

## A novel method for evaluating decision making units based on fuzzy sets type two

R. SANEIFARD\*      N. FEIZIPOUR

Department of Mathematics, Urmia Branch, Islamic Azad University, Oroumieh, Iran.

\*Corresponding author:

E-mail: srsaneefard@yahoo.com

Received: September 29, 2012

Accepted: November 10, 2012

---

### Abstract

A review of previous studies reveals that recently definition of quality has been an important issue in capture of trade. Furthermore, because of the improvements in quality level of products and offering services as main causes of surpassing the rivals, it could be concluded that it would take a major contribution of market. Therefore, today the meaning of quality is developing as a public culture. Fuzzy set theory has been applied to many areas which need to manage vague data such as decision making, optimization, control and so on. In the decision analysis of the fuzzy environment fuzzy numbers need to be compared and discriminated with decision makers. Thus, in this paper researchers proposed a novel approach to evaluate fuzzy quantities based on interval valued. It combines the concept of geometric distance with areas and COG points of interval valued belonged to generalized trapezoidal fuzzy numbers. We also proposed an adjustment algorithm for interval valued fuzzy numbers. Finally some numerical Examples are represented.

**Keywords:** Ranking; Fuzzy Numbers; Defuzzification; DMU; Fuzzy two sets.

## INTRODUCTION

Recently it has been claimed that ranking fuzzy numbers is an important issue in decision process. Since 1970, various fuzzy ranking methods have been proposed [7, 8, 15, 16]. Some of these ranking methods have been compared and reviewed by [2]. A type of method is to map fuzzy numbers to the real axis through appropriate transforms and then compares and sorts it [12]. Lee and Li proposed the comparison of fuzzy numbers for which they considered mean and standard deviation and original points. In approach  $\alpha$  – cut set and decision makers' preference are used to construct ranking function [6]. Jain [10, 11] proposed a method using the concepts of maximizing set to order fuzzy numbers, mentioning that decision maker considers only the right side membership function. Among the existing ranking methods, centroid index methods are studied and applied to many decision making problems. Recently, Saneifard [12] pointed out the drawbacks of the existing centroid index ranking method and proposed a new centroid index method for ranking fuzzy numbers based on Center Of Gravity (COG) point. However, the COG point based on ranking method presented by Wang [16] stated that the results of Saneifard [12, 13] and Chu [8] were lack of accuracy. Thus in this paper

we proposed a new method for ranking fuzzy numbers to overcome the shortcomings of the previous studies about ranking trapezoidal fuzzy numbers based on interval valued. In this approach we used COG points' method for defuzzifying the generalized trapezoidal fuzzy numbers based on interval valued. Then by using the geometric distance we compared the fuzzy numbers.

The rest of this paper is organized as following. In Section 2 we briefly reviewed basic concepts of generalized fuzzy numbers, [5], interval valued fuzzy numbers [18] and their arithmetic operations, [3-5]. In Section 3, we presented a new method for ranking fuzzy number based on interval valued. In Section 4, the method illustrated with numerical Examples. The conclusion is discussed in Section 5.

### preliminaries

Here we review some basic concepts of fuzzy sets.

**Definition 1** Let  $X$  be a universe set. A fuzzy subset  $A$  of  $X$  is define with a membership function  $\mu_A(x)$  that maps each element  $x$  in  $A$  to a real number in the interval  $[0,1]$ . The function value of  $\mu_A(x)$  signifies the grade

of membership of  $x$  in  $A$ . When  $\mu_A(x)$  is large, its grade of membership of  $x$  in  $A$  is strong.

**Basic concepts of fuzzy numbers**

Here we briefly review basic concepts of generalized fuzzy numbers. Chen [5] represented a generalized trapezoidal fuzzy number  $A = (a, b, c, d; w)$ , where  $a, b, c$  and  $d$  are real values and  $0 < w \leq 1$  as shown in Fig. 1.

The membership function  $\mu_A(x)$  of generalized fuzzy numbers  $A$  satisfies the following conditions:

1.  $\mu_A(x)$  is a continuous mapping from the universe of discourse  $X$  to the closed interval in  $[0,1]$ ,
2.  $\mu_A(x)=0$ , for all  $-\infty < x \leq a$ ,
3.  $\mu_A(x)$  is monotonically increasing in  $[a, b]$ ,
4.  $\mu_A(x)=w$ , for all  $b \leq x \leq c$ ,
5.  $\mu_A(x)$  is monotonically decreasing in  $[c, d]$ ,
6.  $\mu_A(x)=0$ , for all  $d \leq x < \infty$ .

If  $w = 1$  then, the generalized fuzzy numbers  $A$  is a normal fuzzy number denote as  $A = (a, b, c, d)$ . If  $a = b$  and  $c = d$  the generalized fuzzy number  $A$  is a crisp interval. If  $a < b = c < d$  then,  $A$  is a triangular fuzzy number. If  $a < b < c < d$  then,  $A$  is a generalized trapezoidal fuzzy number. If  $a = b = c = d$  and  $w = 1$  then,  $A$  is a crisp value.

Here there are two generalized trapezoidal fuzzy numbers  $A_1$  and  $A_2$ , where  $A_1 = (a_1, b_1, c_1, d_1; w_1)$  and  $A_2 = (a_2, b_2, c_2, d_2; w_2)$ . The arithmetic operations between the generalized trapezoidal fuzzy numbers  $A_1$  and  $A_2$  are as reviewed from [5] as follows:

**1. Generalized Fuzzy Numbers Addition  $\oplus$  :**

$$A_1 \oplus A_2 = (a_1, b_1, c_1, d_1; w_1) \oplus (a_2, b_2, c_2, d_2; w_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(w_1, w_2)),$$

Where  $a_1, b_1, c_1, d_1, a_2, b_2, c_2$  and  $d_2$  are real numbers.

**2. Generalized Fuzzy Numbers Subtraction  $\ominus$  :**

$$A_1 \ominus A_2 = (a_1, b_1, c_1, d_1; w_1) \ominus (a_2, b_2, c_2, d_2; w_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2; \min(w_1, w_2)),$$

where  $a_1, b_1, c_1, d_1, a_2, b_2, c_2$  and  $d_2$  are real numbers.

**3. Generalized Fuzzy Numbers Multiplication  $\otimes$  :**

$$A_1 \otimes A_2 = (a, b, c, d; \min(w_1, w_2)),$$

$$a = \text{Min}(a_1 \times a_2, a_1 \times d_2, d_1 \times a_2, d_1 \times d_2),$$

$$b = \text{Min}(b_1 \times b_2, b_1 \times c_2, c_1 \times b_2, c_1 \times c_2),$$

$$c = \text{Max}(b_1 \times b_2, b_1 \times c_2, c_1 \times b_2, c_1 \times c_2),$$

$$d = \text{Max}(a_1 \times a_2, a_1 \times d_2, d_1 \times a_2, d_1 \times d_2).$$

If  $a_1, b_1, c_1, d_1, a_2, b_2, c_2$  and  $d_2$  are positive real number, then

$$A_1 \otimes A_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2; \min(w_1, w_2)).$$

**4. Generalized Fuzzy Numbers Division  $\oslash$**

Let  $A_1$  and  $A_2$  be two generalized trapezoidal fuzzy numbers, where  $A_1 = (a_1, b_1, c_1, d_1; w_1)$ ,  $A_2 = (a_2, b_2, c_2, d_2; w_2)$ ,  $a_1, b_1, c_1, d_1, a_2, b_2, c_2$  and  $d_2$  are nonzero positive real numbers then the division between  $A_1$  and  $A_2$  define as follows:

$$A_1 \oslash A_2 = (a_1, b_1, c_1, d_1; w_1) \oslash (a_2, b_2, c_2, d_2; w_2) = (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2; \min(w_1, w_2)).$$

**Interval valued fuzzy numbers and their arithmetic operations**

Gorzalczany [9] proposed the concept of interval valued fuzzy sets. Then, Yao [17] represented the interval valued trapezoidal fuzzy number  $A = [A^L, A^U] = [(a_1^L, a_2^L, a_3^L, a_4^L; w_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; w_A^U)]$ , as showing in Fig. 2 where  $A^L$  denotes the lower interval valued trapezoidal fuzzy number,  $A^U$  denotes the upper interval valued trapezoidal fuzzy number, and  $A^L \subset A^U$ .

From Fig. 2 we can see that interval valued trapezoidal fuzzy number  $A$  consists of the lower generalized trapezoidal fuzzy number  $A^L$  and the upper generalized trapezoidal fuzzy number  $A^U$ . It is obvious that,

- 1) If  $A^L = A^U$ , then interval valued trapezoidal fuzzy number  $A$  will becomes, a generalized trapezoidal fuzzy number.
- 2) If  $a_1 = a_2 = a_3 = a_4$  and  $w^L = w^U$  then the interval valued trapezoidal fuzzy number  $A$  will become a crisp value.
- 3) If  $a_1 < a_2 = a_3 < a_4$  then the interval valued trapezoidal fuzzy number  $A$  becomes a triangular interval valued fuzzy number.

Assume that there are two interval valued trapezoidal fuzzy numbers  $A$  and  $B$  where  $A =$

$\left[ (a_1^L, a_2^L, a_3^L, a_4^L; w_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; w_A^U) \right]$  and  
 $B = \left[ (b_1^L, b_2^L, b_3^L, b_4^L; w_B^L), (b_1^U, b_2^U, b_3^U, b_4^U; w_B^U) \right]$   
 $a_i^L, a_i^U, b_i^L, b_i^U; i = 1 \dots 4$  are real values,  
 $0 \leq w_A^L \leq w_A^U \leq 1$  and  $0 \leq w_B^L \leq w_B^U \leq 1$ . Then  
 arithmetic  
 Operations between interval valued trapezoidal fuzzy  
 numbers  $A$  and  $B$  are reviewed from Chen [3, 4] as  
 follows:

1) Interval Valued Fuzzy Numbers Addition  $\oplus$  :

$$\begin{aligned}
 A \oplus B = & \left[ (a_1^L, a_2^L, a_3^L, a_4^L; w_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; w_A^U) \right] \oplus \\
 & \left[ (b_1^L, b_2^L, b_3^L, b_4^L; w_B^L), (b_1^U, b_2^U, b_3^U, b_4^U; w_B^U) \right] = \\
 & \left[ (a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; \min(w_A^L, w_B^L)), \right. \\
 & \left. (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; \min(w_A^U, w_B^U)) \right]
 \end{aligned}$$

2) Interval Valued Fuzzy Numbers Subtraction  $\ominus$ :

$$\begin{aligned}
 A \ominus B = & \left[ (a_1^L, a_2^L, a_3^L, a_4^L; w_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; w_A^U) \right] \ominus \\
 & \left[ (b_1^L, b_2^L, b_3^L, b_4^L; w_B^L), (b_1^U, b_2^U, b_3^U, b_4^U; w_B^U) \right] = \\
 & \left[ (a_1^L - b_1^L, a_2^L - b_2^L, a_3^L - b_3^L, a_4^L - b_4^L; \min(w_A^L, w_B^L)), \right. \\
 & \left. (a_1^U - b_1^U, a_2^U - b_2^U, a_3^U - b_3^U, a_4^U - b_4^U; \min(w_A^U, w_B^U)) \right]
 \end{aligned}$$

3) Interval Valued Fuzzy Numbers Multiplication  $\otimes$  :

$$\begin{aligned}
 A \otimes B = & \left[ (a_1^L, a_2^L, a_3^L, a_4^L; w_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; w_A^U) \right] \otimes \\
 & \left[ (b_1^L, b_2^L, b_3^L, b_4^L; w_B^L), (b_1^U, b_2^U, b_3^U, b_4^U; w_B^U) \right] = \\
 & \left[ (a_1^L \times b_1^L, a_2^L \times b_2^L, a_3^L \times b_3^L, a_4^L \times b_4^L; \min(w_A^L, w_B^L)), \right. \\
 & \left. (a_1^U \times b_1^U, a_2^U \times b_2^U, a_3^U \times b_3^U, a_4^U \times b_4^U; \min(w_A^U, w_B^U)) \right]
 \end{aligned}$$

4) Interval Valued Fuzzy Numbers Division  $\oslash$ :

$$\begin{aligned}
 A \oslash B = & \left[ (a_1^L, a_2^L, a_3^L, a_4^L; w_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; w_A^U) \right] \oslash \\
 & \left[ (b_1^L, b_2^L, b_3^L, b_4^L; w_B^L), (b_1^U, b_2^U, b_3^U, b_4^U; w_B^U) \right] = \\
 & \left[ (a_1^L / b_4^L, a_2^L / b_3^L, a_3^L / b_2^L, a_4^L / b_1^L; \min(w_A^L, w_B^L)), \right. \\
 & \left. (a_1^U / b_4^U, a_2^U / b_3^U, a_3^U / b_2^U, a_4^U / b_1^U; \min(w_A^U, w_B^U)) \right]
 \end{aligned}$$

Calculating COG points of fuzzy numbers based interval value

Let two interval valued trapezoidal fuzzy numbers  $A = \left[ (a_1^L, a_2^L, a_3^L, a_4^L; w_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; w_A^U) \right]$  and

$$B = \left[ (b_1^L, b_2^L, b_3^L, b_4^L; w_B^L), (b_1^U, b_2^U, b_3^U, b_4^U; w_B^U) \right].$$

Calculate COG points  $(\bar{x}_{A^L}, \bar{y}_{A^L}), (\bar{x}_{A^U}, \bar{y}_{A^U})$ ,

$(\bar{x}_{B^L}, \bar{y}_{B^L})$  and  $(\bar{x}_{B^U}, \bar{y}_{B^U})$  of  $A^L, A^U, B^L$  and

$B^U$  as follows:

$$\begin{aligned}
 \bar{y}_{A^L} = & \begin{cases} \frac{w_A^L \times \left( \frac{a_3^L - a_2^L}{a_4^L - a_1^L} + 2 \right)}{6}, & \text{if } a_1^L \neq a_4^L \text{ and } 0 < w_A^L \leq 1, \\ \frac{w_A^L}{2}, & \text{if } a_1^L = a_4^L \text{ and } 0 < w_A^L \leq 1, \end{cases} \\
 & (1)
 \end{aligned}$$

$$\begin{aligned}
 \bar{y}_{A^U} = & \begin{cases} \frac{w_A^U \times \left( \frac{a_3^U - a_2^U}{a_4^U - a_1^U} + 2 \right)}{6}, & \text{if } a_1^U \neq a_4^U \text{ and } 0 < w_A^U \leq 1, \\ \frac{w_A^U}{2}, & \text{if } a_1^U = a_4^U \text{ and } 0 < w_A^U \leq 1, \end{cases} \\
 & (2)
 \end{aligned}$$

$$\begin{aligned}
 \bar{y}_{B^L} = & \begin{cases} \frac{w_B^L \times \left( \frac{b_3^L - b_2^L}{b_4^L - b_1^L} + 2 \right)}{6}, & \text{if } b_1^L \neq b_4^L \text{ and } 0 < w_B^L \leq 1, \\ \frac{w_B^L}{2}, & \text{if } b_1^L = b_4^L \text{ and } 0 < w_B^L \leq 1, \end{cases} \\
 & (3)
 \end{aligned}$$

$$\begin{aligned}
 \bar{y}_{B^U} = & \begin{cases} \frac{w_B^U \times \left( \frac{b_3^U - b_2^U}{b_4^U - b_1^U} + 2 \right)}{6}, & \text{if } b_1^U \neq b_4^U \text{ and } 0 < w_B^U \leq 1, \\ \frac{w_B^U}{2}, & \text{if } b_1^U = b_4^U \text{ and } 0 < w_B^U \leq 1. \end{cases} \\
 & (4) \quad \text{Calculate } \bar{x}_{A^L},
 \end{aligned}$$

$\bar{x}_{A^U}, \bar{x}_{B^L}$  and  $\bar{x}_{B^U}$  as follows:

$$\bar{x}_{A^L} = \frac{\bar{y}_{A^L}(a_3^L + a_2^L) + (a_4^L + a_1^L)(w_A^L - \bar{y}_{A^L})}{2w_A^L}, \tag{5}$$

$$\bar{x}_{A^U} = \frac{\bar{y}_{A^U}(a_3^U + a_2^U) + (a_4^U + a_1^U)(w_A^U - \bar{y}_{A^U})}{2w_A^U}, \tag{6}$$

$$\bar{x}_{B^L} = \frac{\bar{y}_{B^L}(b_3^L + b_2^L) + (b_4^L + b_1^L)(w_B^L - \bar{y}_{B^L})}{2w_B^L}, \tag{7}$$

$$\bar{x}_{B^U} = \frac{\bar{y}_{B^U}(b_3^U + b_2^U) + (b_4^U + b_1^U)(w_B^U - \bar{y}_{B^U})}{2w_B^U}. \tag{8}$$

Now calculate COG points  $(\bar{x}_A, \bar{y}_A)$  of interval valued fuzzy number  $A$  where:

$$\bar{x}_A = \begin{cases} \frac{A(A^U) \times \bar{x}_{A^U} - A(A^L) \times \bar{x}_{A^L}}{A(A^U) - A(A^L)}, & \text{if } A(A^U) - A(A^L) \neq 0, \\ 0, & \text{otherwise,} \end{cases} \tag{9}$$

$$\bar{y}_A = \begin{cases} \frac{A(A^U) \times \bar{y}_{A^U} - A(A^L) \times \bar{y}_{A^L}}{A(A^U) - A(A^L)}, & \text{if } A(A^U) - A(A^L) \neq 0, \\ 0, & \text{otherwise,} \end{cases} \tag{10}$$

In the same way calculate the COG point  $(\bar{x}_B, \bar{y}_B)$  as follows:

$$\bar{x}_B = \begin{cases} \frac{A(B^U) \times \bar{x}_{B^U} - A(B^L) \times \bar{x}_{B^L}}{A(B^U) - A(B^L)}, & \text{if } A(B^U) - A(B^L) \neq 0, \\ 0, & \text{otherwise,} \end{cases} \tag{11}$$

$$\bar{y}_B = \begin{cases} \frac{A(B^U) \times \bar{y}_{B^U} - A(B^L) \times \bar{y}_{B^L}}{A(B^U) - A(B^L)}, & \text{if } A(B^U) - A(B^L) \neq 0, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

**An interval valued fuzzy numbers adjustment algorithm**

However, sometimes the upper trapezoidal fuzzy number of the evaluating result cannot contain the lower trapezoidal fuzzy number of the evaluating result. In the following, we use an example to show how this situation occurs.

**Example**

Let  $A = [(0,0.05,0.15,0.2;0.5), (0,0.1,0.1,0.2;1)]$ ,

$B = [(0,0.05,0.15,0.2;0.5), (0,0.1,0.1,0.2;1)]$

we can get;

$$A \otimes B = [(0,0.05,0.15,0.2;0.5), (0,0.1,0.1,0.2;1)] \otimes [(0,0.05,0.15,0.2;0.5), (0,0.1,0.1,0.2;1)] = [(0,0.0025,0.0225,0.04;0.5), (0,0.01,0.01,0.04;1)]$$

We can see that the upper trapezoidal fuzzy number cannot contain the lower trapezoidal fuzzy number, as shown in Fig. 3. In the following, we propose an algorithm to adjust the evaluation result to let the upper trapezoidal fuzzy number contain the lower trapezoidal fuzzy number. Assume that there is an interval valued trapezoidal fuzzy number  $A = [(a_1^L, a_2^L, a_3^L, a_4^L; w_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; w_A^U)]$ , and the universe of discourse is between zero and one. The algorithm is presented as follows:

**Step1:** calculate the values of  $x_l$  and  $x_r$  which are the two end points of the  $w_A^L$ -cut (i.e.,  $[x_l, x_r]$ ) of  $A^U$ . The values of  $x_l$  and  $x_r$  are calculated as follows:

$$x_l = \frac{w_A^L}{w_A^U} (a_2^U - a_1^U) + a_1^U, \tag{25}$$

$$x_r = \frac{w_A^L}{w_A^U} (a_3^U - a_4^U) + a_4^U. \tag{26}$$

**Step 2:** If the value of  $a_2^L$  is greater than then  $x_r$ , let  $a_2^L = x_r$  and  $a_3^L = x_r$ . Otherwise, go to step 3.

**Step 3:** If the value of  $a_2^L$  is smaller than  $x_l$  then let  $a_2^L = x_l$ .

**Step 4:** if the value of  $a_3^L$  is smaller than  $x_l$  then let  $a_3^L = x_l$ . Otherwise, if the value of  $a_3^L$  is greater than  $x_r$  then, let  $a_3^L = x_r$  and stop.

After applying the algorithm, an illegal interval valued fuzzy number  $A$  become a legal interval valued fuzzy number.

**A new method for ranking generalized trapezoidal fuzzy number based interval valued**

Some researchers defined a distance and then compared the fuzzy numbers [13,14]. In this section, we present a new method for ranking trapezoidal fuzzy number based on interval value. In this method we consider the COG point of a generalized trapezoidal fuzzy number based on interval value, and also the geometric distance of these fuzzy numbers. Thus the proposed method combines the concepts of geometric distance the areas and the COG points of interval valued fuzzy numbers.

$$A = [(a_1^L, a_2^L, a_3^L, a_4^L; w_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; w_A^U)],$$

$$B = [(b_1^L, b_2^L, b_3^L, b_4^L; w_B^L), (b_1^U, b_2^U, b_3^U, b_4^U; w_B^U)]$$

Let are the generalized trapezoidal fuzzy numbers.

**Step 1:** calculate the areas  $A(A^L)$  of the lower trapezoidal fuzzy number  $A^L$  and  $A(A^U)$  of the upper trapezoidal fuzzy number  $A^U$  respectively, shown as follows:

$$A(A^L) = \frac{(a_4^L + a_3^L - a_2^L - a_1^L) \times w_A^L}{2}, \tag{19}$$

$$A(A^U) = \frac{(a_4^U + a_3^U - a_2^U - a_1^U) \times w_A^U}{2}, \tag{20}$$

Also in the same way, calculate the areas  $A(B^L)$  of the lower trapezoidal fuzzy number  $B^L$  and  $A(B^U)$  of the upper trapezoidal fuzzy number  $B^U$ , respectively as follows:

$$A(B^L) = \frac{(b_4^L + b_3^L - b_2^L - b_1^L) \times w_B^L}{2}, \tag{21}$$

$$A(B^U) = \frac{(b_4^U + b_3^U - b_2^U - b_1^U) \times w_B^U}{2}, \tag{22}$$

**Step 2:** calculate the COG points  $(\bar{x}_A, \bar{y}_A)$  and  $(\bar{x}_B, \bar{y}_B)$  by the formula (1-12).

**Step 3:** Use the new points  $M = \min(a_i^u, b_i^u)$  for  $i = 1, \dots, 4$ . Use geometric distance for calculating the distance between COG points and the crisp points we define as minimum points of fuzzy numbers to calculate the ranking value of numbers.

$$d(M, A) = \sqrt{(\bar{x}_A - M)^2 + (\bar{y}_A)^2}, \tag{23}$$

$$d(M, B) = \sqrt{(\bar{x}_B - M)^2 + (\bar{y}_B)^2}.$$

(24)  
If  $d(M, A) > d(M, B)$  then we have  $A \succ B$ .

**Some Numerical Examples**

In the following we use some Examples to illustrate the ranking process of generalized trapezoidal fuzzy number based on interval valued. Table 1 shows calculating results of the proposed method.

**Example** Consider the fuzzy numbers

$$A = [(0.3, 0.35, 0.45, 0.5; 0.8), (0.1, 0.25, 0.6, 0.7; 1)]$$

and

$$B = [(0.25, 0.3, 0.4, 0.45; 0.8), (0.05, 0.2, 0.5, 0.65; 1)],$$

based on proposed method calculate COG points  $(\bar{x}_A, \bar{y}_A)$  and  $(\bar{x}_B, \bar{y}_B)$  by formulas (1-12). Thus we have  $(\bar{x}_A, \bar{y}_A) = (0.415, 0.463)$  and  $(\bar{x}_B, \bar{y}_B) = (0.331, 0.424)$  use

point  $M = (0.05, 0)$  then, by geometric distance calculate the rank of fuzzy number, We have:  $d(A, M) = 0.589$ ,  $d(B, M) = 0.508$ .

**Example** Consider the fuzzy numbers

$$C = [(0.3, 0.35, 0.45, 0.5; 0.8), (0.1, 0.25, 0.6, 0.7; 1)]$$

and

$$D = [(0.33, 0.4, 0.5, 0.55; 0.8), (0.05, 0.2, 0.5, 0.65; 1)]$$

based on proposed method calculate COG points  $(\bar{x}_C, \bar{y}_C)$  and  $(\bar{x}_D, \bar{y}_D)$ , by formulas (1-12). Thus we have  $(\bar{x}_C, \bar{y}_C) = (0.415, 0.563)$  and  $(\bar{x}_D, \bar{y}_D) = (0.313, 0.396)$  use

the point  $M = (0.05, 0)$  then, by using geometric distance calculate the rank of fuzzy number, We have:  $d(C, M) = 0.589$ ,  $d(D, M) = 0.475$ .

**Example** Consider the fuzzy numbers

$$E = [(0.3, 0.35, 0.45, 0.5; 0.8), (0.2, 0.3, 0.5, 0.6; 1)]$$

and

$$F = [(0.35, 0.4, 0.5, 0.55; 0.8), (0.2, 0.3, 0.5, 0.6; 1)],$$

as shown above we have:  $d(E, M) = 0.494$ ,  $d(F, M) = 0.523$ .

**CONCLUSIONS**

In the present study, we suggested a new method for ordering fuzzy numbers based on interval valued. According to this method we proposed a new ranking index to classify fuzzy numbers based on interval valued. Moreover, we used

some numerical Examples to illustrate the advantages of the method. By using this method, we solved some shortcomings of ordering interval valued fuzzy numbers.

[18] Zhang W. R. 1986. Knowledge representation using linguistic fuzzy relations. Ph.D. Dissertation University of South Carolina, USA.

## REFERENCES

- [1] Abbasbandy S, Asady B. 2006. Ranking of fuzzy numbers by sign distance. *Inform Sciences*. 176: 2405-2416.
- [2] Bortlan G, Degani R. 1985. A review of some methods for ranking fuzzy numbers. *Fuzzy Sets and Systems*. 15: 1-19.
- [3] Chen S. M. 1995. Arithmetic operations between vague sets. In proceeding of international joint conference of CFSA/IFIS/SOFT'95 on fuzzy theory and applications. 206-211, Taipei, Taiwan, Republics of China.
- [4] Chen S. M. 1997. Fuzzy system reliability analysis based on vague set theory. In proceedings of the IEEE International Conference on systems, man and cybernetics. 2: 12-15. Orlando, USA.
- [5] Chen S. H. 1999. Ranking generalized fuzzy number with graded mean integration. In proceedings of the eighth international fuzzy systems association world congress. 2: 899-902, Taiwan, Republic of China.
- [6] Cheng C. H. 1993. Fuzzy system reliability analysis by confidence interval. *Fuzzy Sets and Systems*. 56: 29-35.
- [7] Cheng C. H. 1998. A new approach for ranking fuzzy numbers by distance method. *Fuzzy Sets and System*, 95: 307-317.
- [8] Chu T. C, Tsao C. T. 2002. Ranking fuzzy numbers with an area between the centroid point and the original point, *Computers and Mathematics with Applications*. 43: 111-117.
- [9] Gorzalczany M. B. 1987. A method of inference in approximate reasoning based on interval-valued fuzzy sets. *Fuzzy Sets and Systems*. 21: 1-11.
- [10] Jain R. 1977. A procedure for multi-aspect decision making using fuzzy set. *Int. J. System. Sciences* 8: 1-7.
- [11] Jain R. 1976. Decision-making in presence of fuzzy variable. *IEEE Trans. System. Man Cybern.* 6: 698-703.
- [12] Li R J, 2002. Fuzzy multi-attribute decision theory and application. Science Press, Beijing.
- [13] Saneifard R. 2012. Anteriority indicator for managing fuzzy dates based on maximizing and minimizing sets. *International Journal of Natural and Engineering Sciences*. 2: 29-32.
- [12] Saneifardd R, Noori E. 2011. A novel approach for defuzzification based on ambiguity\_preserving. *International Journal and Engineering Sciences*. 5: 21-26.
- [13] Saneifard R, Ezzati R. 2010. Defuzzification through a Bi-symmetrical weighted function, *Australian Journal of Basic and applied Sciences*. 10: 4976-4984.
- [14] Saneifard R. 2009. A method for defuzzification by weighted distance, *international Journal of Industrial Mathematics*. 3: 209-217.
- [15] Saneifard R, Allahviranloo T, Hosseinzadeh F, Mikaeilvand N. 2007. Euclidean ranking DMUS with Fuzzy data in DEA, *Applied Mathematical Sciences*. 60: 2989-2998.
- [16] Wang Y. M, Elhang T. M.S. 2006. Fuzzy Topsis method based on alpha level sets with an application to bridge risk assessment *Expert Systems With applications*. 28: 547-556.
- [17] Yao J. S, Lin F. T. 2002. Constructing a fuzzy flow-shop sequencing model based on statistical data, *International journal of approximate reasoning*. 29: 215-234.