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Power Series Solution Method for Elastic Analysis of FGM Thick Cylindrical Pressure Vessels

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Abstract

In this paper, elastic analysis for thick cylindrical pressure vessels made of functionally graded material (FGM) in the plane strain condition by using power series solution method (PSSM) is carried out. Material properties are considered as a function of the radius of the pressure vessel to a power law function and the Poisson's ratio is assumed as constant. Following this, profiles are plotted for different values of the powers of the module of elasticity for the radial displacement, radial stress, and circumferential stress, as a function of radial direction. A numerical solution using finite element method (FEM) is also presented and good agreement was found between the analytical solutions and the solutions carried out through the FEM. The values used in this study are arbitrary chosen to demonstrate the effect of inhomogeneity parameter on displacements, and stresses distributions.

Keywords: Thick Cylindrical Pressure Vessel, Functionally Graded Material (FGM), Power Series Solution Method (PSSM), Finite Element Method (FEM).

INTRODUCTION

Scientists have paid an enormous amount of attention to shells, resulting in numerous theories about their behavior. Of different kinds of shells, due to their extensive use in industrials, thick cylindrical pressure vessels have been of especially importance. Plane strain and plane stress analytical solutions of pressurized thick hollow cylindrical shells have been available for many years in standard and advanced textbooks [1,2,3,4]. Recently, a new class of composite materials known as functionally graded materials (FGMs) has drawn considerable attention. FGMs are composite materials that are microscopically inhomogeneous, and the mechanical properties vary smoothly or continuously from one surface to another. The FGMs concept is applicable to many industrial fields such as aerospace, nuclear energy, chemical plant, electronics, biomaterials and so on. FGMs was proposed by a group of materials scientists [5]. In the last two decades, FGMs have been widely used in engineering applications, particularly in high-temperature environment, microelectronic, power transmission equipment, etc. Horgan and Chan [6] analyzed a pressurized hollow cylinder in the state of plane strain. Assuming that the material has a graded modulus of elasticity, while the Poisson's ratio is a constant, Tutuncu and Ozturk [7], investigated the stress distribution in the axisymmetric structures. They obtained the closed-form solutions for stresses and displacements in functionally graded cylindrical and spherical vessels under internal pressure. Shi et al. [8], studied two different kinds of heterogeneous elastic hollow cylinders. One was multi-layered, and the second had continuously graded properties. They found the exact solutions for an N-layered elastic hollow cylinder subjected to uniform pressures on the inner and outer surfaces. Given the assumption that the material is isotropic with constant Poisson's ratio and exponentially varying modulus of elasticity through the thickness, Naki Tutuncu [9], obtained power series solutions for stresses and displacements in functionally-graded cylindrical vessels subjected to internal pressure alone. In a recent study by Chen and Lin [10], assuming that the property of FGMs is exponential function form, they conducted the elastic analysis for both a thick cylinder and a spherical pressure vessel which were made of functionally graded materials. Assuming that the material properties vary nonlinearly in the radial direction and the Poisson's ratio is constant, Zamani Nejad and Rahimi [11], obtained closed form solutions for one-dimensional steady-state thermal stresses in a rotating functionally graded pressurized thick-walled hollow circular cylinder. A complete and consistent 3-D set of field equations has been developed by tensor analysis to characterize the behavior of FGM thick shells of revolution with arbitrary curvature and variable thickness along the meridional direction [12]. Ghannad and Zamani Nejad [13], obtained the elastic solution of clamped-clamped thick-walled cylindrical shells by an analytic method. Assuming that the material properties change as graded in radial direction to a power law function, deformations and stresses are obtained

[14] for an isotropic FGM rotating cylindrical pressure vessel by the application of the elasticity theory. Assuming the volume fractions of two phases of a FG material vary only with the radius, Nie et al. [15], obtained a technique to tailor materials for linear elastic hollow cylinders and spheres to attain through the thickness either a constant circumferential stress or a constant in-plane shear stress. Based on basic equations of elasticity and power series solution method (PSSM), a simple and efficient method is proposed to elastic analysis of rotating internally pressurized thick-walled cylindrical pressure vessels in plane strain and plane stress conditions by Gharibi and Zamani Nejad [16].

The main objective of this paper is to use of PSSM for analytical solution of FGM cylindrical pressure vessels.

ELASTIC ANALYSIS of PROBLEM

The stress distribution in a FGM thick-walled cylindrical pressure vessel with an inner radius a, and an outer radius b, subjected to an internal pressure P that is axisymmetric in the conditions of plane strain will be calculated (Fig. 1).

The material properties are assumed to be radially dependent. Elastic modulus for the isotropic material is assumed to vary as

$$E = E_i\left(\frac{r}{a}\right)^n \quad (1)$$

where E_i and n are modulus of elasticity in inner surface and inhomogeneity parameter, respectively.

The equilibrium equation of the FGM thick cylinder in the absence of body forces, is expressed as follows

$$\frac{d}{dr}(r\sigma_{rr}) - \sigma_{\theta\theta} = 0 \quad (2)$$

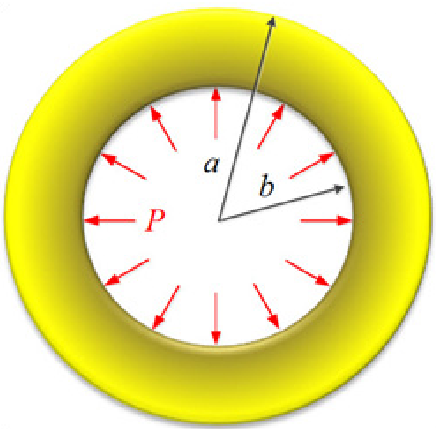

Fig.1. FGM thick-walled cylindrical pressure vessel

The radial strain ε_{rr} and circumferential strain $\varepsilon_{\theta\theta}$ are related to the radial displacement u_r by

$$\varepsilon_{rr} = \frac{du_r}{dr} \quad (3)$$

$$\varepsilon_{\theta\theta} = \frac{u_r}{r} \quad (4)$$

The stress and strain relations for non-homogeneous and isotropic materials are

$$\begin{bmatrix}\sigma_{rr}\\ \sigma_{\theta\theta}\end{bmatrix} = E\begin{bmatrix}A & B\\ B & A\end{bmatrix}\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\end{bmatrix} \quad (5)$$

where E is modulus of elasticity and A and B are related to Poisson's ratio ν as

$$\begin{cases} A = \dfrac{1-\nu}{(1+\nu)(1-2\nu)} \\ B = \dfrac{\nu}{(1+\nu)(1-2\nu)} \end{cases} \quad (6)$$

Using Eqs. (1) to (5), the Navier equation in terms of the radial displacement is

$$\begin{cases} r^2\dfrac{d^2u_r}{dr^2} + r(1+n)\dfrac{du_r}{dr} + (n\nu_1 - 1)u_r = 0 \\ \nu_1 = \dfrac{\nu}{1-\nu} \end{cases} \quad (7)$$

Eq. (7) can be solve by using power series method as follows

$$u_r = \sum_{j=0}^{\infty} a_j r^{j+s} \quad (8)$$

$$\frac{du_r}{dr} = \sum_{j=0}^{\infty} (j+s) a_j r^{j+s-1} \quad (9)$$

$$\frac{d^2u_r}{dr^2} = \sum_{j=0}^{\infty} (j+s)(j+s-1) a_j r^{j+s-2} \quad (10)$$

Substituting Eqs. (8) to (10) into Eq. (7), we have

$$\sum_{\lambda=0}^{\infty} a_\lambda\left[(\lambda+s)(\lambda+s-1) + (\lambda+s)(n+1) + n\nu_1 - 1\right] r^{\lambda+s} = 0 \quad (11)$$

Thus

$$\begin{cases} \lambda = 0 \\ s^2 + ns + n\nu_1 - 1 = 0 \end{cases} \quad (12)$$

Therefore

$$\begin{cases} s_1 = -\dfrac{n}{2} + \dfrac{\sqrt{\Delta}}{2} \\ s_2 = -\dfrac{n}{2} - \dfrac{\sqrt{\Delta}}{2} \\ \Delta = n^2 - 4n\nu_1 + 4 > 0 \end{cases} \quad (13)$$

With substituting Eq. (13) into Eq. (8), we have

$$u_r = C_1 u_1 + C_2 u_2 \quad (14)$$

where

$$\begin{cases} u_1 = r^{s_1} \\ u_2 = r^{s_2} \end{cases} \quad (15)$$

Then

$$u_r = C_1 r^{s_1} + C_2 r^{s_2} \quad (16)$$

Substitution Eq. (16) into first of Eqs. (5)

$$\sigma_{rr} = E\left(C_1\left[As_1 + B\right]r^{s_1-1} + C_2\left[As_2 + B\right]r^{s_2-1}\right) \tag{17}$$

Integration constants C_1 and C_2 are determined by using the following boundary conditions

$$\begin{cases} \sigma_{rr}\Big|_{r=a} = -P \\ \sigma_{rr}\Big|_{r=b} = 0 \end{cases} \tag{18}$$

Thus

$$\begin{cases} C_1 = -\dfrac{Pb^{s_2-1}}{E_i\left(As_1 + B\right)\left[b^{s_2-1}a^{s_1-1} - b^{s_1-1}a^{s_2-1}\right]} \\ C_2 = \dfrac{Pb^{s_1-1}}{E_i\left(As_2 + B\right)\left[b^{s_2-1}a^{s_1-1} - b^{s_1-1}a^{s_2-1}\right]} \end{cases} \tag{19}$$

Hence, non-dimensional radial displacement, radial stress and circumferential stress are found as follows

$$u_r = \frac{Pa}{E_i\left(K^{s_2-s_1} - 1\right)}\left[\frac{1}{As_2 + B}R^{s_2} - \frac{K^{s_2-s_1}}{As_1 + B}R^{s_1}\right] \tag{20}$$

$$\sigma_{rr} = \frac{P}{K^{s_2-s_1} - 1}\left[R^{n+s_2-1} - K^{s_2-s_1}R^{n+s_1-1}\right] \tag{21}$$

$$\sigma_{\theta\theta} = \frac{P}{K^{s_2-s_1} - 1}\left[\left(\frac{\nu_1 s_1 + 1}{s_1 + \nu_1}\right)b^{s_2-s_1}R^{n+s_2-1} - \left(\frac{\nu_1 s_2 + 1}{s_2 + \nu_1}\right)a^{s_1-s_2}R^{n+s_1-1}\right] \tag{22}$$

Where $R = r/a$ and $K = b/a$.

RESULTS AND DISCUSSION

In the previous section, using the PSSM, stresses and radial displacement for FGM thick-walled cylindrical pressure vessels for plane strain condition subjected to internal pressure is obtained. In this section, distribution of radial displacement, radial and circumferential stresses along normalized radius for different values of inhomogeneity parameter in the form of graphs are plotted.

Consider a thick cylindrical pressure vessel under the internal pressure of 80 MPa. The pressure vessel has the inner and outer radius of 4 cm and 6 cm, respectively. In addition, it is assumed that values of the Poisson's ratio, ν , and modulus of elasticity in inner surface, E_i , are 0.3 and 200 GPa, respectively.

In this section, in order to numerical analysis of problem, a geometry specimen was also modeled using a commercial finite element code, ANSYS 12, for a comparative study. In the FEM model, an 8-node axisymmetric quadrilateral element was used to represent the non-homogeneous specimen.

Fig. 2 shows the distribution of tensile radial displacement along the normalized radial direction. It is seen from the curve that at the same position $\left(1 < R < 1.5\right)$, the radial displacement decreases as n increases.

Fig. 3 shows the distribution of the compressive radial stress along normalized radial direction. It is observed that for higher values n , the radial stress increases.

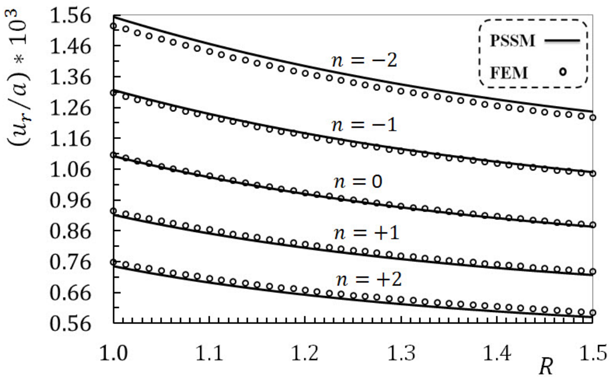

Fig.2. Distribution of radial displacement along normalized radial direction

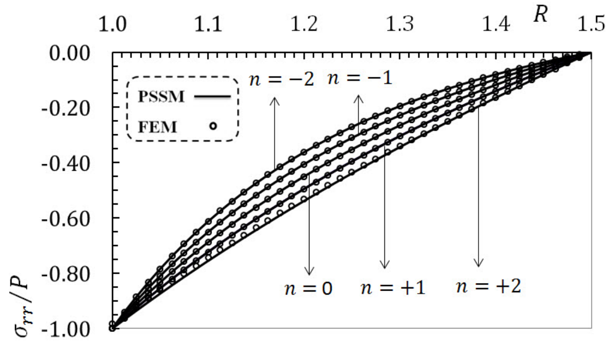

Fig.3. Distribution of radial stress along normalized radial direction

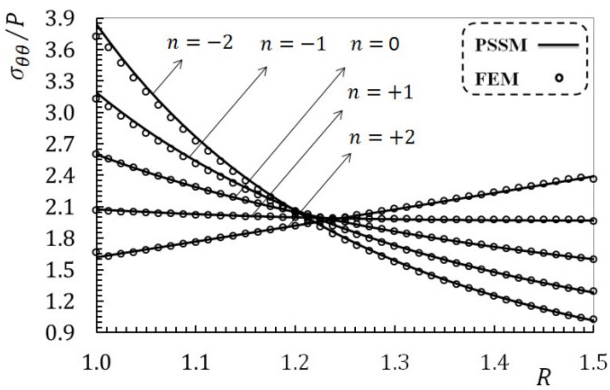

Fig.4. Distribution of circumferential stress along normalized radial direction

The tensile circumferential stress along the normalized radial direction for different values of n is plotted in Fig. 4. It must be noted from this figure that at the same position, almost for $R < 1.22$, there is an decrease in the value of the circumferential stress as n increases, whereas for $R > 1.22$ this situation was reversed. Besides, along the radial direction, almost for $n < 1$ the circumferential stress decreases, while almost for $n > 1$, the circumferential stress increases.

CONCLUSIONS

Using PSSM and assuming the Young's modulus vary nonlinearly in the radial direction, and the Poisson's ratio is constant, , the governing equations for axisymmetric FGM thick cylindrical pressure vessels subjected to internal pressure in plane strain condition is derived. Following this, radial displacement, radial and circumferential stresses are obtained. The radial displacement, radial stress and circumferential stress and distributions depending on an inhomogeneity constant are compared with the solution using finite element method (FEM) and presented in the form of graphs. Good agreement was found between the analytical solutions and the solutions carried out through a commercial finite element code. The presented results show that the material inhomogeneity has a significant influence on the mechanical behaviors of the thick cylindrical pressure vessels made of functionally graded material with radially varying properties.

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