

## A Study in Characteristic Impedance of a Microstrip Transmission Line by COMSOL Model

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### Abstract

In this paper, a sample of microstrip transmission line has been presented. Microstrip transmission lines have been widely applied in microwave purposes. The characteristic impedance of this sample was studied. This characteristic is an important parameter, especially in impedance matching. Then the equations are proposed to realize the effective parameters on the characteristic impedance. By ADS software these parameters are validate and finally, A COMSOL model for a microstrip is proposed. The electrical field through a microstrip is plotted and magnetic flux density through a microstrip sample is presented. The results by this modeling method, proof the ADS simulations and the equations.

**Keywords:** ADS, COMSOL, Microstrip, Microwave, Circuits

### INTRODUCTION

Microstrip transmission lines have been widely used in microwave integrated circuits (MIC), such as filters couplers and mixers, power dividers, and etc. [1].

The general structure of a microstrip is illustrated in Fig (1). A conducting strip (microstrip line) with a width  $W$  and a thickness  $t$  is on the top of a dielectric substrate that has a relative dielectric constant  $\epsilon_r$  and a thickness  $h$  and the bottom of the substrate is a ground (conducting) plane [2].

Transmission characteristics of microstrips are described by two parameters: the effective dielectric constant  $\epsilon_{re}$  and characteristic impedance  $Z_c$ , which may then be obtained by quasistatic analysis [3]. For very thin conductors, the closed-form expressions are given [2] as follows:

For  $W/h \leq 1$ :

$$\epsilon_{re} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left\{ \left( 1 + 12 \frac{h}{w} \right)^{-0.5} + 0.04 \left( 1 - \frac{w}{h} \right)^2 \right\} \quad (1)$$

$$Z_c = \frac{\eta}{2\pi\sqrt{\epsilon_{re}}} \ln \left( \frac{8h}{W} + 0.25 \frac{W}{h} \right) \quad (2)$$

where  $\eta = 120\pi$  ohms is the wave impedance in free space.

For  $W/h \geq 1$ :

$$\epsilon_{re} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left( 1 + 12 \frac{h}{w} \right)^{-0.5} \quad (3)$$

$$Z_c = \frac{\eta}{\sqrt{\epsilon_{re}}} \left\{ \frac{W}{h} + 1.393 + 0.677 \ln \left( \frac{W}{h} + 1.444 \right) \right\}^{-1} \quad (4)$$

From the equations (2) and (4), it is seen that the characteristic impedance  $Z_c$  depends on the width of microstrip ( $W$ ), the thickness of substrate ( $h$ ) and dielectric constant ( $\epsilon_r$ ).

In this paper, the other side effects like frequency have been considered. The mentioned equations have been proofed by modeling a microstrip with COMSOL software. Firstly, a sample microstrip line has proposed to study and then the effects on parameters are mentioned.

#### Effective Parameters on Characteristic Impedance

As seen from equations (1), (2), (3) and (4), the effects of thickness of substrate and the width of strip line and dielectric constant on a microstrip line is obvious. A strip line with specified width (1mm) and length (2.5mm) on a specified RT/duriod 5880 substrate with ( $\epsilon_r=2.22$ , loss tangent of 0.0009) and variable thickness is considered to show these effects on a prototype microstrip line. Then the sample is simulated with ADS software. Fig (2) shows the characteristic impedance through the thicknesses changes.

As seen from the equations, characteristic impedance doesn't depend on frequency changes, but simulation by ADS using mesh in electromagnetic domain shows that characteristic impedance changes with frequency variations. This change in  $Z_c$  is more sensible in lines with more thickness. For given transmission line the magnitude of  $Z_c$  which is get from the equation (2) are: 47.75, 65.91 and 85.15 for  $h = 0.3$  mm,  $h = 0.5$  mm and  $h = 0.8$  mm, respectively. These amount of  $Z_c$  are different from the result of simulation.

Also, the effect of frequency has been detected as dispersion in microstrips. Namely, its phase velocity is not a constant but

depends on frequency. It follows that its effective dielectric constant  $\epsilon_{re}$  is function of frequency and can be defined as the frequency dependent effective dielectric constant  $\epsilon_{re}(f)$  in general[2]. The formulas reported in [4] and [5] can be used, and is given as follows:

$$\epsilon_{re}(f) = \epsilon_{r\infty} - \frac{\epsilon_r - \epsilon_{r\infty}}{1 + (f/f_{50})^m} \tag{5}$$

where:

$$f_{50} = \frac{f_{TM_0}}{0.75 + (0.75 - 0.332 \epsilon_r^{-1.78})W/h} \tag{6}$$

$$f_{TM_0} = \frac{c}{2\pi h \sqrt{\epsilon_r - \epsilon_{r\infty}}} \tan^{-1} \left( \epsilon_r \sqrt{\frac{\epsilon_{r\infty} - 1}{\epsilon_r - \epsilon_{r\infty}}} \right) \tag{7}$$

$$m = m_0 m_c \leq 2.32$$

$$m_0 = 1 + \frac{1}{1 + \sqrt{W/h}} + 0.32 \left( \frac{1}{1 + \sqrt{W/h}} \right)^3 \tag{8}$$

$$m_c = \begin{cases} 1 + \frac{1.4}{1 + W/h} \left\{ 0.15 - 0.235 \exp\left(\frac{-0.45f}{f_{50}}\right) \right\} & \text{for } \frac{W}{h} \leq 0.7 \\ 1 & \text{for } \frac{W}{h} \geq 0.7 \end{cases} \tag{9}$$

Finally:

$$Z_c(f) = Z_c \frac{\epsilon_{re}(f) - 1}{\epsilon_{r\infty} - 1} \sqrt{\frac{\epsilon_{r\infty}}{\epsilon_{re}(f)}} \tag{10}$$

The impedance with the effect of dispersion was calculated in 1GHz, 10GHz and 20GHz for sample transmission line with variable thickness, which are:

$Z_c(f=1 \text{ GHz}) = 47.5339$ ,  $Z_c(f=10 \text{ GHz}) = 47.6795$ ,  
 $Z_c(f=20 \text{ GHz}) = 47.9128$ ,  $Z_c(f=1 \text{ GHz}) = 66.0791$ ,  
 $Z_c(f=10 \text{ GHz}) = 66.5860$ ,  $Z_c(f=20 \text{ GHz}) = 67.4287$ ,  
 $Z_c(f=1 \text{ GHz}) = 85.3141$ ,  $Z_c(f=10 \text{ GHz}) = 86.7266$ ,  
 $Z_c(f=20 \text{ GHz}) = 89.0972$  for  $h = 0.3 \text{ mm}$ ,  $h = 0.5 \text{ mm}$  and  $h = 0.8 \text{ mm}$ , respectively. By comparison with Fig. (2), it can realize that these amounts are approximately equals, especially in lower frequencies that are so applicative frequency ranges. The equation exists for the group delay form [2]. For example the group delay for our sample microstrip line with  $h = 0.5 \text{ mm}$  has been plotted, which is shown in Fig (3).

As seen from the Fig (3), the group delays in 1GHz, 10GHz and 20GHz are  $1.27 \times 10^{-11}$ ,  $1.14 \times 10^{-11}$  and  $0.97 \times 10^{-11}$ , respectively. The group delays dispensable in higher frequencies in comparison with the lower frequencies. These differences are, because of physical structure of microstrip line that is explained in flowing. As seen from the equations and results one of these physical properties is dielectric constant ( $\epsilon_r$ ) which has a reverses relation with characteristic impedance in opposite of the effectiveness of the thickness ( $h$ ). For three substrate with variable material (RT/duorid 5880 with  $\epsilon_r=2.2$ , FR4 with  $\epsilon_r=4.4$  and RO 3010 with  $\epsilon_r=10.2$ ), the characteristic impedance is shown in Fig (4), for a transmission line with ( $W = 1\text{mm}$  and  $h = 0.5\text{mm}$ ).

Other effective factors that can mention about the amount of the characteristic impedance are the thickness of the strip line, loss tangent of substrate material and etc which are dispensable for a miniaturized transmission line like our sample.

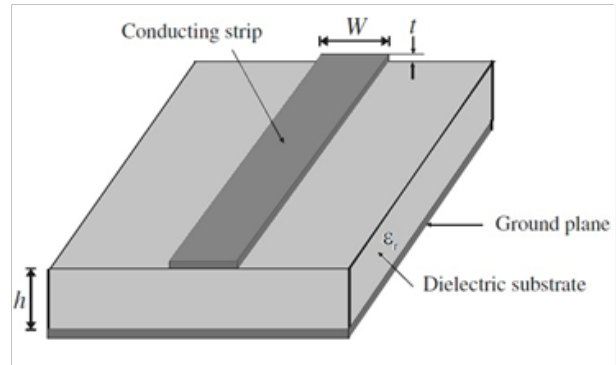


Fig.1. The general structure of a microstrip

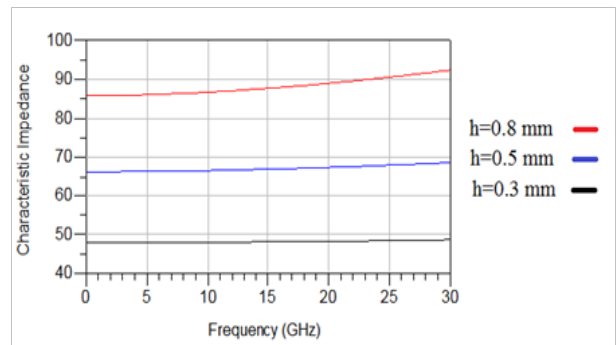


Fig.2. the characteristic impedance through the thicknesses changes

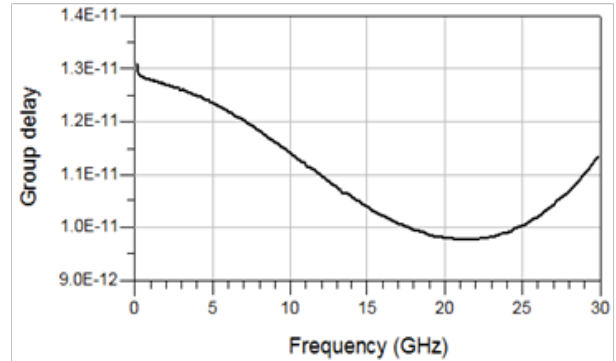


Fig.3. The group delay for sample microstrip line with h = 0.5 mm

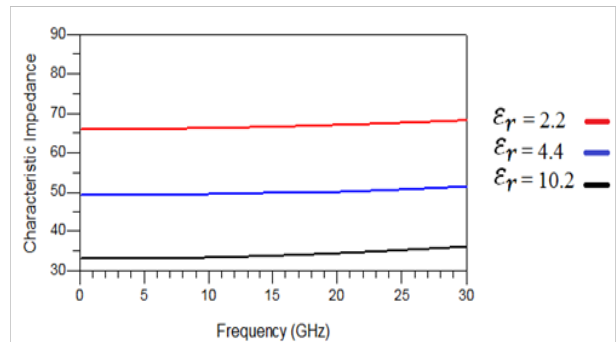


Fig.4. The characteristic impedance through the variable dielectric constants

**COMSOL Model of a Microstrip**

Fig (5) show the electrical potential distribution through a microstrip with applies an electrical forced voltage to the strip line.

The electrical field through the sample and electrical energy density with magnetic flux density are shown in Fig (6) and Fig (7), respectively.

The study on electrical field shows that the fields in center are perpendicular to the strip line and in the other sides behave like two capacitors which are parallel together. The electric energy density plot proofs this case. As seen from the Fig (7), the magnetic flux does not follow from the specific pattern and is so dispensible and it has not inductance behavior. So we can model a microstrip like Fig (8):

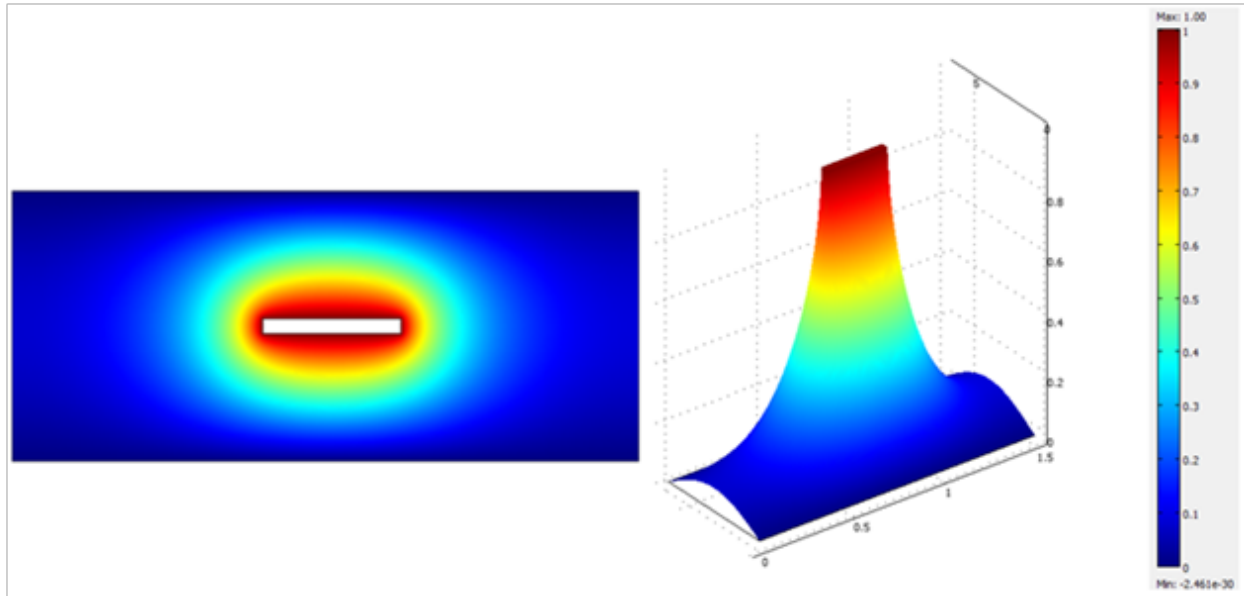


Fig.5. Show the electrical potential distribution through a microstrip

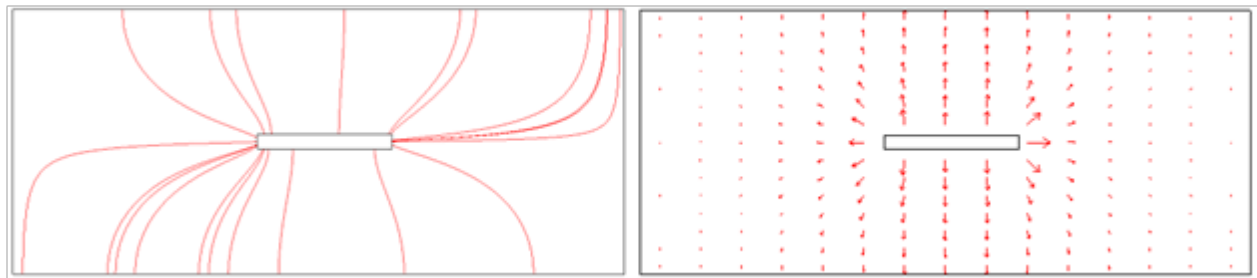


Fig.6. The electrical field through a microstrip

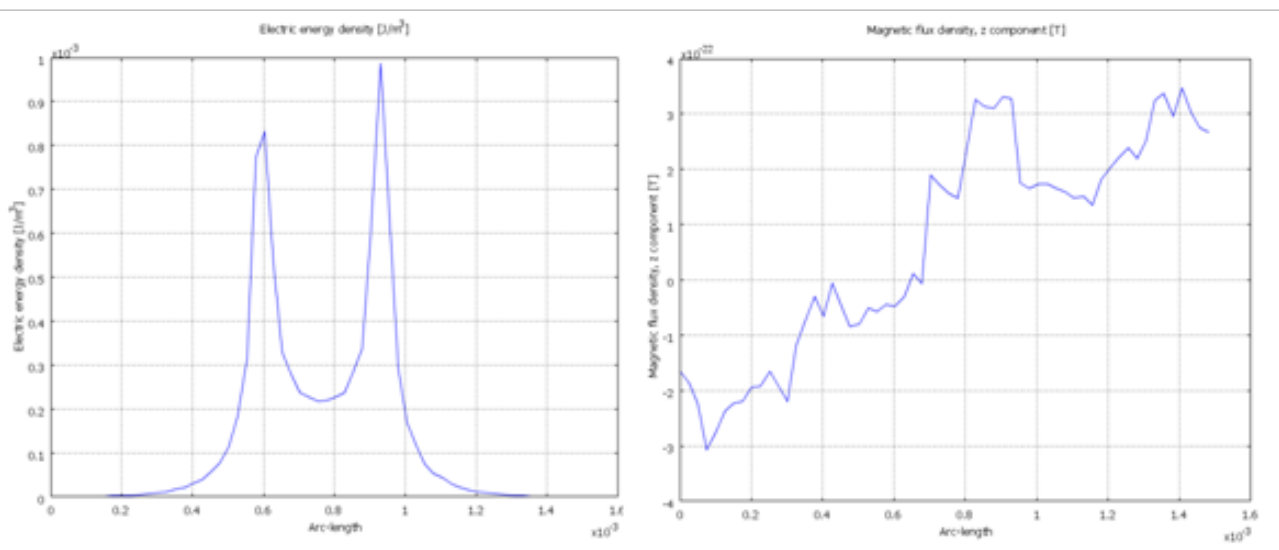


Fig. 7. Electrical energy density with magnetic flux density through a microstrip sample

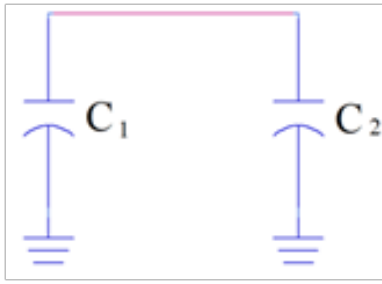


Fig.8. a model for a microstrip transmission line

We know:

$$Z_c = Z_{c1} + Z_{c2} \quad (11)$$

And which symmetrical assume:  $C_1 = C_2 = C$ . For an open-end microstrip line this parameter has calculated in [6] and [7]:

$$\Delta l = \frac{cZ_c C_p}{\sqrt{\epsilon_{re}}} \quad (12)$$

where  $c$  is the light velocity in free space. A closed-form expression for  $\Delta l/h$  is given:

$$\frac{\Delta l}{h} = \frac{\xi_1 \xi_3 \xi_5}{\xi_4} \quad (13)$$

where

$$\xi_1 = 0.434907 \frac{\epsilon_{re}^{0.81} + 0.26 \left(\frac{W}{h}\right)^{0.8544} + 0.236}{\epsilon_{re}^{0.81} - 0.189 \left(\frac{W}{h}\right)^{0.8544} + 0.87} \quad (14)$$

$$\xi_2 = 1 + \frac{\left(\frac{W}{h}\right)^{0.371}}{2.35\epsilon_r + 1} \quad (15)$$

$$\xi_3 = 1 + \frac{0.5274 \tan^{-1} \left[ 0.084 \left(\frac{W}{h}\right)^{\frac{1.9413}{\xi_2}} \right]}{\epsilon_{re}^{0.9236}} \quad (16)$$

$$\xi_4 = 1 + 0.037 \tan^{-1} \left[ 0.067 \left(\frac{W}{h}\right)^{1.456} \right] \cdot \{6 - 5 \exp[0.036(1 - \epsilon_r)]\} \quad (17)$$

$$\xi_5 = 1 - 0.218 \exp\left(-\frac{7.5W}{h}\right) \quad (18)$$

The accuracy is better than 0.2% for the range of  $0.01 \leq w/h \leq 100$  and  $\epsilon_r \leq 128$ . It emphasize that the capacitance and characteristic impedance have dependence with the width of strip line, the thickness of substrate and the dielectric constant.

## CONCLUSION

A COMSOL model of a microstrip is presented in this paper. The microstrip transmission lines are widely used in microwave applications and this model and study can help to reach a true concept of a sample microstrip line. The discussion about the effective parameters on the characteristic impedance is illustrated and the results from COMSOL are presented to validate the equations. The magnetic flux density through our sample has  $0 - 3 \times 10^{-22}$  magnitude ranges, which is negligible. Our sample can be modeled with a capacitance element.

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