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Analysis of Deflection of Rectangular Plates Under Different Loading Conditions

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Abstract

In this paper, bending of an isotropic rectangular plate under various loading conditions is studied using MATLAB code and commercial finite element software ANSYS. Classical plate theory (CPT) and plane stress assumption are used. Also, four node rectangular nonconforming elements are used. The presented code is able to consider residual stresses, self-strains, body forces, distributed and concentrated forces and distributed moments. Furthermore, it can analyze different boundary conditions for the plate. The results of finite element analysis are compared with each other and also by CPT results and a good agreement between them is noticed. On the other hand, the convergence of results is investigated by increasing the number of elements.

Keywords: Rectangular plate; classical plate theory; stress; deflection.

INTRODUCTION

Many structures like bridge deck, pier cap, concrete pavement, shear walls, floor slabs, water retaining structures, ship hulls, chopper blades etc., appearing in various engineering fields, such as civil, mechanical, aeronautical and naval engineering can be modeled as plates. In continuum mechanics, plate theories are mathematical descriptions of the mechanics of flat plates that draws on the theory of beams. Plates are defined as plane structural elements with a small thickness compared to the planar dimensions [1]. The typical thickness to width ratio of a plate structure is less than 0.1. A plate theory takes advantage of this disparity in length scale to reduce the full three-dimensional solid mechanics problem to a two-dimensional problem. The aim of plate theory is to calculate the deformation and stresses in a plate subjected to loads. Of the numerous plate theories that have been developed since the late 19th century, two are widely accepted and used in engineering. These are: The kirchhoff–love theory of plates (classical plate theory)and the mindlin–reissner theory of plates (first-order shear plate theory).

Bending of plates or plate bending refers to the deflection of a plate perpendicular to the plane of the plate under the action of external forces and moments. The amount of deflection can be determined by solving the differential equations of an appropriate plate theory. The stresses in the plate can be calculated from these deflections. Once the stresses are known, failure theories can be used to determine whether a plate will fail under a given load.

Semie [2] used Kirchoff's thin plate theory to study static behaviour of a plate. He developed a finite element computer program based on the assumption of the theory in FORTRAN using a linear triangular element. Imrak et al. [3] presented an exact solution for the deflection of a clamped rectangular plate under uniform load in which each term of the series is trigonometric and hyperbolic, and identically satisfies the boundary conditions on all four edges.

This paper deals with investigating bending of an isotropic rectangular plate under various loading conditions using MATLAB code and simulation with ANSYS. Classical plate theory and plane stress assumption are used. The results of finite element analysis are compared with each other and also by numerical results and a good agreement between them is noticed.

ANALYTICAL FORMULATION

Classical plate theory is used in this paper. It is based on four assumptions as following: 1) Plate's deflection is negligible compared to its thickness, 2) The middle layer is without stress, 3) Each plane normal to the middle layer remains normal after deflection, 4) Normal stress along the plate thickness is zero. They will have good accuracy if the plate deflection will be low. Given the above assumptions the differential equation of plate will be as follows:

$$(L\nabla)^T D \ \nabla w + q = 0 \tag{1}$$

Where, w and q is the plate deflection and applied load, respectively [4]. For this problem, a four-node element is used and three degrees of freedom are at each node which one is for plate deflection at the node and the other two are for rotation of node along the x and y directions (Figure 1). Now for the displacement function regarding using of four-node element, a 3-degree polynomial with twelve terms is utilized.

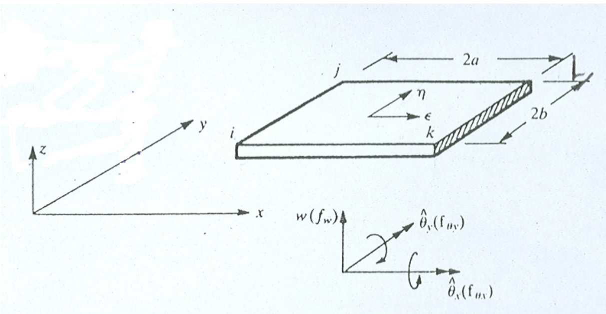

Fig.1. Used Element [1]

$$w = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 x^2 y + \alpha_9 xy^2 + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3 \equiv P\alpha \tag{2}$$

Therefore, the degrees of freedom for each element are:

$$a^e = \begin{Bmatrix} a_i \\ a_j \\ a_l \\ a_k \end{Bmatrix} \qquad a_i = \begin{Bmatrix} w \\ \hat{\theta}_x \\ \hat{\theta}_y \end{Bmatrix}$$

The unknown coefficients in the Eq. (2) should be written based on the node values:

$$a^e = C\alpha \tag{3}$$

Where, C is a 12×12 matrix and will be based on the coordinates of the nodes inside the element. Therefore, we can write:

$$L\nabla w = Q\alpha = Q^{-1} a^e = Ba^e \tag{4}$$

Three boundary conditions are considered here:

1) Free edge: in this case none of the deflections or slopes is zero and torque and shear forces can be predefined or may be zero.

2) Simply support edge: in this case the deflection IS ZERO at the support and the torque is zero in the direction of the axis.

3) Clamped boundary: in this case the deflection at the edge in the direction normal to the edge will be zero.

For calculation of strains, the following equation is used:

$$\varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_x \end{Bmatrix} = -z \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix} \tag{5}$$

Also, the following equation is used to obtain stresses:

$$\sigma_x = \frac{12 M_x z}{h^3} \tag{6}$$

ASSUMPTIONS

In this paper, an arbitrary dimensions rectangular plate under a distributed load and three boundary conditions (free edge, one-edge clamped, four-edge clamped) is taken into account. The way of numbering elements, nodes and edges is depicted in Figure 2 and 3. The other conditions are as follows:

$$T = T_1 + \frac{z}{h} T_0 \tag{7}$$

1) Thermal strain forces: It is assumed that temperature function is linear and it is only changed toward the plate thickness as following:

$$\sigma = \sigma_1 + \frac{z}{h} \sigma_0 \tag{8}$$

2) Residual stress forces: For the residual stress as the temperature function a linear function is used:

3) Linear moments: A constant moment along a line segment which is parallel to the edges of the plate, is applied.

4) Boundary conditions: Three different boundary conditions are applied as explained in previous section.

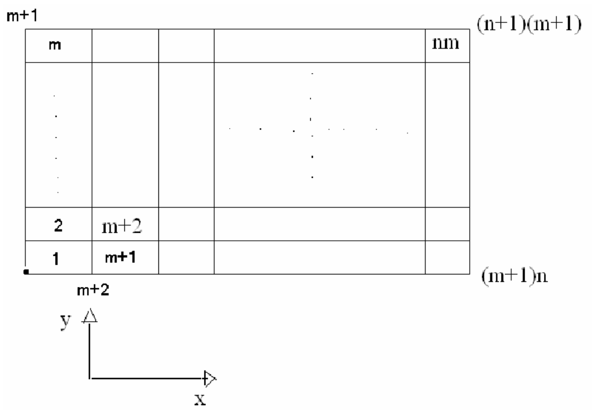

Fig.2. Numbering of elements and nodes

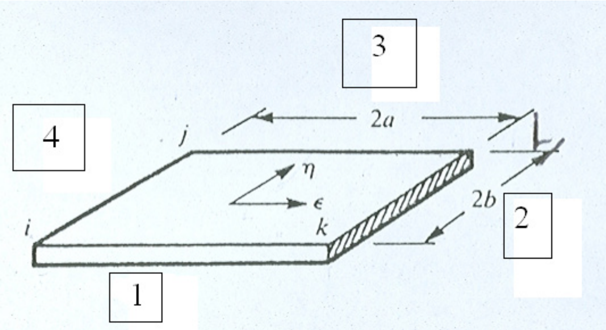

Fig.3. Numbering of edges

RESULTS AND DISCUSSION

First Case

The boundary conditions are four-edge clamped and the plate is under uniform and distributed load. Other characteristics of the plate and the applied load are as following (Table 1):

The results of first case are represented in Table 2. According to Table 2, it is evident that the deflection value obtained by MATLAB code has slight difference with the ones achieved by ANSYS and theoretical analysis. It can be improved by increasing the number of elements. But in maximum stress, the error is higher because of two approximations made for strain and elements results for each element. The results for this case are shown in Figure 4 to Figure 11.

Second Case

In this case, the characteristics of the plate will be in accordance with Table 1 and the boundary conditions are as following: edge 1: clamped, edges 2&4: simply support and edge 3: free. The obtained results for this case are shown in Table 3. As the same to the case 1, it is evident that the deflection value obtained by MATLAB code has slight difference with the ones achieved by ANSYS and theoretical analysis. Further refining the elements does not have much effect on the deflection. As shown in Figures 15 and 16 the supporting forces increase at the edge but it is zero at the edge 3 because of existence of free edge. The results for this case are depicted in Figure 12 to Figure 20.

Table.1. Characteristics of the studied plate

plate width	1 M
plate length	1 M
plate thickness	0.005 M
number of elements in thickness	20
number of elements in length	20
distributed load	10 N/m^2
Young's modulus	207 GPA
Poisson's ratio	0.3
number of elements in MATLAB	400
number of elements in ANSYS	10000

Table 2. Comparison of results in case 1

	results of present study	results of ANSYS with rectangular element	results of ANSYS with triangular element	theoretical results
maximum deflection	0.000536	0.000534	0.000506	0.000531
maximum stress	9.88 Mpa	11.9 Mpa	12.1 Mpa	12.31 Mpa

Table 3. Comparison of results in case 2

	Results of Present Study	Results of Ansys With Rectangular Element	Results of Ansys With Triangular Element
maximum deflection	0.004763	0.00473	0.004741
maximum stress	26.7 Mpa	23.5 Mpa	28.3 Mpa

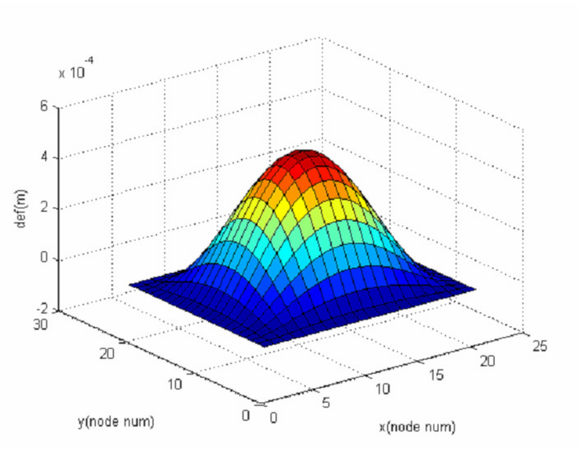

Figure 4. Plate deflection (MATLAB)

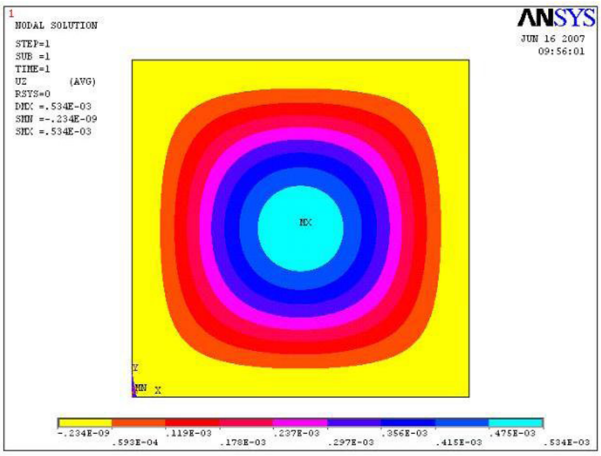

Figure 5. Plate deflection (ASNSYS)

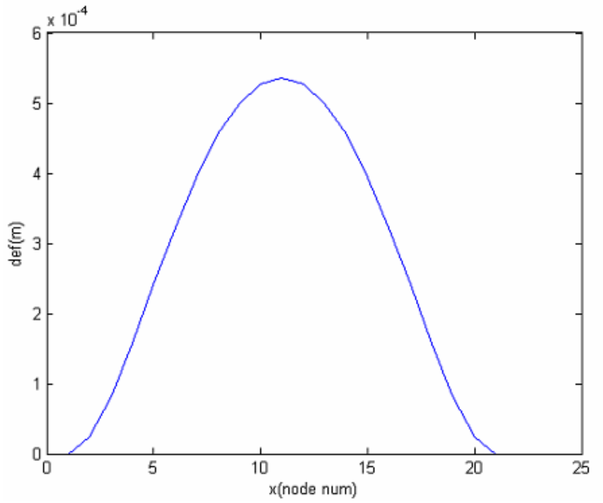

Figure 6. Plate deflection on the central line parallel to x axis

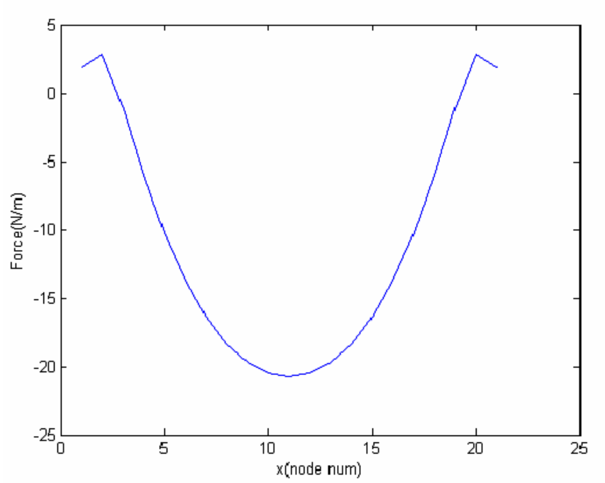

Figure 7. The supporting force along the edge 1

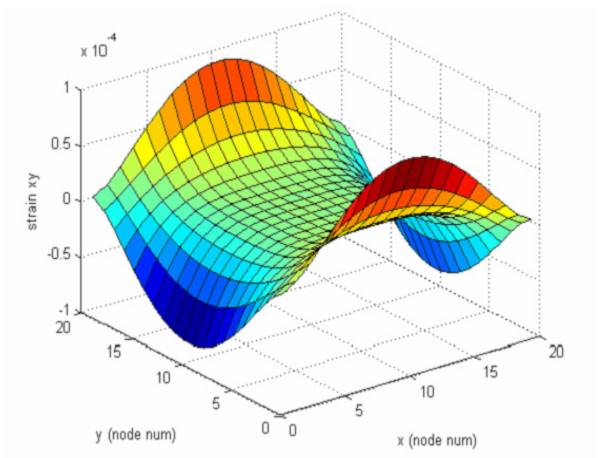

Figure 8. Shear strain along xy

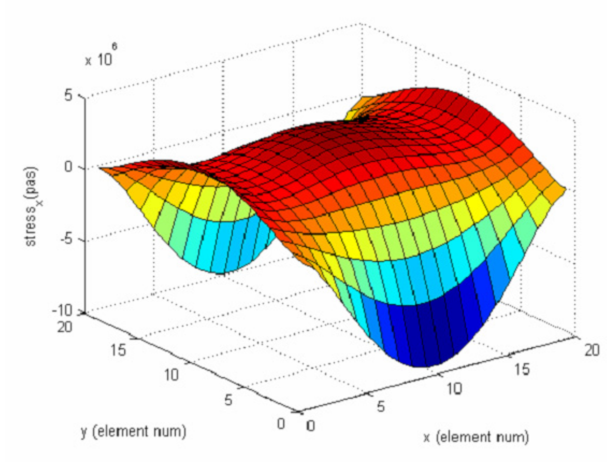

Figure 9. Stress along x direction

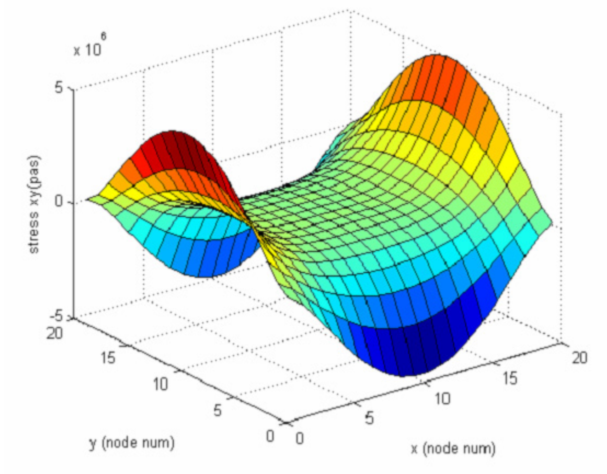

Figure 10. Shear stress along xy

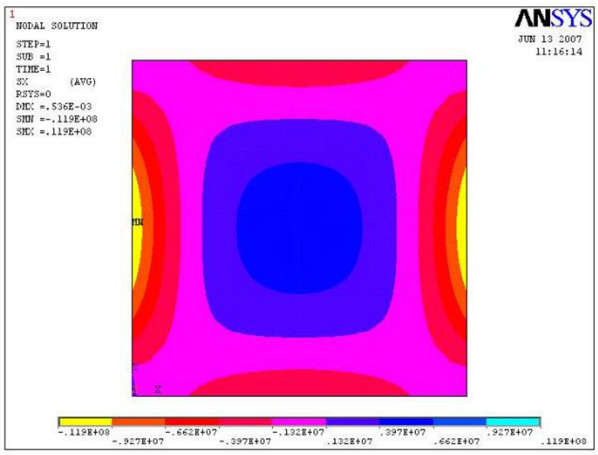

Figure 11. Stress along x direction with rectangular elements (ANSYS)

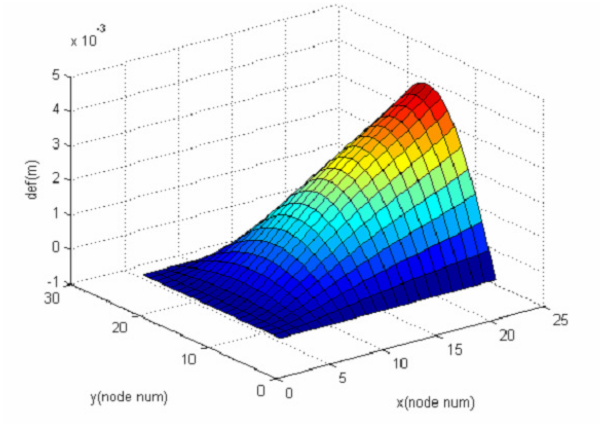

Figure 12. Plate deflection (MATLAB)

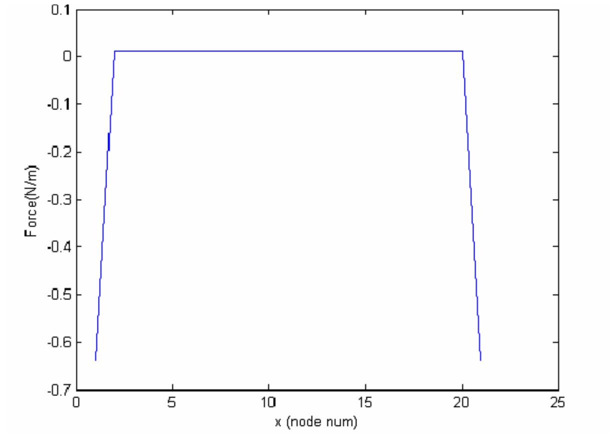

Figure 15. Supporting force along the edge 3

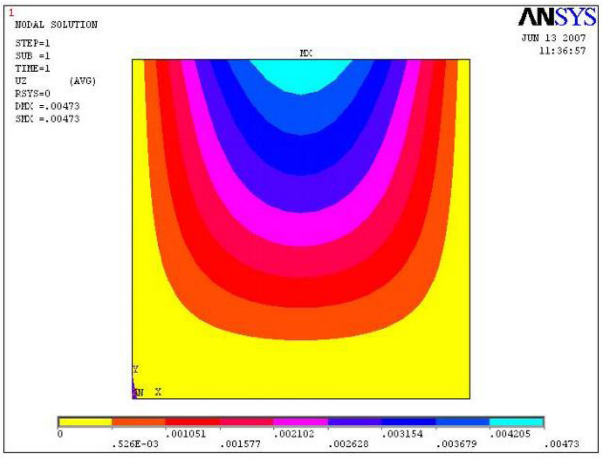

Figure 13. Plate deflection (ASNSYS)

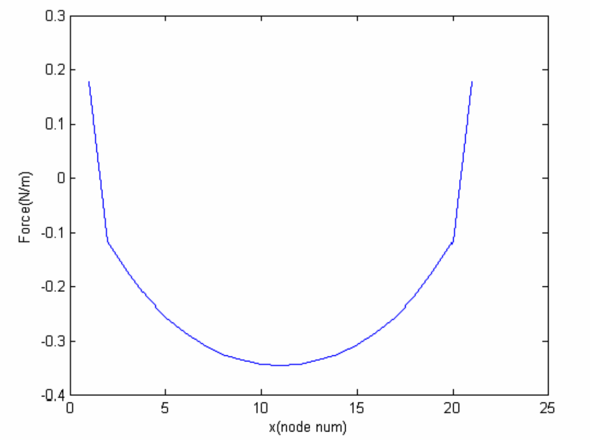

Figure 16. Supporting force along the edge 1

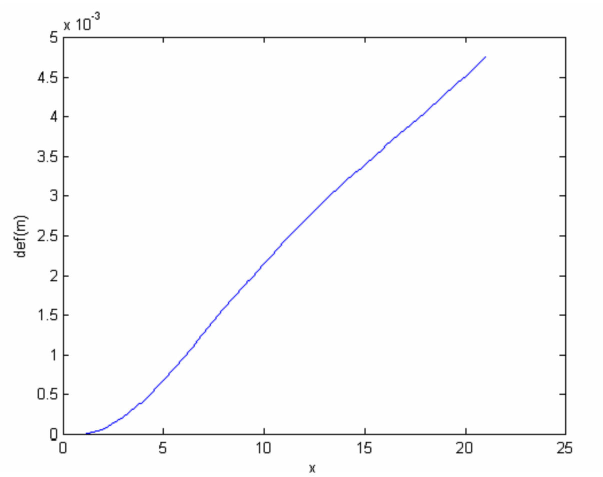

Figure 14. Plate deflection on the central line parallel to x axis

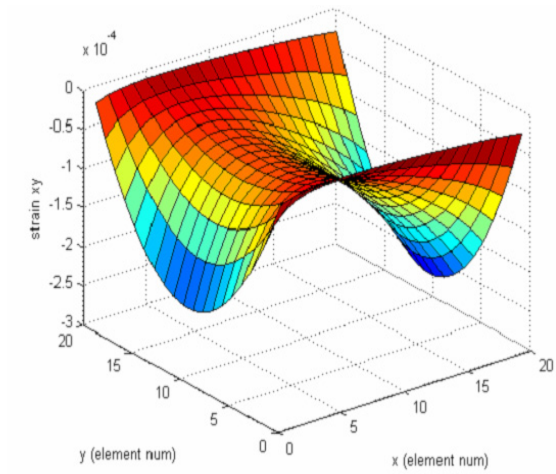

Figure 17. Shear strain along xy

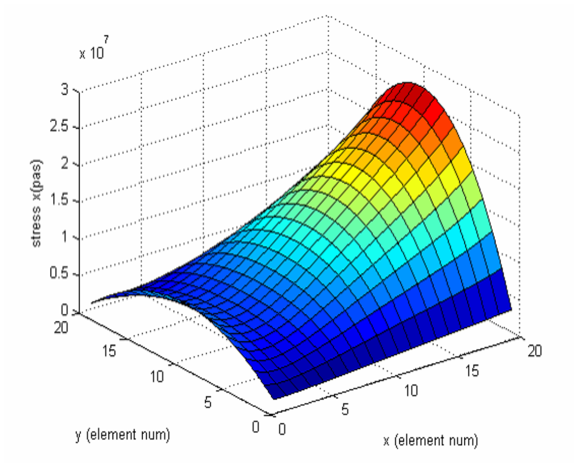

Figure 18. Stress along x direction

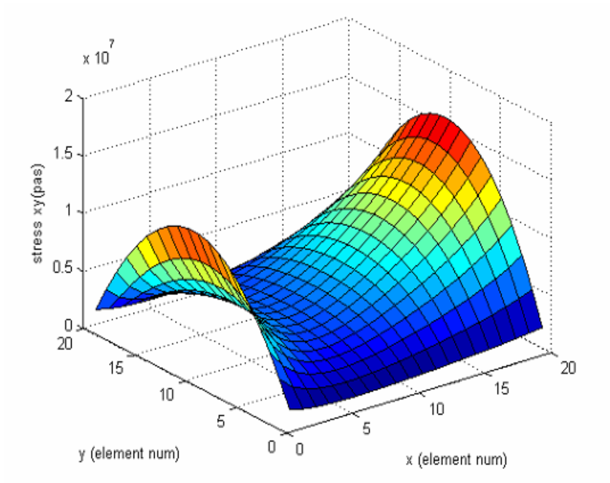

Figure 19. Shear stress along xy

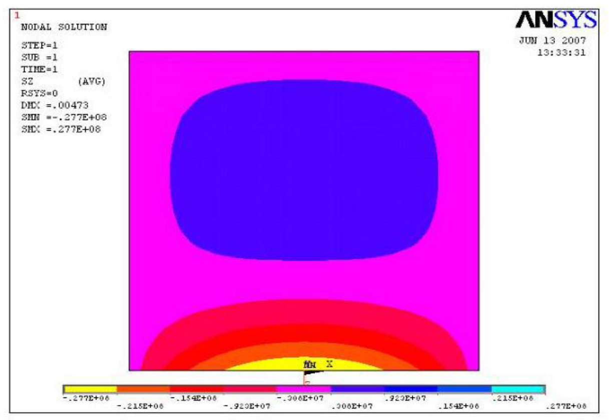

Figure 20. Stress along x direction with rectangular elements (ANSYS)

CONCLUSIONS

In this paper, bending of an isotropic rectangular plate under various loading conditions is studied using MATLAB code and commercial finite element software ANSYS. Classical plate theory and plane stress assumption are used. The obtained results reveal that the deflection value obtained by MATLAB code has slight difference with the ones achieved by ANSYS and theoretical analysis and further refining the elements does not have much effect on the deflection value.

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