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Fractional Order Controller Design for Fractional Order Chaotic Synchronization

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Abstract

This paper describes the fractional order proportional integrated derivation (FOPID) controller for synchronization between two fractional order chaotic systems. For performance test of the synchronization controller, controller was applied to chaotic communication system. Chaotic masking that one of the chaotic communication methods has two chaotic systems, and the systems must be synchronized. FOPID controller has some disadvantages that tuning of parameters and orders are difficult. For the disadvantage, FAPSO can used to tune the parameters of FOPID. The FAPSO method utilizes fuzzy set theory, in order to determine acceleration coefficient adaptability in particle swarm optimization. FAPSO is one of the fast optimization methods in literature. According to the results in the paper and the other papers in the literature, FOPID control method and the algorithm are able to produce correct results. The advantage of the proposed method is provided fast synchronization and recovery time.

Keywords: Chaotic Communication, Fractional order chaotic systems, Fractional order Proportional Integrated Derivation (FOPID), Adaptive Fuzzy Particle Swarm Optimization (FAPSO).

INTRODUCTION

Synchronization of fractional chaotic systems is a novel topic in nonlinear science due to applications in control, chaotic communication, signal processing, and so on [1-4]. In 2003, chaotic behavior in fractional order Lorenz systems was investigated by Grigorenko et al. [5]. Since chaotic systems are sensitive to initial conditions, the systems generate dissimilar signals in different initial conditions. Therefore, the purpose of synchronization is to provide to generate similar waveforms for two identical chaotic systems (these are called master and slave) [6]. Since the 1990s, applications of chaotic systems in communication have used different methods such as chaotic masking method, chaotic shift keying, etc. In the chaotic masking method, this is based on chaotic synchronization [7-8]. One approach for the chaotic masking is by adding the information signal to the noise like chaotic signal. The detection is accomplished by regenerating and subtracting the chaotic signal from the received signal [9]. Then, the information signal can be obtained from the difference between the received signal and one of the variables of the slave chaotic system that is synchronized with the master chaotic system. In order to provide the synchronization of chaotic systems, different control methods can be used such as classic PID, neural fuzzy, fuzzy logic [10-13]. The purpose of these control methods is to reduce the time taken to reach synchronization. The fractional order controller is one of these methods.

Although fractional order calculus is an old mathematical method dating back to the 1600s, the first applications of this method in engineering were realized in 1900s. Recently, fractional order systems and controllers that are based on fractional calculus have been the focus of attention [14-17]. Fractional order proportional integrated derivation (FOPID) is a control method also known as $PI^{\lambda}D^{\mu}$. Here, μ and λ can be any real numbers [18]. A significant issue in the FOPID method is to determine the FOPID parameters. In the literature, there are studies on tuning the parameters and some of these solutions are based on intelligent systems [16]. Particle swarm optimization (PSO) can also be used to tune and optimize the parameters.

PSO is a search technique based on populations to get optimal solutions [19]. Some of the features of PSO are that it improves the capability of solving complex problems and there is high convergence speed for different problems [20]. Population size is an important factor for the search period and optimal solution. Recently, different methods have been developed by researchers. One of the methods is the fuzzy adaptive particle swarm optimization (FAPSO) method [21]. In this method, acceleration constants and inertia weights are adaptively changed by algorithms in any iteration.

In this paper, we present a control method that has several advantages for synchronization. The FOPID control method is used for synchronization and provides high speed synchronization. However, FOPID has a problem with tuning the parameters. FAPSO is used to solve the problem and this optimization method has proved to be more efficient and accurate than the classic PSO method.

Fractional Order Chaotic Systems And Fopid

Fractional calculus**:** Fractional order systems are based on fractional calculus. Although fractional order calculus is old mathematical method, researchers are still studying solutions using this method. Fractional order systems have many solution techniques. These techniques are the Grunwald–Letnikov (GL) method (Eq. 1) and the Riemann–Liouville (RL) method (Eq. 2). The GL equation is described as follows:

$$ {}_aD_t^\alpha f(t) = \lim_{h\to\infty} \frac{1}{\Gamma(\alpha)h^\alpha} \sum_{k=0}^{[(t-a)/h]} \frac{\Gamma(\alpha+k)}{\Gamma(k+1)} f(x-kh) \tag{1} $$

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{d}{dt}\right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} d\tau \tag{2} $$

Another technique is the Laplace transformation method. Laplace transformation can also be used for numerical solutions of fractional systems [23-25]. In zero initial conditions, the Laplace transforms of fractional systems are defined as follows:

$$ L\left\{\frac{d^\mu f(t)}{dt^\mu}\right\} = s^\mu F(s) \qquad \mu \in R \tag{3} $$

There are many approaches to solving s^μ Laplace operators. Some of these approaches are the Crone, Carlson and Matsuda methods. The most widely used approximation of s^μ is Crone.

Crone is a French acronym (non-integer order robust control — Commande Robuste d'Ordre Non-Entier) [26]. In fractional order systems, Laplace and Crone approximations are useful methods. Therefore, several research studies use these approximations.

Fractional Order Lorenz Chaotic System: Lorenz systems, which are adopted for chaotic communications, are used in many research articles [27-28]. Firstly, the Lorenz equation is used for weather forecasts [29]; it is also used in different applications.

In the fractional order Lorenz system [5], the conventional derivative is replaced by the fractional derivative, as

$$ \begin{aligned} \frac{d^{0.9}x}{dt^{0.9}} &= 10(-x+y) \\ \frac{d^{0.9}y}{dt^{0.9}} &= 28x - y - xz \\ \frac{d^{0.9}z}{dt^{0.9}} &= -\frac{8}{3}z + xy \end{aligned} \tag{4} $$

where the order of the system is equal to α=0.9.

Figure 1 demonstrates the chaotic attractor for the fractional order Lorenz.

Synchronization of Fractional Order Chaotic Lorenz System **:** If two identical chaotic systems have different initial conditions, the systems generate different waveforms. The purpose of synchronization is to generate similar waveforms using two identical chaotic systems in different initial conditions. Many kinds of synchronization have been found, such as generalized synchronization, phase synchronization, lag synchronization and partially synchronization [30-34].

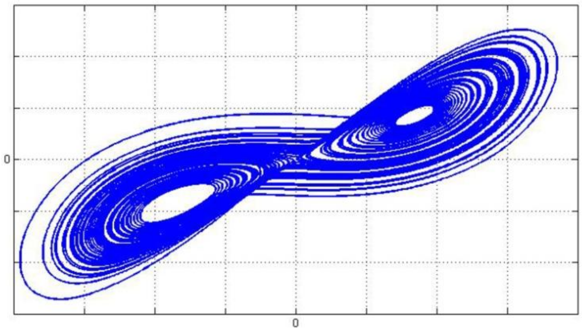

Fig.1. Fractional Lorenz chaotic attractor.

In generalized synchronization, a control input is added to the slave chaotic system [31]. The mathematical equations of the slave systems of fractional order Lorenz are given below:

$$ \begin{aligned} \frac{d^{0.9}x_s}{dt^{0.9}} &= 10.(-x_s + y_s) \\ \frac{d^{0.9}y_s}{dt^{0.9}} &= 28.x_s - y_s - x_s.z_s + u \\ \frac{d^{0.9}z_s}{dt^{0.9}} &= -\frac{8}{3}z_s + x_s.y_s \end{aligned} \tag{5} $$

where u is the control variable.

FOPID : A PID controller based on the feedback control system is widely used in industrial control systems. The PID controller has the following transfer function:

$$ G(s) = K_p + K_d s + \frac{K_i}{s} \tag{6} $$

In the transfer function of PID, there are three constant parameters: the proportional constant (K_p), derivate time constant (K_d) and integral time constant (K_i). In the PID controller, the derivation and integration orders are equal to 1. We can go for a generalization for the PID controller, which can be called the $PI^{vi}D^{vd}$ controller, because v_i and v_d belong to the set of real numbers [18].

The FOPID controller has the following transfer function:

$$ \begin{aligned} G(s) &= K_p + K_d s^{v_d} + K_i s^{-v_i} \\ v_i, v_d &\in R \\ v_i, v_d &\geq 0 \end{aligned} \tag{7} $$

where if v_i and v_d are both equal to 1, we obtain a classic PID controller. The FOPID controller has five parameters: proportional (K_p), derivative time (K_d), integral time (K_i), constants and orders of the derivative (v_d) and integral (v_i). In FOPID, the determination of these parameters is an important problem. The PSO method is one of the useful methods for solving the problem.

FAPSO

Review of PSO: Particle swarm optimization was first introduced by Kennedy and Eberhart in the mid-1990s. This method is similar to the process used by a flock of birds when searching for food [19]. The PSO algorithm comprises a swarm of particles moving in the D-dimensional search space and includes all possible candidate solutions. In the search space, P_{best} denotes the personal best position the ith particle has found so far, and g_{best} is the global best position discovered by the swarm. Velocities and positions are updated as

$$V_i^{d+1} = V_i^d + c_1.rand_1^d.(Pbest_i^d - X_i^d) + c_2.rand_2^d.(gbest^d - X_i^d) \tag{8}$$

$$X_i^{d+1} = X_i^d + V_i^d \tag{9}$$

where *$rand_1$ and $rand_2$* are two random number in [0 1]; c1 and c2 are called acceleration constants and are two positive constants. Researchers have modified Eq. (8) as

$$V_i^{d+1} = \omega.V_i^d + c_1.rand_1^d.(Pbest_i^d - X_i^d) + c_2.rand_2^d.(gbest^d - X_i^d) \tag{10}$$

where ω is the inertia weight and is defined as

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{i_{\max}}.k \tag{11}$$

where i_{max} is the maximum iteration, ω_{max} and ω_{min} are the limits of the inertia weight and k is the current iteration number [35]. Empirically, these values are taken as

ω_{max}=0.9, ω_{min}=0.4, c_1 and c_2=2.

FAPSO **:** In standard PSO, some values (ω_{max}, ω_{min}, c_1 and c_2) are taken as constants that have been inspired by other studies. For the best results, these values can be different for each iteration and each study. Therefore, it is thought [36-38] that, if the inertia weight and learning constants are adaptive variables, the results will be better than those for classic PSO.

In PSO, c_1 pulls the particle to its own historical best position and maintains the diversity of the swarm; c_2 pushes the swarm to the current globally best region with fast convergence. There are four states: the exploration state, exploitation state, convergence state and jumping-out state. In the exploration state, c_1 is increased and c_2 is decreased. In the exploitation state, c_1 is slightly increased and c_2 is slightly decreased. In the convergence state, c_1 and c_2 are slightly increased. In the jumping-out state, c_1 is decreased and c_2 increased [39].

In order to calculate the acceleration constants (c1 and c2), the fuzzy method is thought to be very successful. dg denotes the difference between the g_{best} for two consecutive iterations at iteration t:

$$dg^t = \frac{gbest^{t-1} - g_{\min}}{g_{\max} - g_{\min}} \tag{12}$$

where g_{max} and g_{min} are the maximum and minimum values of the best fitness function. For this paper, these values are selected as g_{max}=150, g_{min}=10. $c_1 \leq 1$, $c_2 \leq 2$ and ω is;

$$\omega(dg) = \frac{1}{1 + \sigma.e^{\varepsilon.dg}} \tag{13}$$

In [33], the σ and ε constants are respectively taken as 1.5 and -2.6. The membership functions of the four states are depicted in Figure 2.

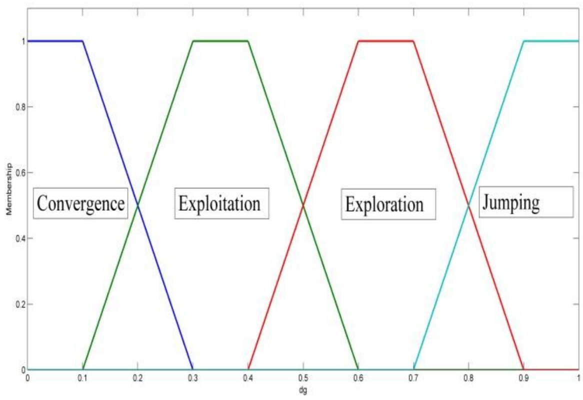

Fig.2. Membership function.

The fuzzy logic system, used to calculate the values of c1 and c2, has one input and two outputs. At the initial condition, the value of c1 is selected as large and the value of c2 is selected as small. The fuzzy system has the following rules:

Rule 1: if dg is convergence, then c1 and c2 are slightly decreased.
Rule 2: if dg is exploitation, then c_1 is slightly increased and c_2 is slightly decreased
Rule 3: if dg is exploration, then c_1 is increased and c_2 is decreased
Rule 4: if dg is jumping-out, then c_1 is decreased and c_2 is increased.

The acceleration constant and weight inertia are constants in standard PSO. However, in the FAPSO method, these values are changed adaptively by the fuzzy system as follows:

Step 1: randomly generate initial positions and velocities for all generations. The value of c_1 is selected as large; the value of c_2 is selected as small.
Step 2: adjust adaptively the acceleration constant and weight inertia using equations (12) and (13) and the rules.
Step 3: store the positions of each particle and select the best position (P_{best}) and global best position (g_{best}) of the particle according to the fitness function
Step 4: the position and velocity of each particle is updated using equations (8) and (9)
Step 5: the fitness function is calculated for the new positions. If the new positions achieve the better result, the P_{best} is updated by the position
Step 6: g_{best} is updated according to P_{best}
Step 7: if the number of iterations reaches the maximum or the result reaches the desired value, the latest g_{best} is the optimal solution. Otherwise, go to step 2 and repeat the other steps until the desired value is reached.

Chaotic Communication And Control of Synchronization

There are many kinds of chaotic communication, such as chaotic masking, chaotic shift keying, etc. In the chaotic masking method, the information signal is added to the chaotic signal at the transmitter. In the receiver, the signal received from the public channel is removed from the chaotic signal that is generated by the slave chaotic system (synchronized with the master system in the transmitter). A block diagram of chaotic masking is shown in Figure 3

[22]. In chaotic masking, to remove masking, master and slave chaotic systems must be synchronized. Therefore, synchronization control is required.

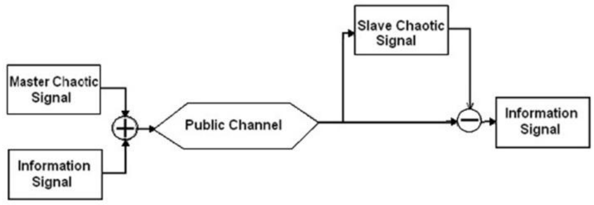

Fig.3. Block diagram of chaotic masking.

Chaotic communication systems have two chaotic systems, the master and slave chaotic systems, and both of them must be synchronized for chaotic communication. A block diagram of the FOPID that is used for synchronization control is shown in Figure 4. The synchronization error between the state variable of the master and slave chaotic systems is applied to the FOPID controller to produce the control signal.

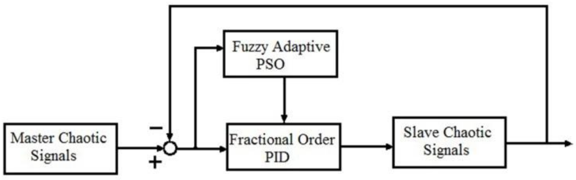

Fig.4. Block diagram of FOPID for chaotic communication.

Transfer function of the FOPID controller is;

$$G(s) = K_p + K_d \frac{a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_3 s^3 + b_2 s^2 + b_1 s + b_0} + K_i \frac{c_3 s^3 + c_2 s^2 + c_1 s + c_0}{d_3 s^3 + d_2 s^2 + d_1 s + d_0} \qquad (14)$$

where (a_n, a_{n-1},…, a_1, a_0), (b_n, b_{n-},…, b_1, b_0) are dependent on μ, (c_n, c_{n-1},…, c_1, c_0) and (d_n, d_{n-1},…, d_1, d_0) are dependent on λ.

Determining the parameters is great importance for the FOPID controller. In this paper, we are used FAPSO method to solve this problem.

SIMULATION RESULTS

Optimization of FOPID using FAPSO: Intelligent optimization methods like the genetic algorithm and PSO have been used to determine the parameters of the controller since the discovery of intelligent optimization methods. FAPSO is used for the rapid optimization of the FOPID parameters (K_p, K_d, K_i, v_d and v_i).

The flowchart for the FAPSO algorithm is given in Figure 5. At the first step, the algorithm randomly determines the initial value of the parameters (K_p, K_d, K_i, v_d and v_i). The initial values of the acceleration constants (c1 and c2) and weight inertia (w) are also determined randomly. Then the value of the fitness function is calculated using the function of ISE (the integral of the squared error), which is determined as

$$ISE = \int_0^T e^2(t)dt \qquad (15)$$

where T is period of the integral.

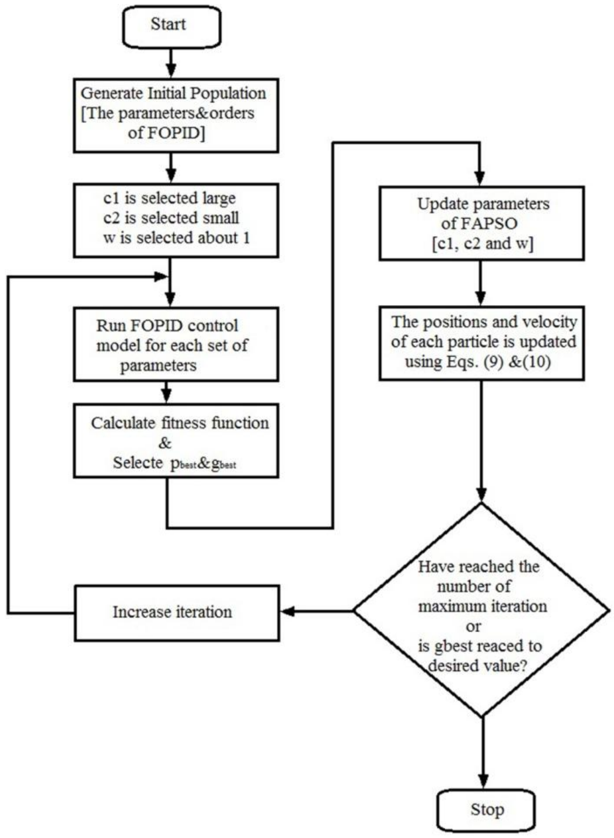

Fig.5. Flowchart of the FAPSO algorithm.

By using FAPSO, we get good results for the coefficient optimization of FOPID and this result can be seen as a convergence curve (Figure 6).

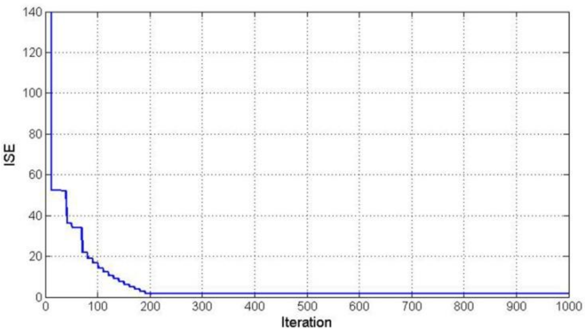

Fig.6. Convergence curve of FAPSO.

Synchronization and application of chaotic communication using FOPID**.** With the classic PID controller, the synchronization error reaches the point of zero after a long period. After the parameters of FOPID are optimized, the synchronization error has reached the zero point after quite a short period. This result, namely that the system reaches synchronization after quite a short period, is seen Figure 7.

The synchronization errors of FOPID and the classic PID that are optimized by FAPSO are shown in Figure 7 and Figure 8 respectively. It is evident from Figure 7 and Figure 8 that FOPID performs better than classic PID does. The optimized parameters of FOPID are

K_p=24.89, K_i=6.82, K_d=2.21 v_d=0.51, v_i=0.11

According to parameters of reference [10] and optimized parameters of classic PID and FOPID, the synchronization error is shown in Figure 9.

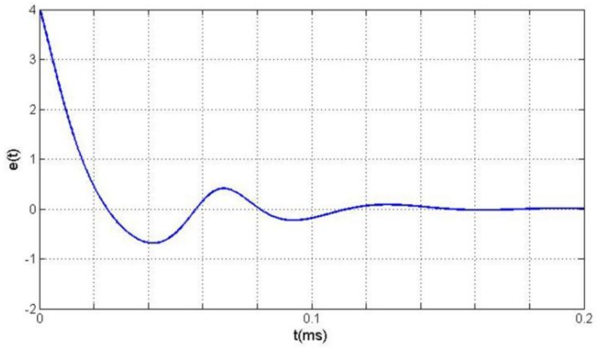

Fig.7. Synchronization error of the FOPID control method for optimization by FAPSO.

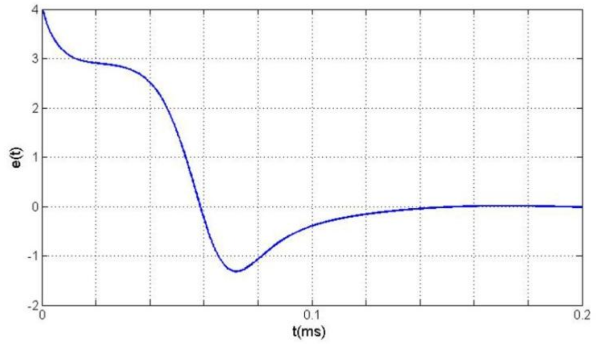

Fig.8. Synchronization error of the classic PID control method.

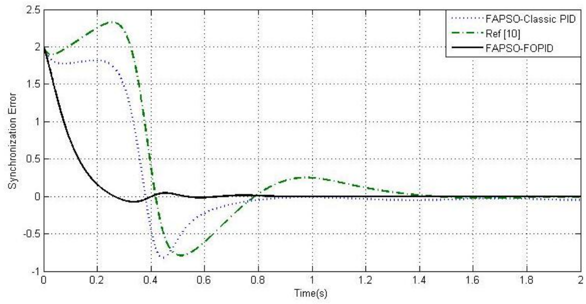

Fig.9. Synchronization error of Ref [10], optimized of the parameters Classic PID and FOPID.

Effects of the FOPID control method on the chaotic communication system The effects of the classic PID and FOPID methods on the chaotic communication system are examined; the synchronization occurs after a short period and ensures minimum data loss. The transmitted and received data are analyzed; the data loss of the chaotic communication system that uses the FOPID control method and Lorenz chaotic system is very low and this is shown in Figure 10 and Figure 11.

The results are given in Table 1 for different PID synchronization algorithms. It is clearly seen that FOPID control method has better performance than the control methods algorithms given in the Reference [10, 13, 22] for the synchronization control of nonlinear systems.

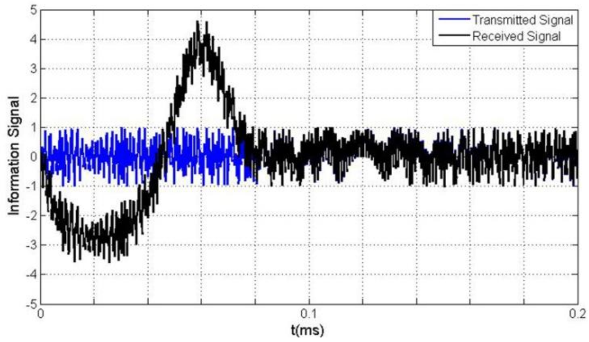

Fig.10. Transmitted and receiver signals using the classic PID controller and fractional Lorenz chaotic system.

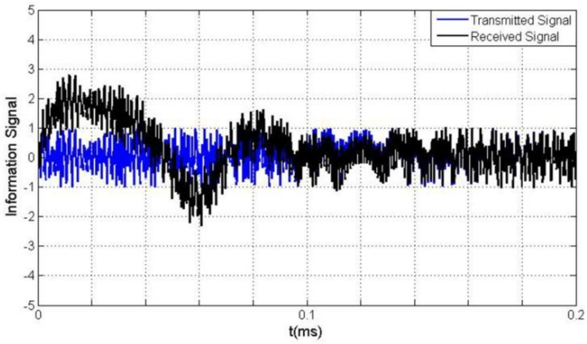

Fig.11. Transmitted and receiver signals, using the FOPID controller and fractional Lorenz chaotic system.

Table 1. Synchronization time and received signal time in this paper and references [10], [13] and [22].

	Chaotic system	Control Method	Synchronization Time (sec)	Recovered Signal (sec)
The paper	Fractional order Lorenz	Fractional Order PID	0.38	0.4
Ref [10]**	Sprott	EP-Based PID	~ 2	~ 2
Ref [13]**	Gyros	Self-learning PID	~0.8	Not given
Ref [22]	Integer Order	Extended Kalman Filter*	<0.4 >0.13	0.43
Ref [22]	Fractional Order Lorenz	Extended Kalman Filter*	<0.72 >0.4	1.611

* state estimation
**The data have been approximately obtained from figures in the Ref [10] and [13]

DISCUSSION AND CONCLUSIONS

In this paper, an alternative control method for the synchronization of fractional Lorenz systems has been proposed. The FOPID control method has been applied to the chaotic communication system and the parameters of FOPID are determined using the FAPSO intelligent method.

According to the obtained results, the FOPID control method has provided faster synchronization than the other

PID method in the [10, 13]. It is also seen that the FOPID control method performs better than the methods in the literature. Effective data transfer has been obtained using the FOPID control method for the control of synchronization due to less data loss at the start compared with in other intelligence methods and other control methods [10, 22]. The parameters of FOPID are determined by FAPSO after quite a short time. Thanks to the optimized parameters of FOPID, the efficiency of the chaotic communication system is increased.

In future work we intend to realize the chaotic secure communication system using fractional order multi-scroll chaotic systems and examine the noise performance of this method.

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