerically and analytically that the performance of SSSL estimator is improved in the non-uniform power allocation. With some calculations, the MSE (45) is minimized in this case as

$$\begin{aligned} J_{Multiple\ SSSL\ (min)} &= tr \{ \mathbf{C}_h \} \left(1 - p \frac{tr \{ \mathbf{C}_h \}}{N} \sum_{n=1}^N \frac{k_n^2}{k_n p tr \{ \mathbf{C}_h \} + rt^2} \right)^2 \\ &\quad + p r t^2 \left(\frac{tr \{ \mathbf{C}_h \}}{N} \right)^2 \sum_{n=1}^N \frac{k_n^3}{(k_n p tr \{ \mathbf{C}_h \} + rt^2)^2} \end{aligned} \quad (53)$$

when $k_n = 1$, (53) reduces to (52).

Multiple MMSE Estimation

Using (3) and (23), the MMSE channel estimator can be rewritten as

$$\hat{\mathbf{h}}_{MMSE} = \mathbf{m} + \mathbf{A} \tilde{\mathbf{X}} (\mathbf{h} - \mathbf{m}) + \mathbf{A} \mathbf{v} \quad (54)$$

Using (32) and (54), the MSE of multiple MMSE channel estimator is expressed as

$$\begin{aligned} J_{Multiple\ MMSE} &= E \left\{ \left\| \mathbf{h} - \sum_{n=1}^N a_n \hat{\mathbf{h}}_n \right\|_F^2 \right\} \\ &= E \left\{ \left\| \mathbf{h} - \sum_{n=1}^N a_n (\mathbf{m} + \mathbf{A}_n \tilde{\mathbf{X}}_n (\mathbf{h} - \mathbf{m}) + \mathbf{A}_n \mathbf{v}_n) \right\|_F^2 \right\} \\ &= E \left\{ \left\| (\mathbf{I}_{rt} - \sum_{n=1}^N a_n \mathbf{A}_n \tilde{\mathbf{X}}_n) (\mathbf{h} - \mathbf{m}) - \sum_{n=1}^N a_n \mathbf{A}_n \mathbf{v}_n \right\|_F^2 \right\} \\ &= tr \{ (\mathbf{I}_{rt} - \sum_{n=1}^N a_n \mathbf{A}_n \tilde{\mathbf{X}}_n) \mathbf{C}_h (\mathbf{I}_{rt} - \sum_{m=1}^N a_m^* \tilde{\mathbf{X}}_m^H \mathbf{A}_m^H) \\ &\quad + \sum_{m=1}^N \sum_{n=1}^N a_n a_m^* \mathbf{A}_n \mathbf{E} \{ \mathbf{v}_n \mathbf{v}_m^H \} \mathbf{A}_m^H \} \end{aligned} \quad (55)$$

where

$$\begin{aligned} \mathbf{A}_n &= \mathbf{C}_h \tilde{\mathbf{X}}_n^H (\tilde{\mathbf{X}}_n \mathbf{C}_h \tilde{\mathbf{X}}_n^H + \mathbf{I}_{rt})^{-1} \\ &= (\mathbf{C}_h^{-1} + \tilde{\mathbf{X}}_n^H \tilde{\mathbf{X}}_n)^{-1} \tilde{\mathbf{X}}_n^H \end{aligned} \quad (56)$$

Using (36), (56), and with some calculations, the MSE (55) can be expressed as

$$\begin{aligned} J_{Multiple\ MMSE} &= tr\{\mathbf{C}_h\} - \sum_{n=1}^N a_n tr\{\mathbf{A}_n \tilde{\mathbf{X}}_n \mathbf{C}_h\} \\ &+ \sum_{n=1}^N (|a_n|^2 - a_n^*) tr\{\mathbf{C}_h \tilde{\mathbf{X}}_n^H \mathbf{A}_n^H\} \\ &+ \sum_{m=1}^N \sum_{n=1, n \neq m}^N a_n a_m^* tr\{\mathbf{A}_n \tilde{\mathbf{X}}_n \mathbf{C}_h \tilde{\mathbf{X}}_m^H \mathbf{A}_m^H\} \end{aligned} \quad (57)$$

The optimization problem is

$$\min_{a_1, \dots, a_N} J_{Multiple\ MMSE} \quad ST \quad \sum_{n=1}^N a_n = 1 \quad (58)$$

The MMSE estimator is biased. The constraint in (58) results in that the multiple MMSE estimation is also biased. The Lagrange multiplier method is used as

$$L(a_1, \dots, a_N, \eta) = J_{Multiple\ MMSE} + \eta \left\{ \sum_{n=1}^N a_n - 1 \right\} \quad (59)$$

The partial derivatives of (59) are obtained with respect to a_i ($i = 1, 2, \dots, N$), then, the result is set equal to zero as

$$\begin{aligned} \frac{\partial L}{\partial a_i} &= -tr\{\mathbf{A}_i \tilde{\mathbf{X}}_i \mathbf{C}_h\} + a_i^* tr\{\mathbf{C}_h \tilde{\mathbf{X}}_i^H \mathbf{A}_i^H\} \\ &+ \sum_{n=1, n \neq i}^N a_n^* tr\{\mathbf{A}_i \tilde{\mathbf{X}}_i \mathbf{C}_h \tilde{\mathbf{X}}_n^H \mathbf{A}_n^H\} + \eta = 0 \end{aligned} \quad (60)$$

The optimal training condition in MMSE channel estimator, $\tilde{\mathbf{X}}_i^H \tilde{\mathbf{X}}_i = ((tr\{\mathbf{C}_h^{-1}\} + r p_i) / rt) \mathbf{I}_{rt} - \mathbf{C}_h^{-1}$, is used and with some calculations it is seen that (60) reduces to

$$r^2 t^2 \left(\frac{tr\{\mathbf{C}_h^{-1}\}}{tr\{\mathbf{C}_h^{-1}\} + r p_i} - 1 \right) \sum_{n=1, n \neq i}^N \frac{a_n^*}{tr\{\mathbf{C}_h^{-1}\} + r p_n} + \eta = 0 \quad (61)$$

In the uniform power allocation, $p_1 = \dots = p_N = p_{tot} / N = p$,

using $\sum_{n=1}^N a_n = 1$, (61) reduces to

$$\frac{r^3 t^2 p}{(tr\{\mathbf{C}_h^{-1}\} + r p)^2} (a_i^* - 1) + \eta = 0 \quad (62)$$

Using (62) and $\sum_{n=1}^N a_n = 1$, the Lagrange multiplier can be obtained as

$$\eta = \frac{r^3 t^2 p (N - 1)}{N (tr\{\mathbf{C}_h^{-1}\} + r p)^2} \quad (63)$$

Substituting (63) back into (62), it is shown that in the uniform power allocation a_n is same as (51). Using (51) and under optimal training, the MSE (57) is minimized in the uniform power allocation as

$$J_{Multiple\ MMSE\ (min)} = \frac{r^2 t^2}{r p + tr\{\mathbf{C}_h^{-1}\}} \left(\frac{1}{N} + \frac{N-1}{N} \frac{tr\{\mathbf{C}_h^{-1}\}}{r p + tr\{\mathbf{C}_h^{-1}\}} \right) \quad (64)$$

When $N=1$, (64) reduces to the special case of (30) for single channel estimation with the MMSE estimator. According to (64), it is seen that the error decreases when the number of sub-blocks N increases.

In the non-uniform power allocation, $p_n = k_n p$, $p_{tot} = \sum_{n=1}^N p_n = N \times p$, $\sum_{n=1}^N k_n = N$, suppose that $a_n = k_n / N$ for $n = 1, 2, \dots, N$. With some calculations, it is shown that the MSE (57) is minimized as

$$\begin{aligned} J_{Multiple\ MMSE\ (min)} &= \frac{r^2 t^2}{N^2} (tr\{\mathbf{C}_h^{-1}\}) \left(\sum_{n=1}^N \frac{k_n}{(r p_n + tr\{\mathbf{C}_h^{-1}\})} \right)^2 \\ &+ \sum_{n=1}^N \frac{k_n^2}{(r p_n + tr\{\mathbf{C}_h^{-1}\})} \\ &- tr\{\mathbf{C}_h^{-1}\} \sum_{n=1}^N \frac{k_n^2}{(r p_n + tr\{\mathbf{C}_h^{-1}\})^2} \end{aligned} \quad (65)$$

When $k_n = 1$, (65) reduces to (64).

SIMULATION RESULTS

In this section, the performance of the ML, SSLS, and MMSE estimators is numerically examined in the case of SE and ME. As a performance measure, it is considered that the channel MSE is normalized by the average channel energy as

$$NMSE = \frac{E\{\|\mathbf{h} - \hat{\mathbf{h}}\|_F^2\}}{E\{\|\mathbf{h}\|_F^2\}} \quad (66)$$

Same as [21], the correlation matrix, $\mathbf{R}_{h_{Ray}}$, is assumed to be

$$\mathbf{R}_{h_{Ray}} = \mathbf{R}_t \otimes \mathbf{R}_r \quad (67)$$

Where \mathbf{R}_t and \mathbf{R}_r are the spatial correlation matrix at the transmitter and receiver sides, respectively. In simulation process, it is assumed that the elements of the spatial correlation matrices are [22]

$$[\mathbf{R}_t]_{i,j} = \alpha^{|i-j|}, \quad |\alpha| \leq 1, \quad [\mathbf{R}_r]_{i,j} = \beta^{|i-j|}, \quad |\beta| \leq 1 \quad (68)$$

Fig. 2 shows Normalized MSE (NMSE) of the ML channel estimator with optimal training versus SNR in the case of SE and ME. According to this figure, increasing the number of the sub-blocks N results in a lower error of the estimation. In other words, the performance of the ML estimator in ME case is better than SE case. Clearly, the performance of the ML estimator is independent of the channel Rice factor, K , and the correlation coefficients, α and β .

Figs. 3 and 4 indicate the NMSE of the SSLS channel estimator in the case of SE [8] and ME for $K=2, 10$ dB, respectively. As depicted in these figures, the SSLS estimator has better performance in ME case than SE especially at high SNRs and low Rice factors. However, at low SNRs, the NMSEs of the estimator for various numbers of sub-blocks N are analogous particularly for high values of K . It is notable

that the performance of the SSLS channel estimator is independent of the correlation coefficients, α and β . It is not shown here because of low space and its clarity (see [8]).

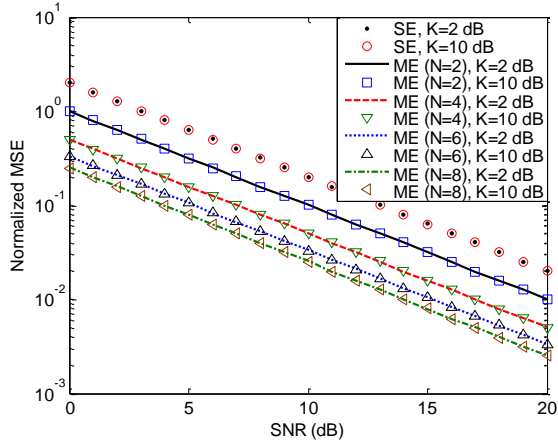


Figure 2. Normalized MSE of the ML estimator in the case of SE ($N = 1$) and ME ($N = 2, 4, 6, 8$) for $K = 2, 10$ dB ($r = t = 2, \alpha = \beta = 0.5$)

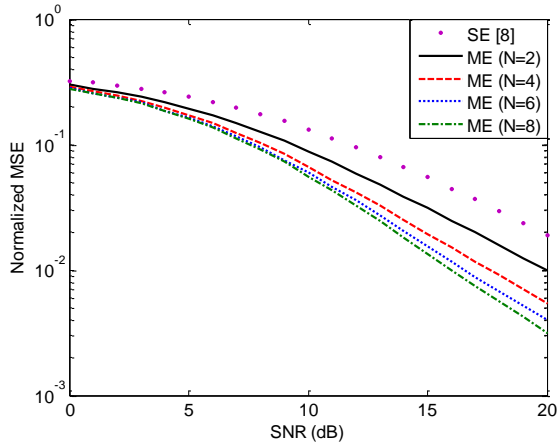


Figure 3. Normalized MSE of the SSLS estimator in the case of SE ($N = 1$) and ME ($N = 2, 4, 6, 8$) for $K = 2$ dB ($r = t = 2, \alpha = \beta = 0.5$)

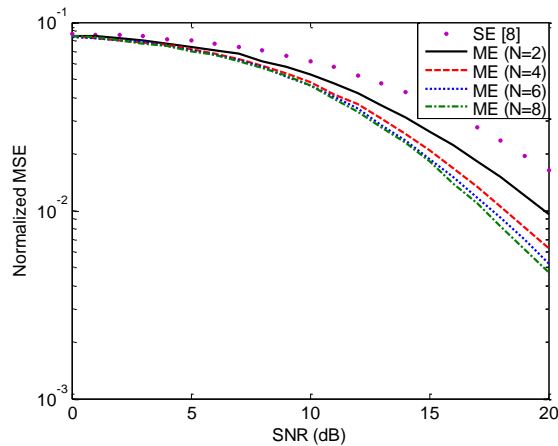


Figure 4. Normalized MSE of the SSLS estimator in the case of SE ($N = 1$) and ME ($N = 2, 4, 6, 8$) for $K = 10$ dB ($r = t = 2, \alpha = \beta = 0.5$)

In Figs. 5 and 6, the NMSE of the MMSE channel estimator is also shown in the case of SE [7] and ME for $K = 2, 10$ dB, respectively. The results are same as Figs. 3 and 4 for

SSLS estimator. Moreover, the performance of MMSE estimator is better than SSLS at low SNRs and high K and low number of N . On the other hand, it is seen that in the ME case for large numbers of N and at high SNRs, the error of SSLS estimator is lower than MMSE especially for low K . These results are also confirmed by Figs. 7 and 8.

In Figs. 7 and 8, the performance of the ML, SSLS and MMSE estimators is compared for various SNRs, Rice factors, and the number of sub-blocks N . It is seen that at low SNRs and high Rice factors as well as for small numbers of N , the SSLS estimator is better than ML. Moreover, the MMSE estimator is better than both estimators in this case because the MMSE estimator can employ more a priori knowledge about the channel than the SSLS estimator [7]. The NMSEs of the estimators coincide at high SNRs and low Rice factors. On the other hand, at high SNRs and low Rice factors and for large numbers of N , the ML estimator is better than SSLS (and MMSE). However, at low SNRs the performance of the SSLS and MMSE estimators is still better than ML particularly for high values of K . Therefore, in the Rician channels with a long coherence time and hence large N , the ML estimator is generally an appropriate method but in channels with a short coherence time and hence small N , the SSLS and MMSE are mainly better than ML.

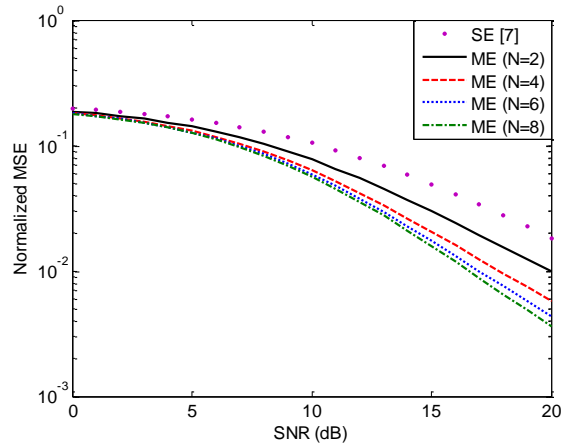


Figure 5. Normalized MSE of the MMSE estimator in the case of SE ($N = 1$) and ME ($N = 2, 4, 6, 8$) for $K = 2$ dB ($r = t = 2, \alpha = \beta = 0.5$)

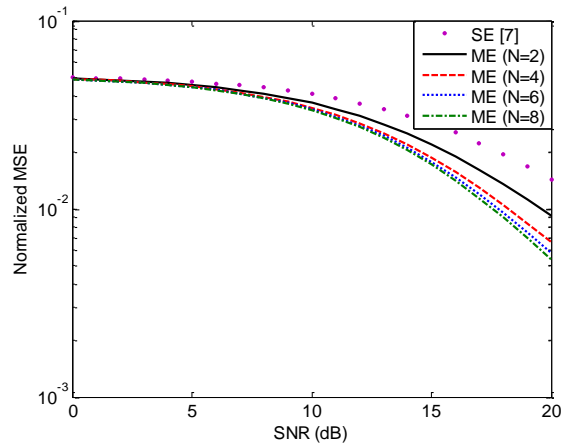


Figure 6. Normalized MSE of the MMSE estimator in the case of SE ($N = 1$) and ME ($N = 2, 4, 6, 8$) for $K = 10$ dB ($r = t = 2, \alpha = \beta = 0.5$)

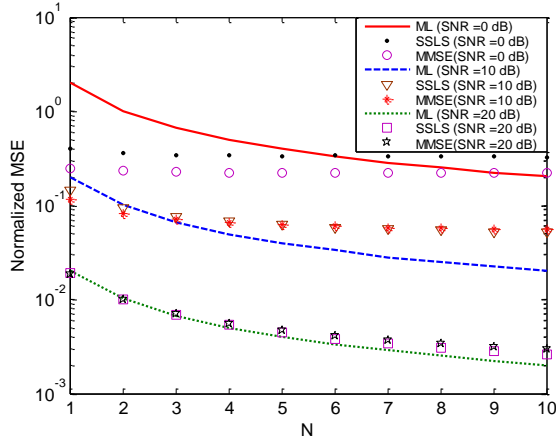


Figure 7. Normalized MSE of the ML, SSLS, and MMSE estimators vs. the number of sub-blocks for various SNRs ($r = t = 2$, $K = 0$ dB, $\alpha = \beta = 0.5$)

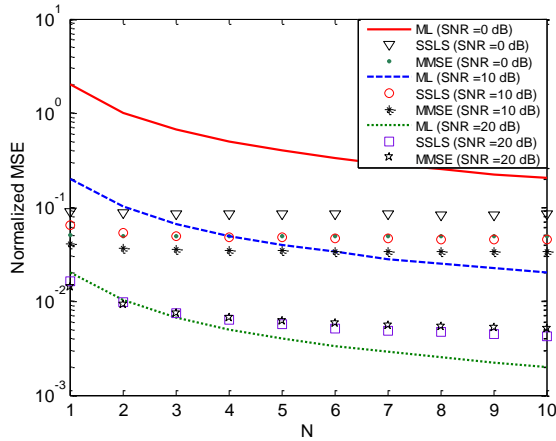


Figure 8. Normalized MSE of the ML, SSLS, and MMSE estimators vs. the number of sub-blocks for various SNRs ($r = t = 2$, $K = 10$ dB, $\alpha = \beta = 0.5$)

The results in Figs. 9 and 10 are obtained considering non-uniform power allocation during N sub-blocks that are used for channel estimation. In these figures, the SSLS and MMSE channel estimators are used respectively. The proposed non-uniform power allocations are linear and exponential schemes as follows:

$$p_n = k_n P = \frac{2n}{N+1} P, \quad n = 1, \dots, N \quad (69)$$

$$p_n = k_n P = \frac{N(1-e^{-1})}{e^{-1}-e^{-N-1}} e^{-n} P, \quad n = 1, \dots, N \quad (70)$$

It means that the optimal weight coefficients, a_n , have the linear and exponential distribution, respectively. In Figs. 9 and 10, the results are compared with uniform power allocation. It is seen that the error with linear power allocation is very close to the error with uniform power allocation. However, the exponential power allocation has lower error than the uniform power allocation with both estimators.

In practice, to obtain the best result in channel estimation, one of the ML, SSLS, or MMSE methods can be used considering the values of K , SNR, the number of antennas and N (or channel coherent time) in (42), (53), and (65). In order

to choose the best estimator among the ML, SSLS, and MMSE channel estimators, the NMSEs of (42), (53), and (65) can be computed and compared at the receiver.

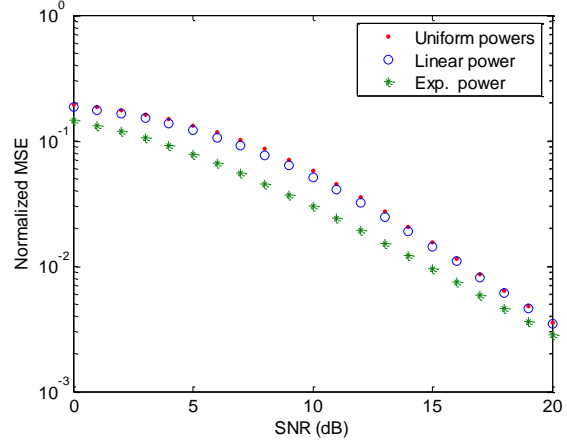


Figure 9. Normalized MSE of the SSLS estimator in uniform power and non-uniform power allocation ($r = t = 2$, $N = 8$, $K = 5$ dB, $\alpha = \beta = 0.5$)

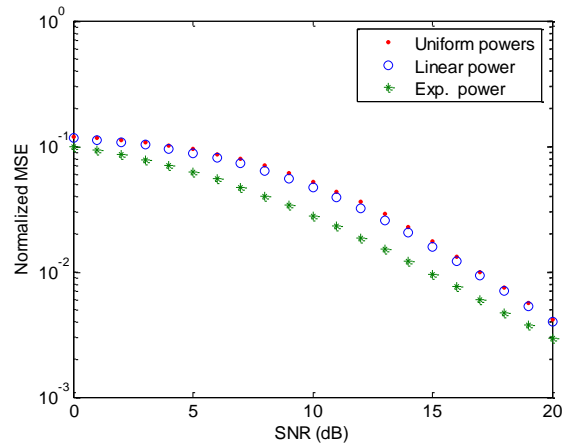


Figure 10. Normalized MSE of the MMSE estimator in uniform power and non-uniform power allocation ($r = t = 2$, $N = 8$, $K = 5$ dB, $\alpha = \beta = 0.5$)

CONCLUSION

The performance of ML, SSLS, and MMSE estimators in the case of SE and ME has been probed in the Rician frequency-flat fading MIMO channels. In the both SE and ME cases, the channel estimation errors have been obtained under optimal training. In the case of ME, the optimal weight coefficients and MSE are achieved for aforementioned estimators with uniform and non-uniform power allocations.

Analytical and numerical results show that the MSE of all estimators decreases when the number of sub-blocks N is increased. For small values of N , suitable for estimation of the channel with fast fading, the MMSE estimator is better than SSLS (ML) especially at low SNRs and high Rice factors. However, for large values of N , proper for estimation of the channel with slow fading, the ML estimator is better than SSLS (MMSE) particularly at high SNRs and low Rice factors. It is also shown that the performance of the SSLS and MMSE estimators in the un-equal power allocation is better than equal power allocation.

As a result, in slow/fast fading MIMO channels with a long/short coherence time, one of the above mentioned estimators can be used considering the MIMO channel parameters same as the channel Rice factor, the correlation matrix of the channel, the channel coherence time, and SNR.

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